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THREE ESSAYS ON SEASONAL INTEGRATION AND CAUSALITY RELATIONS IN TOURISM TIME SERIES

Andrii Bodnar

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Doctoral Program of Tourism and Environmental Economics

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RESUM EN CATALÀ

Aquesta tesi inclou tres assajos publicables, referits a la construcció d'eines pràctiques per a la modelització de sèries de turisme altament estacional i les relacions de causalitat. A n'aquest sentit, els dos primers assajos contrasten l'estacionalitat determinista amb la ecotàstica amb els corresponents contrastes desenvolupats als paquets economètrics Gauss y Stata. Els mateixos van ser disenyats amb subrutines S=4 i S=12 corresponent-se amb les freqüències trimestrals I mensuals de les sèries temporals. Aquestes subrutines es troben disponibles als annexes d'aquesta tesi.

Aquest estudi considera tant el turisme receptiu com el turisme emissiu. Les dades inclouen la informació mensual de passatgers arribats a l'aeroport de Palma de Mallorca, u destí que es caracteritza per l'estacionalitat, amb les altes afluències de visitants a l'estiu i baixes a l'hivern.

Aquesta tesi té tres objectius principals, el primer d'ells es trobar les superfícies de resposta i pvalues pels contrastos d'arrels unitàries estacionals. El segon, es crear una eina flexible i documentada pels contrastos d'estacionalitat en el paquet estadístic STATA. Finalment, revelar les relacions de causalitat entre els viatges de vacacions i de negocis i determinar el seu comportment a llarg termini.

Per aconseguir aquests objectius el treball s'ha dividit en cinc capítols, el primer és la introducció del tema, mentres que el segon es desenvolupa una superfície de resposta pels contrastos d'arrels unitàries estacionals amb OLS i GLs "de-trending". Com el resultat d'aquest estudi es va obtenir una matriu de coeficients i una subrutina, que determina el p-valor per t i els estadístics F. Aquesta subrutina té sis opcions de termes deterministes, dues freqüències de dades i dos tipus de procediments per lidiar amb la part determinista de les sèries temporals. Per aquesta raó s'utilitzen les dades dels passatgers arribats a Palma de Mallorca des d'Alemanya, on es troba una arrel unitària en freqüència zero i un parell d'arrels complexes conjugades en la freqüència $\pi/6$ que correspon al llarg termini i estacionalitat d'un any respectivament.

El tercer capítol planteja un test d'arrel unitària estacional HEGY amb OLS i GLS "de-trending" en Stata, seguint la metodologia descrita pel segon capítol. La subrutina inclou un nombre d'opcions útils, com són la possibilitat de treball amb dos mètodes diferents per bregar amb la part determinista (OLS i GLS "de-trending"), quatre criteris diferents per determinar l'ordre d'augment òptim en la regressió HEGY, sis especificacions distintes de la part determinística i un test de correlació pels residus. Aquests són presentats amb els valors crítics que varen ser calculats amb la corresponent superfície de resposta, a nivells de significació 1%, 5% i 10%.

El quart capítol utilitza les dades de 27 països europeus per contrastar la hipòtesi sobre les relacions de causalitat entre el turisme d'oci i el de negocis. Les sèries de temps previnents de les partides de l'aeroport són dividides en dues categories per motiu de viatge. Mitjançant un test de causalitat de Granger es mostra la relació entre ambdós i un gràfic d'impuls de resposta demostra la seva dinàmica.

Finalment, en el quint capítol es presenten les principals conclusions i contribucions d'aquesta tesi, així com també una breu descripció dels resultats trobats als capítols 2, 3 i 4.

RESUMEN EN CASTELLANO

Esta tesis incluye tres ensayos publicables, referidos a la construcción de herramientas prácticas para la modelización de series de turismo altamente estacional y las relaciones de causalidad. En este sentido los dos primeros ensayos contrastan la estacionalidad determinística con la estocástica con los correspondientes contrastes desarrollados en los paquetes econométricos Gauss y Stata. Los mismos fueron diseñados con subrutinas S=4 y S=12 correspondiéndose con las frecuencias trimestrales y mensuales de la series temporales. Estas subrutinas se encuentran disponibles en los anexos de esta tesis.

En el estudio consideramos tanto el turismo receptivo como el turismo emisivo. Los datos incluyen la información mensual de pasajeros llegados al aeropuerto de Palma de Mallorca, un destino que se caracteriza por la estacionalidad, con altas afluencias de visitantes en verano y bajas en invierno.

Esta tesis tiene tres objetivos principales, el primero de ellos es encontrar la superficies de respuesta y p-values para los contrastes de raíces untarías estacionales. El segundo es crear una herramienta flexible y documentada para los contrates de estacionalidad en el paquete estadístico STATA. Finalmente, revelar las relaciones de causalidad entre los viajes de vacaciones y de negocios y determinar su comportamiento en el largo plazo.

Para lograr cubrir estos objetivos es que hemos dividido este trabajo en cinco capítulos, el primero es la introducción al tema, mientras que el segundo capítulo se desarrolla una superficie de respuesta para los contrates de raíces unitarias estacionales con OLS y GLS de-trending. Como resultado de este estudio se obtuvo una matriz de coeficientes y una subrutina, que determina el p-valor para t y los estadísticos F. Esta subrutina tiene seis opciones de términos determinísticos, dos frecuencias de datos y dos tipos de procedimientos para lidiar con la parte determinista de las series temporales. Para ello utilizamos los datos de los pasajeros arribados a Palma de Mallorca desde Alemania, donde encontramos que una raíz unitaria en frecuencia cero y un par de raíces complejas conjugadas en la frecuencia $\pi/6$ que corresponde al largo plazo y la estacionalidad de un año respectivamente.

El tercer capítulo plantea un test de raíz unitaria estacional de HEGY con OLS y GLS de-trending en Stata, siguiendo la metodología descripta para el segundo capítulo. La subrutina incluye un número de opciones útiles, como son la posibilidad de trabajo con dos métodos diferentes para lidiar con la parte determinista (OLS y GLS de-treding), cuatro criterios diferentes para determinar el orden de aumentación óptimo en la regresión HEGY, seis especficaciones distintas de la parte determinista y un test de correlación para los residuos. Estos test son presentados con los valores críticos que fueron calculados con su correspondiente superficie de respuesta, a niveles de significación del 1%, 5% y 10%.

El cuarto capítulo utiliza los datos de 27 países europeos para contrastar la hipótesis sobre las relaciones de causalidad entre el turismo de ocio y el de negocios. Las series de tiempo provenientes de las partidas del aeropuerto son divididas en dos categorías por motivo de viaje. Mediante un test de causalidad de Granger se muestra la relación entre ambos y un gráfico de impulso respuesta demuestra su dinámica.

Finalmente en el quinto capítulo se presentan las principales conclusiones y contribuciones de esta tesis así como también una breve descripción de los resultados encontrados en los capítulos 2, 3 y 4.

RESUME IN ENGLISH

The thesis includes three self-contained essays with publishable academic structure. Two essays are aimed to construct practical tools for modeling highly seasonal tourism time series and the third one deals with causality relations. The first two papers contrast deterministic seasonality with stochastic seasonality and corresponding tests are developed in Gauss and Stata econometric packages. The test subroutines are designed for frequencies S=4 and S=12, which corresponds to quarterly and monthly time series. The subroutines are available in the supplementary material of the thesis.

In the study we deal both with inbound and outbound tourism activity. The data set includes monthly airport passenger arrivals to Palma de Mallorca, which is a tourist destination with high peaks during summer period and low peaks in winter.

The thesis is designed in order to accomplish three main tasks. The first objective is to fill in the gap in the numerical distribution studies for seasonality tests. The second objective is to create a flexible tool with supporting documentation for seasonality tests in Stata environment. Finally, the third task is to reveal the causality relations between holiday and business tourism activity and determine its long time behavior.

The thesis consists of five chapters, where the first section is an introduction to the topic. The second chapter develops the response surface for seasonal unit root test with OLS and GLS detrending. As the result of the study we have obtained a matrix of coefficients and a subroutine, which return the p-value for t and F-type test statistics. The subroutine has six options of deterministic terms, two frequencies of the data and two types of detrending procedure. The empirical part of the paper tests the time series of airport passenger arrivals from Germany to Palma de Mallorca. We found out that seasonal unit roots are present at zero and $\pi/6$ frequencies, which corresponds to long-run and one-year seasonality.

The third chapter develops the HEGY seasonal unit root test with OLS and GLS detrending in Stata. It follows the general context and methodology of the second chapter. The subroutine includes a number of useful options, like two methods for dealing with deterministic component (OLS and GLS detrending), four different information criteria for determining the optimal augmentation lag in the HEGY regression, a choice of six possible deterministic terms and a test for serial correlation in residuals. The test statistics is reported together with critical values, which are calculated from the corresponding response surfaces. The subroutine reports the critical values at 1%, 5% and 10% significance level.

The forth chapter deals with a data set of 27 European countries. This research tests the hypothesis about causality relations between holiday and business tourism. The time series of airport departures are split into two categories by the purpose of travel. The Granger causality test reveals the relation and the impulse response graph demonstrates its dynamics.

Finally, the fifth chapter presents the principal conclusions and contributions of the thesis, as well as brief results description of chapters 2, 3 and 4.

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Introduction

Tourism demand modeling is an important part of tourism studies. Measurements of tourist activity at macroeconomic level are mostly done with help of arrival and departure statistics. Such data is collected in time series at certain frequency. The most relevant time frames in terms of a country or a destination are months and quarters. It gives a good overview of tourism activity during one cycle, which is one year.

It happened that people prefer to travel to "sun and sea" destinations in summer and go skiing in winter. This behavioral pattern affects tourism time series and introduces a substantial seasonal component into the data. Its forecasts need to be based on the deep understanding of the essence of such seasonality. Specific statistical tests are developed in order to distinguish between stochastic and deterministic seasonality. Such kinds of tests are under the study of this research.

The thesis includes three self-contained essays with publishable academic structure. Two essays are aimed to construct practical tools for modeling highly seasonal tourism time series and the third deals with causality relations. The first two papers contrast deterministic seasonality with stochastic seasonality and corresponding tests are developed in Gauss and Stata econometric packages. The test subroutines are designed for seasonal periodicity four and twelve, which corresponds to quarterly and monthly time series. Tourism data is mostly collected at these two time frames, while semiannual, bimonthly, weekly, daily or even hourly data is rarely used. The subroutines are available in the supplementary material of the thesis.

In this thesis we deal both with inbound and outbound tourism activity. The data set includes monthly airport passenger arrivals to Palma de Mallorca, which is a tourist destination with high peaks during summer period and low peaks in winter. From the side of outbound tourism we deal with airport departures from 27 European countries. The departures are analyzed on the quarterly basis.

The general objectives of our investigation are:

1. To fill in the gap in the numerical distribution studies for seasonality tests.
2. To create a flexible tool with supporting documentation for seasonality tests in Stata environment.
3. To reveal the causality relations between holiday and business tourism activity and determine its long time behavior.

The thesis consists of five chapters, where the first section is an introduction to the topic and the last section presents the main contributions of the thesis and concludes. A brief description of the chapters 2 to 4 is provided below.

Hylleberg, Engle, Granger and Yoo (1990) (henceforth HEGY) have developed the seasonal unit root tests, which treat all seasonal and zero frequencies separately. The first paper and the second chapter of the thesis implements the approach of MacKinnon (1994) and MacKinnon (1996) in order to estimate the response surface of corresponding *t* and F-type test statistics. The Generalized Least Square (GLS) detrending procedure was initially adopted for seasonal time series by Rodrigues and Taylor (2007). It improves the power of the test for the data with deterministic component. In this research we study the numerical distribution of HEGY seasonal unit root test with GLS detrending as described in del Barrio Castro, Osborn and Taylor (2016) as well as original OLS detrending case.

The contribution of such research is twofold. The first advance is an extension of the response surface methodology to the GLS detrending case. The second contribution is a comparison of our results with the results of previous studies: tables of critical values and response surface functions. Such comparison verifies both our research and previous investigations. Moreover, some minor divergence is revealed. In these cases we suggest the refined versions of critical values. Finally, in the practical part of the paper we illustrate the tests results using monthly airport passenger arrivals to the tourist destination.

The methodology of HEGY seasonal unit root test employs both *t* and F-type tests. Two lower-tailed regression *t* tests control the presence of unit roots at zero and Nyquist frequencies correspondingly. The upper-tailed regression F-type test is employed to control the conjugates unit roots. Similarly, the two hypothesis of unit roots presence at any seasonal frequency and any seasonal plus zero frequencies are tested with help of upper-tailed F-type tests.

Distribution of the test statistics depends on deterministic component of the data. We consider six cases of deterministic terms for HEGY test with OLS detrending: no deterministic terms, only a constant, a constant with linear trend, seasonal intercepts, seasonal intercepts with linear trend and seasonal intercepts with seasonal trends. The last five options are also employed for the case of the test with GLS detrending.

The method of response surface estimation developed by MacKinnon (1994, 1996) employs extensive Monte Carlo simulations and deals with the augmented Dickey-Fuller unit root test statistics. This methodology was adapted to the seasonal unit root context by Harvey and van Dijk (2006) and Diaz-Emparanza (2014). Following these papers our study is based on 5.7 billion simulations of the time series with seasonal “random walk”. The simulations are distributed according to 27 sample size options, six deterministic terms, two frequencies and two detrending procedures (OLS and GLS). It is worth to mention that such extensive number of replications requires huge computing abilities. The task was performed on the server with processor Intel® Xeon® CPU E5-2470 during four weeks and it took nearly 250 GB of disk space to store the results.

After obtaining the Monte Carlo simulation results we proceed to the response surface estimation. On this step we obtain 221 response surfaces for every type of test and deterministic term and then fit it with help of general method of moments. In the appendix of the paper we report the 1%, 5% and 10% response surface coefficients. These tables can be used for simple computation of corresponding critical values.

Finally, the *p*-value is derived from the response surfaces using the polynomial interpolation. A subroutine for GAUSS is developed as a result of this study. It calculates the *p*-value using the inputs of test statistics, effective sample size, deterministic terms and type of detrending procedure.

In the illustrative part of the second chapter we analyze the monthly airport passenger arrivals to Palma de Mallorca (PMI) from Germany. HEGY seasonal unit root test with GLS detrending reveals the seasonal integration of the data in hand.

The third chapter develops HEGY seasonal unit root test with GLS detrending as a subroutine for Stata. It is constructed in the form of *.ado* command and provided together with the supporting documentation, such as help file and illustrative example. The command provides a variety of estimation and post estimation options. Previous subroutines with HEGY test for quarterly data were developed by Baum and Sperling (2001) and Depalo (2009). Our research extends the command to the monthly frequency, provides a possibility of GLS detrending and introduces new optimal lag length selection criteria.

The methodology of HEGY seasonal unit root testing is similar to the second chapter. The command has six available deterministic terms denoted analogically to *hegy4* command by Baum and Sperling (2001): *none* stands for no deterministic terms, *const* – only a constant, *trend* – a constant with linear trend, *seas* – seasonal intercepts, *strend* – seasonal intercepts with linear trend, *mult* – seasonal intercepts with seasonal trends.

The battery of optimal augmentation lag criteria is available for removing serial correlation in the residuals. Among the methods are Akaike information criteria (AIC), modified AIC (MAIC), Bayesian information criteria (BIC) and sequential *t*-test method, where the last statistically not significant augmenting lags are dropped one by one. The MAIC option is developed following the methodology of Perron and Qu (2007) and del Barrio Castro, Osborn and Taylor (2016). There is also a possibility to fix an augmentation lag to a user specified value.

The default maximal augmentation lag in the program is dependent on the sample size of the data. However, user may introduce it with *maxlag()* option. The GLS detrending procedure is available as a generic option and requires deterministic terms to be specified as *const*, *trend*, *seas*, *strend* or *mult*.

The post estimation options allow user to output the regression table, as well as to work with the residuals. The option *noac()* suppress the autocorrelation function (ACF), partial ACF and the Ljung-Box Q-statistics of the residuals, which are reported by default. The option *residuals()* creates a variable with actual residuals.

The critical values for the tests are calculated depending on the sample size. The response surfaces are obtained from the first paper using Tables 2 to 5. The corresponding critical values are reported at 1%, 5% and 10% significance level. Due to scale limitations of Stata it is not possible to introduce the *p*-value calculator into the *.ado* command. The calculator requires around 80 thousands coefficients to be introduced into the matrix that is far beyond the system limits of Stata/IC 11.

In the empirical part we analyze the monthly tourist arrivals to Palma de Mallorca from the United Kingdom. Four cases of HEGY seasonal unit root test with different options are contrasted. The deterministic terms are seasonal dummies with seasonal trends. The maximal augmentation lag is allowed to be a default value. The contrasted options are two detrending methods and two optimal lag selection criteria.

The forth chapter deals with outbound tourism data, particularly airport departures from 27 European countries. The tourism demand is studied more often from the side of destination, particularly arrivals. Outbound tourism is rarely under attention of the investigators. Our research is aimed to spread some light on this side of demand and study tourism in the country of origin.

While treating holiday and business tourism as separate time series, we are trying to improve the forecasts of departures.

The time series are split by the purpose of travel. This research reveals the dependency between these two types of tourism and measures it with help of impulse response functions. Such relations are detected with help of Granger causality test and are used in vector autoregression (VAR) models. The causality relations are revealed in almost one third of the cases.

The data coherence is crucial for this study. Due to consolidated statistics of Eurostat we can claim that the results are also coherent. The comparability of the time series is reported to be "very good" (Eurostat, 2012) both for time scale (between different years) and geographical scale (between countries). The quarterly data is available for the time period since the first quarter of 1994 to the last quarter of 2011. However, the longest time series have 64 observations and some of them have observation gaps.

As result the countries are grouped into the four blocks according to the (i) presence of causality relations and (ii) its direction. An important finding of the research is that most of the time series are driven by different types of deterministic trends. It implies a "short memory" of structural breaks in tourism data.

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Chapter 2: Numerical Distribution Functions for Seasonal Unit Root Tests with OLS and GLS Detrending

Numerical Distribution Functions for Seasonal Unit Root Tests with OLS and GLS Detrending

Abstract

This paper implements the approach introduced by MacKinnon (1994, 1996) to estimate the response surface of the test statistics of seasonal unit root tests with OLS and GLS detrending for quarterly and monthly time series. The Gauss code that is available in the supplementary material of the paper produces *p*-values for five test statistics depending on the sample size, deterministic terms and frequency of the data. A comparison with previous studies is undertaken, and an empirical example using airport passenger arrivals to a tourist destination is carried out. Quantile function coefficients are reported for simple computation of critical values for tests at 1%, 5% and 10% significance levels.

Key words: HEGY test, GLS detrending, response surfaces

1.- Introduction

It is common practice to test for the presence of seasonal unit roots in data recorded at intervals of less than one year. The paper by Hylleberg, Engle, Granger and Yoo (1990) (henceforth HEGY) suggests a set of filters and corresponding tests that consider unit roots separately at each seasonal and zero frequencies for quarterly data. Beaulieu and Miron (1993) and Taylor (1998) extend the HEGY approach to the monthly case, and extensions for an arbitrary number of seasons (S) per year can be found in Smith and Taylor (1999) and Smith et al. (2009). In order to improve the size and power properties of HEGY tests in the presence of deterministic components, Rodrigues and Taylor (2007) adopt the Generalized Least Squares (GLS) detrending procedure for seasonal unit root tests. The procedure was originally introduced by Elliot, Rothenberg and Stock (1996) for zero frequency unit root tests.

The (seasonal) unit root tests have non-standard distributions, and the most common way to calculate the critical values is with Monte Carlo simulations. Harvey and van Dijk (2006) estimate response surface regressions to compute critical values for conventional HEGY tests in the quarterly case considering only two specifications for the deterministic part (seasonal intercepts and seasonal intercepts with a zero frequency deterministic trend). Diaz-Emparanza (2014) extends the method of Harvey and van Dijk to any seasonal frequency of data following MacKinnon (1994, 1996) and obtains *p*-values for the test statistics of the HEGY approach using OLS detrending and considering four specifications of the deterministic part of the seasonal process (zero frequency intercept, zero frequency intercept with trend, seasonal intercepts and seasonal intercepts with zero frequency trend).

In this paper we go further and following the approach suggested by MacKinnon (1994, 1996) estimate response surfaces and obtain *p*-values for the test statistics of the HEGY approach considering OLS and GLS detrending for quarterly and monthly data. Finally, for both OLS and GLS detrending we consider five specifications of the deterministic part: zero frequency intercept, zero frequency intercept with trend, seasonal intercepts, seasonal intercepts with zero frequency trend and seasonal intercepts with trends.

All of the simulations are done in the GAUSS™ programming language. The results are compiled into a GAUSS™ subroutine and are provided in the supplementary material of the paper. Interested persons can use the same subroutine for obtaining *p*-values of HEGY tests.

2.- Seasonal unit root test context

The general model used in seasonal unit root testing is as follows:

$$y_{St+s} = \mu_{St+s} + x_{St+s}$$

$$\alpha(L)x_{St+s} = u_{St+s} \tag{1}$$

$$u_{St+s} = \psi(L)\varepsilon_{St+s},\ s = 1-S,...,0,\ \ t = 1,2,...,N$$

See assumptions A.1, A.2 and A.1' and remark 1 in del Barrio Castro, Osborn and Taylor (2012) for details about the properties of u_{St+s} and ε_{St+s} in (1). It is also assumed that the observed time series y_{St+s} can be decomposed into two parts, the deterministic part μ_{St+s} and the stochastic part x_{St+s}. *S* denotes the number of seasons. In the case of monthly data *S*=12 and in the quarterly case *S*=4; hence, in the rest of the paper we are going to assume that *S* is even. *N* represents the number of years.

$\alpha(L)$ is an AR(S) polynomial, $\alpha(L) = \left(1 - \hat{\alpha}_1 L - \hat{\alpha}_2 L^2 - \cdots - -\hat{\alpha}_S L^S\right)$ where L is the usual lag operator. This polynomial can be factorized as:

$$\alpha(L) = (1-\alpha_0 L)(1+\alpha_{S/2}L)\prod_{j=1}^{S^*}\left(1 - 2\left[\alpha_j \cos\left(\frac{2\pi j}{S}\right) - \beta_j \sin\left(\frac{2\pi j}{S}\right)\right]L + \left(\alpha_j^2 + \beta_j^2\right)L^2\right), \tag{2}$$

with $S^* = S/2-1$. Our focus is to test for the presence of unit roots in the polynomial $\alpha(L)$. Note that the parameter α_0 of $(1-\alpha_0 L)$ is associated with the zero frequency, the parameter $\alpha_{S/2}$ of $(1+\alpha_{S/2}L)$ is associated with the Nyquist frequency (π) and the parameters α_j and β_j of $\left(1 - 2\left[\alpha_j \cos\left(\frac{2\pi j}{S}\right) - \beta_j \sin\left(\frac{2\pi j}{S}\right)\right]L + \left(\alpha_j^2 + \beta_j^2\right)L^2\right)$ are associated with the conjugate (harmonic) seasonal frequencies $\frac{2\pi j}{S}$ and 2π - $\frac{2\pi j}{S}$ for $j = 1,\ldots,S^* = S/2-1$.

Regarding the deterministic part μ_{St+s}, and following Smith and Taylor (1998), Rodriguez and Taylor (2007) and Smith, Taylor and del Barrio Castro (2009), it is possible to define six scenarios: no deterministic terms, zero frequency intercept (one intercept), zero frequency intercept with trend, seasonal intercepts, seasonal intercepts with zero frequency trend and seasonal intercepts with trends. In this paper we do not pay attention to the first case and only consider the following five relevant cases, for $\mu_{St+s} = \delta' z_{St+s,\xi}$:

Case 1: Zero frequency intercept:

$z_{St+s,1} = [1]'$ with $\delta = (\delta_0)$.

Case 2: Zero frequency intercept with trend:

$z_{St+s,2} = [1, St+s]'$ with $\delta = (\delta_0, \bar{\delta}_0)$'.

Case 3: Seasonal intercepts:

$$z_{St+s,3} = [1, \cos(2\pi(St+s)/S), \sin(2\pi(St+s)/S), \ldots$$
$$\ldots, \cos(2\pi S^*(St+s)/S), \sin(2\pi S^*(St+s)/S), (-1)^{St+s}]'$$

with $\delta = (\delta_0, \delta'_1, \ldots, \delta'_{S*}, \delta_{S/2})$', $\delta_k = (\delta_{k,1}, \delta_{k,2})$', $k = 1, \ldots, S^*$ =S.

Case 4: Seasonal intercepts with zero frequency trend:

$z_{St+s,4} = [z'_{St+s,3}, St+s]'$ $\delta = (\delta_0, \delta'_1, \ldots, \delta'_{S*}, \delta_{S/2}, \bar{\delta}_0)$', $\delta_k = (\delta_{k,1}, \delta_{k,2})$', $k = 1, \ldots, S^*$.

Case 5: Seasonal intercepts with trends:

$z_{St+s,5} = [z'_{St+s,3}, (St+s)z'_{St+s,3}]'$

with $\delta = (\delta_0, \delta'_1, \ldots, \delta'_{S*}, \delta'_{S/2}, \bar{\delta}_0, \bar{\delta}'_1, \ldots, \bar{\delta}'_{S*}, \bar{\delta}_{S/2})$', $\delta_k = (\delta_{k,1}, \delta_{k,2})$', , $\bar{\delta}_k = (\bar{\delta}_{k,1}, \bar{\delta}_{k,2})$' $k = 1, \ldots, S^*$.

As shown by Smith and Taylor (1998) and Smith, Taylor and del Barrio Castro (2009), the inclusion of seasonal intercepts allows for tests invariant to the presence of non-zero initial conditions under the null hypothesis of seasonal integration to be obtained, and the inclusion of seasonal intercepts with trends allows for tests invariant to the presence of non-zero initial values and seasonal drifts to be obtained. As will be mentioned later, the deterministic part considered in the seasonal unit root procedures plays an important role in the distribution of the tests.

The overall null hypothesis of seasonal unit roots is $H_0 : \alpha(L) = 1 - L^S = \Delta_S$, hence the time series y_{St+s} is seasonally integrated. This can be partitioned into the following nulls:

$$\begin{aligned} &H_{0,0} : \alpha_0 = 1, \qquad H_{0,S/2} : \alpha_{S/2} = 1 \\ &H_{0,k} : \alpha_k = 1, \beta_k = 0 \quad k = 1, \cdots, S/2-1 \end{aligned} \tag{3}$$

Under $H_{0,0}$ we have a unit root associated with the zero frequency, under $H_{0,S/2}$ we have a unit root associated with the Nyquist Frequency (π). And under $H_{0,k}$ we have a pair of complex conjugate roots associated with seasonal harmonic frequencies $\frac{2\pi k}{S}$ for $k=1,\ldots,S^*=S/2-1$. The alternative hypothesis is of stationarity at one or more of the zero or seasonal frequencies; that is, $H_1 = U_{j=0}^{S/2} H_{1,j}$, where:

$$H_{1,0} : \alpha_0 < 1, \qquad H_{1,S/2} : \alpha_{S/2} < 1$$
$$H_{1,k} : \alpha_k^2 + \beta_k^2 < 1 \quad k = 1,\cdots,S/2-1 \tag{4}$$

It is possible to define a set of filters that remove the presence of unit roots at the zero, Nyquist and seasonal harmonic frequencies $\omega_k = \frac{2\pi k}{S}$ for $k=1,\ldots,S^*=S/2-1$, as follows:

$$\Delta_0^0(L) = \frac{1-L^S}{1-L} = (1+L+L^2+\ldots+L^{S-1}) \tag{5}$$

$$\Delta_{S/2}^0(L) = -\frac{1-L^S}{1+L} = -(1-L+L^2-\ldots-L^{S-1})$$

$$\Delta_k^0(L) = -\frac{1-L^S}{(1-2\cos[\omega_k]L+L^2)} = -\frac{\sum_{j=0}^{S-1}\sin[(j+1)\omega_k]L^j}{\sin[\omega_k]} =$$
$$= -(1-L^2)\sum_{j\neq k, j=1}^{S^*}(1-2\cos[\omega_j]L+L^2)$$

for k=1,…, (S–1)/2.

Following HEGY (1990) and Smith et al. (2009), the regression-based approach for testing for unit roots in $\alpha(L)$ can be developed in two steps. The first step is detrending the data in order to obtain tests that will be invariant to the parameters that characterize the deterministic part μ_{St+s}. The most popular methods for doing this are OLS detrending (see, for example, HEGY (1990) and Smith et al. (2009)) and GLS detrending (see Rodrigues and Taylor (2007)). In the case of OLS detrending, the resulting detrended time series is obtained from $y_{St+s}^{\xi} = y_{St+s} - \hat{\delta}' z_{St+s,\xi}$, where $\hat{\delta}'$ is obtained from the OLS regression of y on z_{ξ}, with y being a vector with the generic element y_{St+s} and z_{ξ} is a matrix with generic row element $z_{St+s,\xi}$. And ξ corresponds to the deterministic part considered. In the case of GLS detrending, the resulting detrended time series is defined as $y_{St+s}^{\xi} = y_{St+s} - \hat{\delta}' z_{St+s,\xi}$ and in this case $\hat{\delta}'$ is obtained from the OLS regression of y_c on $z_{c,\xi}$, where:

$$y_c = (y_{1-S}, y_{2-S} - \alpha_1^c y_{1-S}, y_{3-S} - \alpha_1^c y_{2-S} - \alpha_2^c y_{1-S},\ldots, y_0 - \alpha_1^c y_{-1} - \ldots - \alpha_S^c y_{1-S}, \Delta_c y_1,\ldots,\Delta_c y_T)'$$

$$z_{c,\xi} = (z_{1-S,\xi}, z_{2-S,\xi} - \alpha_1^c z_{1-S,\xi}, z_{3-S,\xi} - \alpha_1^c z_{2-S,\xi} - \alpha_2^c z_{1-S,\xi},\ldots, z_{0,\xi} - \alpha_1^c z_{1,\xi} - \ldots - \alpha_S^c z_{1-S,\xi},$$
$$\Delta_c z_{1,\xi},\ldots,\Delta_c z_{T,\xi})' \tag{6}$$

and

$$\Delta_c = (1-\overline{\alpha}_0 L)(1-\overline{\alpha}_{S/2} L)\prod_{j=1}^{S/2-1}\left(1-2\left[\alpha_j \cos\left(\frac{2\pi j}{S}\right)\right]L+\overline{\alpha}_j^2 L^2\right)=\left(1-\sum_{j=1}^{S}\alpha_j^c L^j\right)$$

$$with: \qquad (7)$$

$$\overline{\alpha}_0 = 1+\frac{c_0}{SN}, \quad \overline{\alpha}_{S/2} = 1+\frac{c_{S/2}}{SN}, \quad \overline{\alpha}_j = 1+\frac{c_j}{SN} \quad j=1,2,\cdots,S/2-1$$

Table 1 collects the detrending parameters suggested by Elliot, Rothenberg and Stock (1996), Gregoir (2006) and Rodrigues and Taylor (2007).

Table 1. The QD detrending parameters

	Case 1 Zero frequency intercept	Case 2 Zero frequency intercept with trend	Case 3 Seasonal intercepts	Case 4 Seasonal intercepts with zero frequency trend	Case 5 Seasonal intercepts with trends
c_0	-7.00	-13.5	-7.00	-13.5	-13.5
c_k	0	0	-3.75	-3.75	-8.65
$c_{S/2}$	0	0	-7.00	-7.00	-13.5

Then, using the detrended data obtained from OLS or GLS detrending, the HEGY (1990) approach is based on expanding $\alpha(L)$ around the zero and seasonal frequency unit roots, $\exp(\pm i2\pi j/S)$, $j=0,\ldots,S/2$, hence the testing equation of the augmented HEGY approach can be written as:

$$\begin{aligned}\Delta_S y_{St+s}^{\xi} &= \pi_0 y_{0,St+s}^{\xi} + \pi_{S/2} y_{S/2,St+s}^{\xi} + \sum_{j=1}^{S/2-1}(\pi_{1j} y_{1j,St+s}^{\xi} + \pi_{2j} y_{2j,St+s}^{\xi}) \\ &\quad + \sum_{j=1}^{k} d_j \Delta_S y_{St+s-j}^{\xi} + e_{St+s,k}^{\xi}\end{aligned} \qquad (8)$$

where

$$y_{0,St+s}^{\xi} = \Delta_0^0(L) y_{St+s-1}^{\xi} = \sum_{i=0}^{S-1} y_{St+s-i-1}^{\xi} \qquad (9)$$

$$y_{S/2,St+s}^{\xi} = \Delta_{S/2}^0(L) y_{Sn+s-1}^{\xi} = \sum_{i=0}^{S-1}\cos[(i+1)\pi] y_{St+s-i-1}^{\xi}$$

$$y^{\xi}_{1j,St+s} = -[\cos(\omega_j) - L]\Delta^0_j(L) y^{\xi}_{Sn+s-i-1} = \sum_{q=0}^{S-1} \cos[(q+1)\omega_j] y^{\xi}_{St+s-q-1}$$

$$y^{\xi}_{2j,St+s} = \sin(\omega_j)\Delta^0_j(L) y^{\xi}_{St+s-1} = -\sum_{q=0}^{S-1} \sin[(q+1)\omega_j] y^{\xi}_{St+s-q-1}$$

$j = 1,\ldots,S/2-1$.

Under the HEGY approach, the possible presence of serial correlation in the innovation u_{St+s} in equation (1) is accommodated by augmenting regression (9) by adding lags of $\Delta_S y^{\xi}_{St+s}$, approximating the possible serial correlation in u_{St+s} with a finite AR(k) process. As show by del Barrio Castro et al. (2014), this approach is valid for innovations that are allowed to follow a general linear process, and hence u_{St+s} allows for causal and invertible ARMA(p,q) representation. See del Barrio Castro et al. (2014) for details regarding assumptions.

As shown in HEGY (1990) and Smith et al. (2009), testing $H_{0,0} : \alpha_0 = 1$ and $H_{0,S/2} : \alpha_{S/2} = 1$ is equivalent to testing $H_{0,0} : \pi_0 = 0$ and $H_{0,S/2} : \pi_{S/2} = 0$, respectively. Note that the coefficients π_0 and $\pi_{S/2}$ in equation (8) are associated with the auxiliary variables $y^{\xi}_{0,St+s}$ and $y^{\xi}_{S/2,St+s}$, respectively, and these same variables refer to the unit root at zero and Nyquist frequencies, respectively. In both cases, the test is carried out using lower tailed regression t-test statistics t_0 and $t_{S/2}$.

When testing the pairs of complex conjugates unit roots $H_{0,k} : \alpha_k = 1, \beta_k = 0$, $k = 1,\cdots,S/2-1$, it is equivalent to test $H_{0,k} : \pi_{1k} = 0, \pi_{2k} = 0$, associated with the auxiliary variables $y^{\xi}_{1j,St+s}$ and $y^{\xi}_{2j,St+s}$. For this purpose, a lower tailed regression t-test statistic for $\pi_{1k} = 0$ and a two tailed regression t-test statistic for $\pi_{2k} = 0$ are proposed in the original HEGY (1990) paper as well as an upper-tailed regression F-type test to test the joint null hypothesis $H_{0,k} : \pi_{1k} = 0, \pi_{2k} = 0$, F_k.

Further, Ghysels et al. (1994) and Smith et al. (2009) consider joint frequency tests, in particular, the F-type test for controlling for the presence of any seasonal unit root by checking the hypotheses $H_{0,S/2} : \pi_{S/2} = 0$ and $H_{0,k} : \pi_{1k} = 0, \pi_{2k} = 0$, F_{SEAS}. Finally, the presence of any unit root is tested jointly by the hypotheses $H_{0,0} : \pi_0 = 0$, $H_{0,S/2} : \pi_{S/2} = 0$ and $H_{0,k} : \pi_{1k} = 0, \pi_{2k} = 0$, F_{ALL}.

Burridge and Taylor (2001) and Smith, Taylor and del Barrio Castro (2009) in the case of autoregressive (AR) innovations, del Barrio Castro and Osborn (2011) for moving average (MA) innovations and del Barrio Castro, Osborn and Taylor (2012) in the case of general linear processes, show that if regression (8) is properly augmented, the limiting null distributions of the t-statistics for unit roots at the zero and Nyquist frequencies and joint F-type statistics are pivotal, while those of the t-statistics at the harmonic seasonal frequencies depend on nuisance parameters which are functions of the parameters associated with the process followed by the innovation.

Then, in practice using only the t-statistics for unit roots at the zero and Nyquist frequencies is recommended, and using joint F-type statistics is recommended for testing other hypotheses.

As shown in Smith and Taylor (1998) and Smith, Taylor and del Barrio Castro (2009), when there is no deterministic part (μ_{St+s}=0), the distribution of the tests is a function of standard Brownian motions. In the case of OLS detrending, when seasonal intercepts are considered (Case 3) the distribution of the tests is a function of demeaned Brownian motions. In the case of a zero frequency intercept (Case 1) only the distribution of the tests associated with the zero frequency is a function of a demeaned Brownian motion. When seasonal intercepts with trends (Case 5) are included, the distribution of the tests is a function of demeaned and detrended Brownian motions. And finally, if seasonal intercepts with a zero frequency trend (Case 4) are considered, the distribution of all the tests is a function of demeaned Brownian motions except the zero frequency test which is a function of demeaned and detrended Brownian motion. Finally, when GLS detrending is considered, the limit distribution of the statistics with standard Brownian motions are replaced by their relevant local GLS detrended analogues; see Theorem 5.1 of Rodrigues and Taylor (2007, pp. 559–560). See also the tables in the appendix.

3.- Simulation experiments

The seminal papers of MacKinnon (1994, 1996) developed the methodology for obtaining numerical distribution functions for the (zero frequency) unit root test statistics. This methodology implies extensive computation, as well as huge matrices of results. MacKinnon (1996) himself reports several months of computing time and around 20 thousand estimated coefficients. Harvey and van Dijk (2006) and Diaz-Emparanza (2014) apply this methodology to the case of seasonal unit root tests considering only OLS detrending.

Here, we also use the methodology of simulation experiments as described in MacKinnon (1996) but applied to the case of seasonal unit roots; we also consider both OLS and GLS detrending and analyze a wider set of specifications for the deterministic part.

In order to obtain results which can be fitted to response surface functions we need to run the Monte-Carlo Experiment discussed hereafter based on the following data generating process:

$$\left(1-L^{S}\right)y_{St+s}=u_{St+s},\ s=1-S,...,0,\ \ t=1,2,...,N \tag{10}$$

with zero initial values $y_{0+s}=0$, $s=1-S,...,0$, and where for quarterly data *S=4* and for monthly data *S=12*. For the response surface estimation we use a Gaussian IID innovations, that in equation (10) u_{St+s} is IID N(0,1). We use 200.000 replications, and instead of repeating each experiment 50 times as in MacKinnon (1996), we do it 48 times because of parallel computing with 12 threads, and 48 is the closest aliquot number. We consider 27 effective sample sizes of *N* (number of years). Density of *N* is higher for the small samples due to the decreased power of the test. The selected values of *N* are: 9, 10, 11, 12, 13, 14, 16, 18, 19, 21, 23, 26, 28, 31, 34, 38, 41, 45, 50, 55, 61, 67, 73, 81, 89, 100 and 150. Equations (8) and (9) without augmentation (that is, with *k*=0) are fitted to process (10), and the value obtained for the t-ratio and F-type statistics of the HEGY procedure are stored using OLS and GLS detrending. For OLS detrending, we consider six sets of deterministic term specifications, the five cases detailed in section 2 plus the case of no deterministic part. And in the case of GLS detrending we consider the five cases mentioned in section 2.

Thus, we performed a total of 28,512 simulation experiments with 200,000 replications each. The number of experiments is the product of 48 repetitions, 27 sample sizes, five and six different specifications for the deterministic part and two values for *S*. It took around 4 weeks of computing time and 250 GB of disk space to store the results. The simulations were performed on Intel® Xeon® CPU E5-2470.

A replication experiment is a good example to highlight the power of parallel computing. Each experiment may be computed separately, which is why we computed 12 experiments at once. Using Amdahl's formula, it is possible to estimate the expected improvement in computing time using the parallelization.

$$T_C = \frac{1}{(1-P)+\frac{P}{C}} \tag{11}$$

Where T_C is the time improvement, C is the number of cores used, and P is the percentage of the algorithm that can be parallelized, and which is calculated as the number of code lines under threading divided by the total number of lines; it is equal to 79.4% in our case. In total, we made a 3.67 times improvement due to parallel computing.

3.1.- Response surface estimation

Gathering the results of all the simulations (the most extensive part of the study) and following MacKinnon (1996), we proceed to estimate the regressions for the quantiles for each test statistic. For each experiment with 200,000 replications we estimate 221 quantiles, which are: .0001, .0002, .0005, .001, .002, … , .010, .015, …, .985, .990, .991, …, .999, .9995, .9998, .9999. These quantiles for a given S and a given specification of the deterministic terms, are denoted as $q^p(N_i)$, where N_i is the sample size of the i^{th} experiment, p is the quantile and $i = 1,\ldots,$ 1296, represents the 27 different samples sizes and 48 repetitions. These quantiles are used as the dependent variable of the following response surface equation:

$$q^p(N_i) = \theta_\infty^p + \theta_1^p N_i^{-1} + \theta_2^p N_i^{-2} + \theta_3^p N_i^{-3} + \varepsilon_i \quad . \tag{12}$$

Note that the explanatory variables of the previous equation are three negative power functions of the sample size. [1] Equation (12) is fitted for the 221 quantiles, for each possible specification of the deterministic part (with OLS and GLS detrending), for each test of the HEGY procedure and finally for quarterly and monthly data.

[1] For the F_k tests (k=1,2 …, 5); in the monthly case we have 6 480 observations. In fact, there are five F-type tests with the same asymptotic distribution, thus we used the test results of the entire simulation in order to increase the efficiency of estimates. The resulting numerical distribution is valid for testing all five $H_{0,k}$ hypothesis.

In order to put model (12), that is, $q^p = Z\theta + \varepsilon$ in matrix notation, we follow MacKinnon (1996) and use a modification of the GMM estimator of Cragg (1983) as an appropriate way to deal with the presence of heteroscedasticity in the errors:

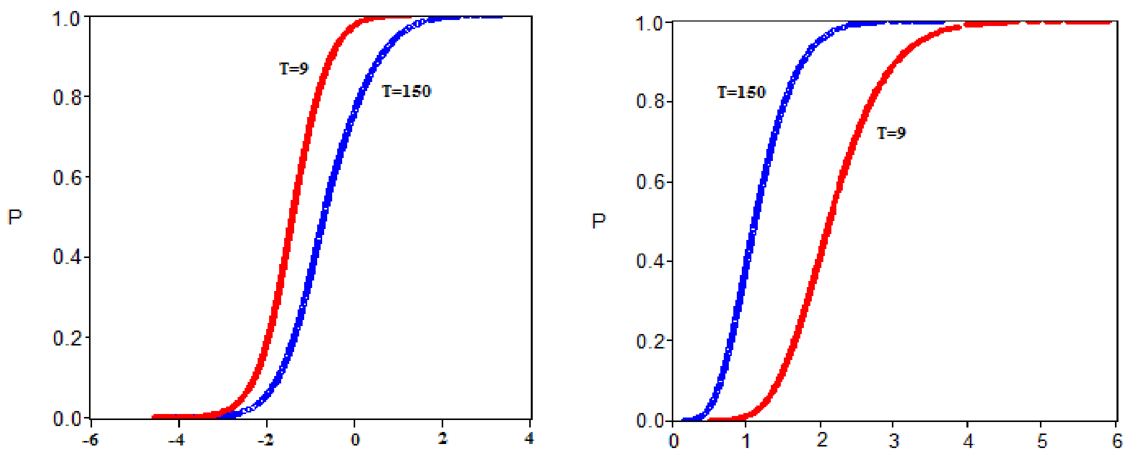
$$\tilde{\theta} = (Z'W(W'\tilde{\Omega}W)^{-1}W'Z)^{-1}Z'W(W'\tilde{\Omega}W)^{-1}W'q^p \tag{13}$$

where W is a matrix of 27 zero-one dummy variables, that is, the first column takes a value of 1 when N_i=9, the second one takes a value of 1 when N_i=10,…; Z is a matrix that collects the regressors of equation (12); and $\tilde{\Omega}$ is a diagonal matrix in which the elements of the principal diagonal are the squares of a fitted endogenous variable obtained from a regression of the absolute value of the residuals from an OLS regression of q^p on W, on a constant and $1/N$.

Figure 1 shows the cumulative numerical distribution function of two test statistics for two fixed sample sizes. The deterministic term specification is seasonal intercepts. The graph on the left hand side represents the distribution of t_0 statistics. The sample size is denoted by N. A curve that corresponds to a 9-year sample size is associated with more negative values than a curve that corresponds to 150 years, which means that hypothesis $H_{0,0}$ is harder to reject for smaller samples. The graph on the right represents the distribution of F-type test statistics for checking the overall null hypothesis.

Figure 1. The cumulative numerical distribution of t_0 and F-all statistics with fixed sample sizes.

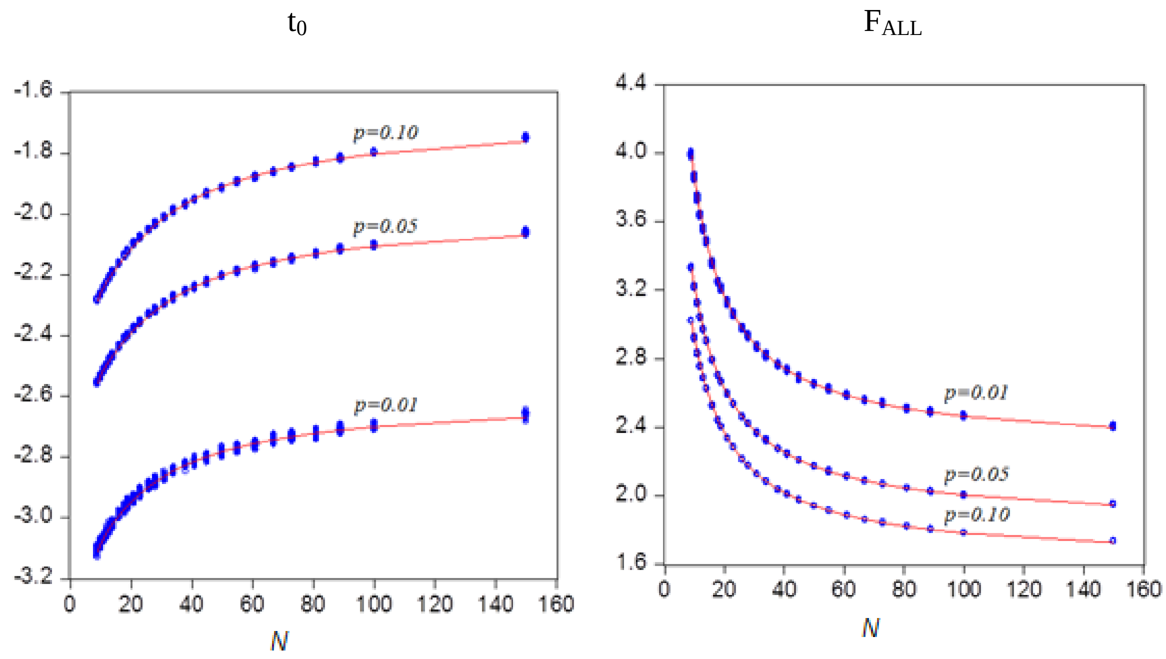
t_0 F_{ALL}



Note: T on the graph is a sample size in years and equal to N in this case. The deterministic term considers seasonal intercepts.

Another way to look at the data in hand is to fix the p-value and plot the distribution depending on N. Figure 2 shows the quantiles that correspond to 1%, 5% and 10% significance levels of the p-value. The graph on the left is associated with t_0 test statistics and the graph on the right is associated with F_{ALL} test statistics. Dots represent the actual numerical distribution of test statistics, and the connecting lines are quantile functions constructed according to equation (12).

Figure 2. Quantile distributions with fitted quantile functions for t_0 and F_{ALL} tests.



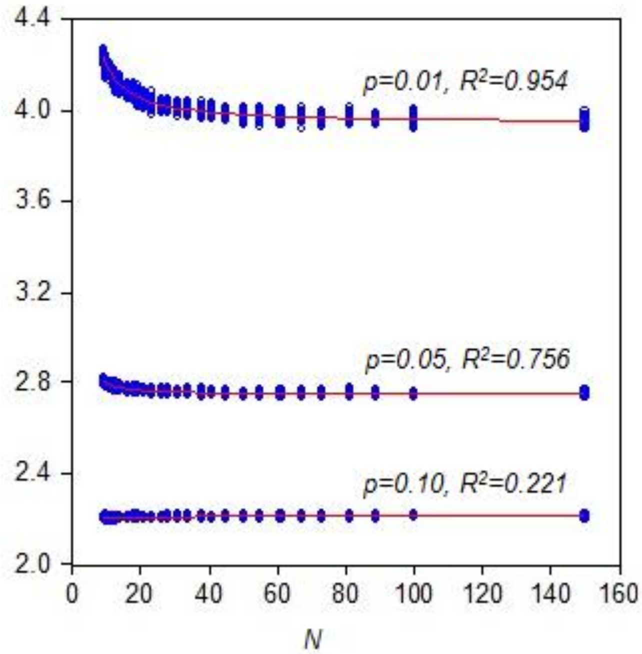
Note: Quantiles correspond to the *p*-levels equal to 1%, 5% and 10%. The deterministic term considers seasonal intercepts. *N* is a sample size in years.

In the appendix we report the response surface coefficients calculated according to equation (12) with *p*-values equal to 1%, 5% and 10%. These functions are associated with the line curves in Figure 2. Tables 2 to 5 report the coefficients for monthly and quarterly frequencies with OLS and GLS detrending. In particular, Tables 2 and 3 display the results when GLS detrending is used for monthly and quarterly data, respectively, and tables 4 and 5 show the equivalent results under OLS detrending.

Note that the first parameter θ_∞^p is the p^{th} quantile of the asymptotic distribution. And, it is possible to check, for example, that the estimated values of θ_∞^p in table 2 case 1 (with GLS detrending) and table 4 with no deterministic part (with OLS detrending) report basically the same values as expected. Note also that in table 2 we obtain the same values for θ_∞^p for cases 1 and 3. Or, the fitted value of θ_∞^p for t_0 for case 2 in table 2 is the same as those observed for t_0 and $t_{S/2}$ for case 4 in table 2. Hence the previous examples, as well as others, show that the reported results in tables 2 to 5 support the theoretical results regarding the distribution of the tests of the HEGY procedure reported in, for example, Rodrigues and Taylor (2007) and del Barrio Castro, Osborn and Taylor (2012).

The eighth column of the table shows the R^2 statistics for the corresponding response surfaces. Most of the values are above 0.90. However, some of the cases are quite low. After careful evaluation of such cases we found that the response surface is less dependent on the sample size. For instance, in Table 3 (GLS detrending and quarterly data), the R^2 for the F_{SEAS} type of test, case 1, we observe that the determination coefficient ranges from 0.95 to 0.22. Figure 2A shows the quantile distribution together with the response surface for the F_{SEAS} type of test. As it can be seen, the low R^2 is due to the low variation of the quantiles as a function of the sample size.

Figure 2A. Quantile distribution with fitted quantile functions for F_{SEAS} type of test, GLS detrending, case 1, quarterly data



Moreover, in table 6 in the appendix the critical values obtained from the response surfaces with low R^2 are compared with the corresponding critical values tabulated in previous studies (Smith and Taylor, 1998; Franses and Hobijn, 1997). Both the values from the previous studies and those calculated from the response surfaces exhibit only small changes as the sample size grows. Moreover, the tabulated critical values sometimes show local maximums or minimums as sample size increases which seems contrary to the expected evolution of the distribution. These results indicate that even in the case that the R^2 is low for a given response surface, the critical values computed with this function are more accurate that the previously tabulated ones.

3.2.- Local approximations for p-values

The resulting 221 estimated response surfaces for the quantiles return the estimated quantile value for any sample size and for a given type of test and specification of the deterministic term. These estimates can be used to obtain approximate *p*-values for any value of the test statistic. As in MacKinnon (1996), we use interpolation to estimate the *p*-value of a particular value of the reported test statistic. The first step is to locate the closest quantiles. The sample size is introduced into 221 response surfaces and the corresponding critical values are obtained. Then, the test statistic is compared sequentially with every critical value and the four closest quantiles are identified: two from above and two from below. After obtaining the closest values, an interpolation function using a polynomial of order three is used. Then, the test statistics of the HEGY seasonal unit root test are introduced into the estimated polynomial, and the *p*-value is reported. The local approximation method to obtain the *p*-values and the estimates of the response surfaces are implemented in a GAUSS™ library and are available from the authors upon request.

3.3.- Comparison of our results with the previous studies

Having the response surface estimates and a method with which to obtain *p*-values by local approximation, we proceed to compare our results with those of previous studies. One of the ways to do so is to use the critical values reported by HEGY (1990), Franses and Hobijn (1997), Smith and Taylor (1998) and Rodrigues and Taylor (2007) as the input for our local approximation method and then compare the resulting *p*-values with the reported ones. Of course, some small discrepancies are expected between results given that they come from estimations and are subject to randomness.

Table 7 reports the critical values that differ from the previous studies by less than two decimal places. All other cases are different out to the third decimal place or more. The first column of the table reports the sample size in quarters or months. The second column reports the type of test: t_0, $t_{S/2}$, F_K, F_{SEAS} or F_{ALL}. In the third column, deterministic terms are represented by CASE 1 to 5, and each of these cases corresponds to the cases described in the methodological part; none stands for no deterministic terms. The next two columns correspond to nominal size and critical values reported in the previous papers. Then, we calculate the *p*-value for the critical value reported ("*p*-value, calculated" column) and show the difference between this computed *p*-value and the reported one. The table is sorted by descending *p*-value difference. Finally, the last column of Table 7 suggests our critical value for the corresponding test statistics based on 9,6 million replications for each sample size (that is 200,000 replications in 48 simulation experiments for each *N*). The authors of the original tables perform from 24 to 100 thousands replications per sample size. The extensive number of simulations in our research implies more accurate estimates.

Originally, the paper of HEGY (1990) works with quarterly data. The authors use 24,000 Monte Carlo replications and report $t_{\pi1}$, $t_{\pi2}$, $t_{\pi3}$, $t_{\pi4}$ and $F_{\pi3,\pi4}$ critical values. In our study we refer to these statistics as t_0, $t_{S/2}$ and F_K, and we do not report $t_{\pi3}$ and $t_{\pi4}$ statistics. We checked the quantiles that correspond to 5% and 10% significance levels. Table 7 reports the 14 of a total of 120 cases where the difference is bigger than 0.02. In the other 88,3% of cases the difference is smaller.

Franses and Hobijn (1997) work with frequencies S=2, S=4, S=6 and S=12, so we use S=4 and S=12 in order to compare results. The authors apply 25,000 replications and report the same five types of tests. For the quarterly case, the results matched 99.5% of the time (difference observed in only 2 of 400 cases) and for the monthly case, the results matched 99% of the time (difference observed in 4 of 400 cases).

Smith and Taylor (1998) report the critical value for quarterly data at 1%, 2.5%, 5% and 10% significance levels. The deterministic terms considered are seasonal intercepts with seasonal trends. The authors use 40,000 Monte Carlo replications and report t_0, $t_{S/2}$, F_K, F_{SEAS} and F_{ALL} statistics. The results of their study perfectly match with our results and none of the 100 critical values differ significantly.

Rodrigues and Taylor (2007) report the critical values for seasonal unit root tests with GLS detrending based on 100,000 replications. The authors consider quarterly data and perform the same five types of tests that we do. Only in 8 of 360 total cases do the *p*-values differ to the second decimal place. The other 97,8% of results differ from the third decimal place or more.

The studies of MacKinnon (1996) and Diaz-Emparanza (2014) report the same response surface functions and routines for computing *p*-values as we do. So, one possible way to compare our results is to introduce the same statistics into the reported functions and compare the *p*-values.

The paper of Diaz-Emparanza (2014) reports response surfaces for HEGY seasonal unit root test for given periodicity as well as *p*-values. The package of functions is developed under the Gretl program, and it is available online. The deterministic terms considered are: only a constant, constant with linear trend, constant with seasonal dummies, and seasonal dummies with linear trend. It is possible to compare *p*-values for monthly and quarterly frequencies using all five types of tests.

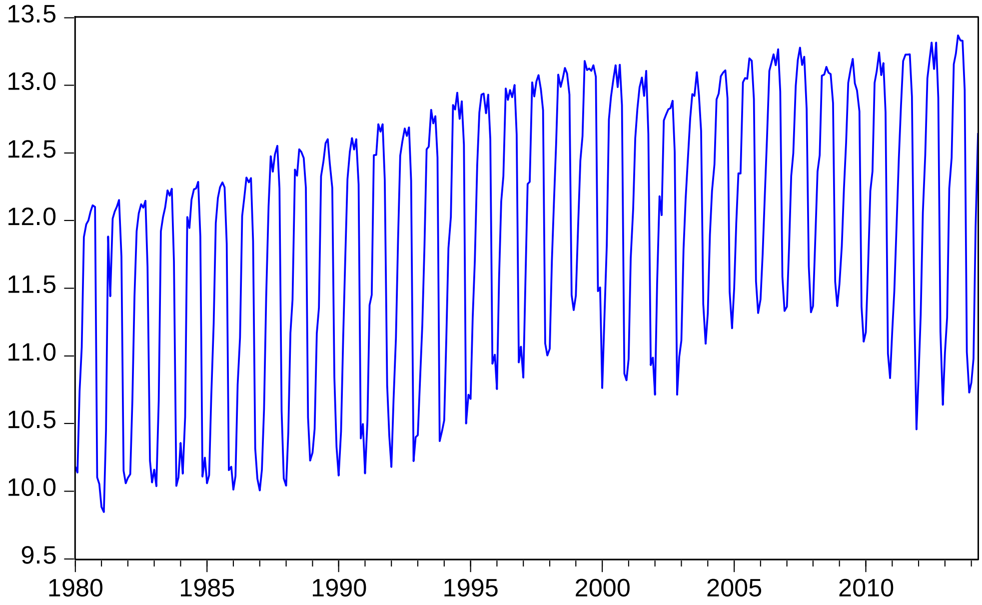
After introducing the same test statistics into two functions, we obtain very similar results for t_0, $t_{S/2}$ and F_K types of tests. The *p*-values for the F_{SEAS} and F_{ALL} statistics are significantly different. That is why we perform cross verification with the critical values published in previous studies. These statistics are only present in the paper of Franses and Hobijn (1997), as HEGY (1990) do not report F_{SEAS} and F_{ALL} types of statistics; Smith and Taylor (1998) report only the critical values for the tests with seasonal intercepts and seasonal trends, which is not covered by Diaz-Emparanza (2014). Finally, Rodrigues and Taylor (2007) report the case with GLS detrending which are not accounted for by Diaz-Emparanza (2014). Table 8 shows the results of this cross verification exercise. Here, we report only the results for the 10% quantile of the largest sample size, which is 40 years in the tables of Franses and Hobijn (1997.

Table 8 demonstrates that there is a significant divergence of results between Diaz-Emparanza (2014) on one hand, and Franses and Hobijn (1997) and our study on the other. This divergence concerns only the F_{SEAS} and F_{ALL} types of test statistics and suggests that our response surfaces are more accurate than the ones reported by Diaz-Emparanza (2014) for these two test statistics. The distribution of the other three test statistics is consistent across all studies.

4.- Empirical example

Tourism arrivals to summer destinations are highly influenced by seasonality. In this part, we analyze airport passenger arrivals from Germany to Mallorca, Spain. The island is considered a “sun and sea” destination with high peaks during the summer period and troughs during the winter. The observation period includes data from January, 1980 through April, 2014, that is 412 months. Before testing for the presence of seasonal unit roots, we apply a natural logarithm in order to reduce heteroscedasticity of the data in hand (Figure 3).

Figure 3. Logarithm of airport passenger arrivals from Germany to Palma de Mallorca, Spain.



Note: Observation period: January, 1980 – April, 2014.

In Table 9 we report the results of HEGY seasonal unit root test with GLS detrending for the data considered. The test statistics are reported together with corresponding *p*-values. We consider three cases of deterministic terms: seasonal intercepts, seasonal intercepts with zero frequency trend and seasonal intercepts with trends. The optimal autoregressive order is estimated by sequentially dropping the last lag until a 10% significance level is reached. The maximal lag for testing is calculated as the integer part of $12(T/100)^{1/4}$, where T is the number of observations (equal to 412). The optimal lag estimated after sequential testing is equal to 13 for all three cases.

Table 9. Results of the HEGY seasonal unit root test with GLS detrending applied to airport passenger arrivals to Palma de Mallorca, Spain.

	Seasonal intercepts		Seasonal intercepts and zero freq. trend		Seasonal intercepts and trends	
	Value	Prob.	Value	Prob.	Value	Prob.
t_0	-0.555	**0.772**	-1.308	**0.867**	-1.284	**0.891**
$t_{S/2}$	-2.967	**0.007**	-2.973	**0.007**	-3.217	**0.042**
$F_{\pi/6}$	0.895	**0.564**	0.947	**0.542**	2.676	**0.775**
$F_{\pi/3}$	3.766	**0.044**	3.865	**0.040**	7.065	**0.073**
$F_{\pi/2}$	3.499	**0.057**	3.271	**0.071**	6.090	**0.141**
$F_{2\pi/3}$	6.657	**0.003**	6.676	**0.003**	10.411	**0.006**
$F_{5\pi/6}$	3.493	**0.057**	3.397	**0.063**	6.386	**0.116**
F_{SEAS}	4.248	**0.000**	4.224	**0.000**	7.241	**0.002**
F_{ALL}	3.922	**0.000**	4.017	**0.000**	6.793	**0.005**

Note: Deterministic terms are: constant with seasonal intercepts, seasonal intercepts with linear trend and seasonal intercepts with seasonal trends. AR order is equal to 13. Effective sample size is equal to 387.

According to the results shown in the previous table, unit roots were found at the zero and $\pi/6$ frequencies, which correspond to long-run and one-year seasonality, respectively. In all three cases which include deterministic terms, both hypotheses $H_{0,0}$ and $H_{0,1}$ cannot be rejected at any usual significance value. The seasonal unit root hypothesis can be rejected at frequencies $\pi/2$ and $5\pi/6$ at a 10% significance level according to the test with seasonal intercepts as well as seasonal intercepts with a linear trend, but not when seasonal trends are included. For the π frequency, which corresponds to a two-month cycle, the unit root hypothesis is rejected at a 1% significance level when no seasonal trends are included but only at a 5% significance level when they are included. The pair of complex unit roots at the $\pi/3$ frequency, has a *p*-value around 4% for the two first specifications of the deterministic terms, so the presence of these unit roots is rejected at the 5% but not at the 1% significance level; when seasonal trends are introduced, the null hypothesis is rejected at a 10% but not at a 5% significance level. Finally, the hypothesis of unit roots associated with the $2\pi/3$ frequency, the null hypothesis of all seasonal unit roots and the null hypothesis of the 12 unit roots are all rejected at a 1% significance level for the three specifications of the deterministic terms.

5.- Conclusions

This research focuses on seasonal unit root testing and extends the response surface analysis to the context of GLS detrending. Previous studies of Harvey and van Dijk (2006) and Diaz-Emparanza (2014) deal only with the response surfaces with OLS detrending in HEGY seasonal unit root test. The present paper is aimed at handling the case of response surfaces and *p*-values for the HEGY test with GLS detrending for monthly and quarterly data.

Validation tests, which compare the results of our investigation with those of previous studies, are undertaken. Most of the results of previous studies are consistent with this study. In some cases, where the difference is significant, we suggest corrected critical values.

As a result of this investigation we generated a Gauss library that reports *p*-values for particular *t* and *F* test statistics depending on sample size, deterministic terms and frequency of the data. Users can find the Gauss code in the supplementary material of this article.

The empirical application of the suggested methodology to passenger arrivals in Mallorca, shows strong evidence of unit roots at zero and one-cycle-per-year frequency as well as strong evidence that not all the seasonal unit roots are present simultaneously.

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Appendix

Table 2. Quantile coefficients for monthly data, p = 1%, 5% and 10%, GLS detrending case

Deter	Type	Nominal size	θ_{∞}^{p}	θ_{1}^{p}	θ_{2}^{p}	θ_{3}^{p}	R^2	$\hat{\sigma}_{\varepsilon}$
CASE 1	t_0	0.01	-2.6064538	-10.175621	107.06159	-402.02016	0.9926	0.0079
		0.05	-1.9928579	-12.360973	127.92231	-477.93486	0.9976	0.0056
		0.10	-1.6750552	-13.950681	143.98809	-538.39363	0.9982	0.0054
	$t_{S/2}$	0.01	-2.565369	1.0884361	-2.5664519	12.626518	0.9308	0.0076
		0.05	-1.9399909	0.8874135	-0.833552	5.3743644	0.9732	0.0042
		0.10	-1.6159129	0.7730544	-0.565721	4.4067282	0.9789	0.0033
	F_K	0.01	4.7332408	-2.6292837	-1.6384555	15.6802	0.9248	0.0223
		0.05	3.1100621	-2.0903709	-2.0046105	16.114836	0.9766	0.0098
		0.10	2.4080405	-1.7772591	-0.7755729	7.3328665	0.9838	0.0068
	F_{SEAS}	0.01	2.3426785	0.4280839	-4.0327865	23.752202	0.5580	0.0064
		0.05	1.8781229	-0.1924286	-0.8683397	5.3513218	0.8307	0.0031
		0.10	1.6547904	-0.2877829	-1.660114	8.7904756	0.9599	0.0023
	F_{ALL}	0.01	2.2801347	3.5575784	-20.034165	55.871893	0.9895	0.0061
		0.05	1.8413546	2.8052868	-18.320792	46.396554	0.9943	0.0030
		0.10	1.6302099	2.5580081	-18.33629	46.626318	0.9954	0.0022
CASE 2	t_0	0.01	-3.4304315	-7.4282461	91.886777	-356.69718	0.9769	0.0071
		0.05	-2.8781125	-8.4054363	104.26608	-402.18423	0.9923	0.0045
		0.10	-2.5934679	-9.1778755	114.42316	-445.30801	0.9944	0.0042
	$t_{S/2}$	0.01	-2.5646271	0.7722459	0.3960046	-2.0871711	0.9020	0.0078
		0.05	-1.9396988	0.6585131	1.0901346	-4.1652911	0.9636	0.0042
		0.10	-1.6155446	0.572219	1.1731581	-4.6940397	0.9706	0.0033
	F_K	0.01	4.7325103	-2.8332203	2.146207	6.5769371	0.8913	0.0262
		0.05	3.1102995	-2.2223455	0.3914827	7.1689207	0.9629	0.0122
		0.10	2.4073859	-1.7890958	-1.0971038	13.233966	0.9741	0.0085
	F_{SEAS}	0.01	2.344197	0.2426348	-0.160438	7.1934005	0.6614	0.0064
		0.05	1.8770934	-0.1630821	-1.4027531	11.669285	0.7809	0.0030
		0.10	1.6550325	-0.3489925	-0.5352557	5.8125377	0.9524	0.0023
	F_{ALL}	0.01	2.5458633	5.4555905	-48.924177	195.25074	0.9908	0.0065
		0.05	2.085075	4.6714437	-49.067748	194.37183	0.9943	0.0034
		0.10	1.8627893	4.3530181	-48.705273	192.25109	0.9947	0.0026
CASE 3	t_0	0.01	-2.6032254	-10.565431	89.99929	-324.73774	0.9964	0.0079
		0.05	-1.990071	-12.720026	114.68568	-416.39375	0.9987	0.0053
		0.10	-1.6723983	-14.264588	132.26462	-483.8779	0.9989	0.0052
	$t_{S/2}$	0.01	-2.6033922	-10.571855	90.11207	-326.24338	0.9966	0.0076
		0.05	-1.9882113	-12.875656	117.75139	-433.64868	0.9987	0.0053
		0.10	-1.6700677	-14.477611	136.40506	-506.43205	0.9990	0.0050
	F_K	0.01	4.7221492	22.17452	-129.9066	361.83506	0.9959	0.0230
		0.05	3.096573	20.490423	-109.26157	251.84211	0.9991	0.0103
		0.10	2.3935497	19.062754	-90.431116	167.21374	0.9995	0.0074
	F_{SEAS}	0.01	2.3262363	19.097837	-44.878795	56.146641	0.9997	0.0075
		0.05	1.8606476	16.958423	-38.28922	16.810367	0.9999	0.0037
		0.10	1.639021	15.815438	-33.022949	-9.2209065	0.9999	0.0028
	F_{ALL}	0.01	2.2664664	20.290791	-51.10663	72.442815	0.9998	0.0072
		0.05	1.8262331	18.238574	-47.567433	47.302576	0.9999	0.0037
		0.10	1.61578	17.136718	-43.799072	28.997292	0.9999	0.0029

(Continued)

Table 2. *(Continued)*

Deter	Type	Nominal size	θ_{∞}^{p}	θ_{1}^{p}	θ_{2}^{p}	θ_{3}^{p}	R2	$\hat{\sigma}_{\varepsilon}$
CASE 4	t_0	0.01	-3.4262229	-7.9358366	72.645255	-264.86438	0.9939	0.0069
		0.05	-2.8737864	-8.9013045	89.260973	-338.73215	0.9975	0.0044
		0.10	-2.590424	-9.5647527	99.205847	-380.26877	0.9980	0.0040
	$t_{S/2}$	0.01	-2.6033096	-10.851371	91.946728	-337.23634	0.9969	0.0076
		0.05	-1.9890582	-12.985181	116.53786	-426.26983	0.9988	0.0053
		0.10	-1.6704546	-14.597529	135.88504	-502.82602	0.9990	0.0051
	F_K	0.01	4.7219074	22.132648	-126.63915	356.32214	0.9961	0.0228
		0.05	3.0964846	20.463844	-106.4717	239.52831	0.9990	0.0108
		0.10	2.3940686	19.002652	-86.878466	147.08193	0.9994	0.0084
	F_{SEAS}	0.01	2.3260625	19.106758	-42.357516	44.817087	0.9997	0.0074
		0.05	1.860186	17.015625	-37.07035	11.463137	0.9999	0.0039
		0.10	1.6382999	15.895696	-32.40938	-12.082042	0.9999	0.0031
	F_{ALL}	0.01	2.5298922	22.052039	-73.86684	189.47025	0.9998	0.0072
		0.05	2.0693866	19.9103	-71.399675	168.54082	0.9999	0.0037
		0.10	1.8476919	18.791946	-68.234817	151.9954	0.9999	0.0027
CASE 5	t_0	0.01	-3.4255075	-10.916232	78.735441	-301.87954	0.9981	0.0073
		0.05	-2.8735277	-11.421414	93.029268	-359.18202	0.9992	0.0045
		0.10	-2.5897606	-11.918101	103.7276	-405.92011	0.9993	0.0041
	$t_{S/2}$	0.01	-3.4306407	-10.619913	73.569174	-276.34133	0.9980	0.0074
		0.05	-2.8764177	-11.285219	90.881955	-348.71474	0.9992	0.0044
		0.10	-2.5927494	-11.781383	101.68756	-396.43024	0.9993	0.0040
	F_K	0.01	8.6680306	41.175588	-206.52703	763.13099	0.9983	0.0326
		0.05	6.5991203	39.046128	-242.56219	878.43064	0.9993	0.0173
		0.10	5.6375963	38.436568	-261.67489	952.63835	0.9995	0.0137
	F_{SEAS}	0.01	5.2745442	46.189689	-199.46349	862.48093	0.9999	0.0118
		0.05	4.6063816	43.236638	-217.73805	876.83087	0.9999	0.0073
		0.10	4.2719648	41.957602	-226.62313	887.03499	0.9999	0.0064
	F_{ALL}	0.01	5.1863965	47.7039	-211.37367	928.73124	0.9999	0.0117
		0.05	4.5515347	44.75336	-231.17955	948.01106	0.9999	0.0076
		0.10	4.233677	43.432724	-240.63387	962.66299	0.9999	0.0067

Note: CASE 1 to 5 refers to deterministic terms, which are described in the methodological part.

Table 3. Quantile coefficients for quarterly data, p = 1%, 5% and 10%, GLS detrending case

Deter	Type	Nominal size	θ_{∞}^{p}	θ_{1}^{p}	θ_{2}^{p}	θ_{3}^{p}	R^2	$\hat{\sigma}_{\varepsilon}$
CASE 1	t_0	0.01	-2.6017744	-11.576394	103.59152	-396.9619	0.9964	0.0084
		0.05	-1.9890882	-13.271645	124.01966	-460.71773	0.9986	0.0055
		0.10	-1.6709987	-14.719768	140.0609	-515.29043	0.9989	0.0052
	$t_{S/2}$	0.01	-2.5633008	-0.1433426	-2.9538673	14.146696	0.5478	0.0085
		0.05	-1.9396597	0.2979791	-3.7835909	21.564794	0.3821	0.0044
		0.10	-1.6154096	0.3350189	-2.5383989	13.526894	0.7540	0.0035
	F_K	0.01	4.7455076	0.0883917	9.2387492	-4.5591302	0.6413	0.0236
		0.05	3.1157636	-0.9173068	2.3154692	0.1649904	0.7895	0.0102
		0.10	2.4101268	-0.9831277	-0.6050786	11.350452	0.9431	0.0068
	F_{SEAS}	0.01	3.9375752	2.048144	4.6598029	8.3126654	0.9538	0.0173
		0.05	2.7463841	0.2255762	2.1941347	0.123433	0.7575	0.0078
		0.10	2.2153354	-0.2166531	1.2369	-0.5729254	0.2210	0.0053
	F_{ALL}	0.01	3.4591416	13.786047	-56.194826	183.58524	0.9972	0.0153
		0.05	2.5003904	10.751192	-52.53282	129.29991	0.9987	0.0068
		0.10	2.0644772	9.6646754	-51.480407	116.29608	0.9990	0.0050
CASE 2	t_0	0.01	-3.42946	-9.7579046	82.481958	-310.64282	0.9962	0.0077
		0.05	-2.8745889	-10.209661	100.4063	-379.1598	0.9981	0.0046
		0.10	-2.5895772	-10.773139	112.83004	-431.11607	0.9982	0.0042
	$t_{S/2}$	0.01	-2.5648603	-0.7220796	-1.6630053	5.185504	0.9064	0.0081
		0.05	-1.9396038	-0.2543905	-1.2392589	9.0240309	0.8037	0.0043
		0.10	-1.6157063	-0.1129541	-1.0298877	7.4431335	0.6158	0.0034
	F_K	0.01	4.731942	0.4340476	8.7469067	23.626831	0.8065	0.0241
		0.05	3.1124684	-1.0782544	5.9701458	2.9993925	0.6250	0.0101
		0.10	2.4105288	-1.2820898	4.6041636	-0.4866935	0.9252	0.0068
	F_{SEAS}	0.01	3.935979	2.1304154	13.614065	-16.254418	0.9697	0.0180
		0.05	2.7454947	0.3471148	4.7715588	5.4579053	0.9297	0.0076
		0.10	2.215263	-0.1597885	3.8089946	-1.9892152	0.6693	0.0052
	F_{ALL}	0.01	4.4016081	20.771288	-121.55971	503.01758	0.9980	0.0172
		0.05	3.3400212	17.111022	-131.87092	508.37418	0.9988	0.0085
		0.10	2.8472804	15.794686	-137.3654	528.29785	0.9989	0.0065
CASE 3	t_0	0.01	-2.5990665	-12.047061	85.687973	-313.47376	0.9982	0.0077
		0.05	-1.983883	-13.928391	115.1281	-418.81782	0.9991	0.0053
		0.10	-1.6671559	-15.262401	132.19935	-478.62541	0.9993	0.0049
	$t_{S/2}$	0.01	-2.5974222	-12.235628	89.523079	-333.16689	0.9980	0.0082
		0.05	-1.9841934	-13.936898	115.37681	-419.67485	0.9991	0.0053
		0.10	-1.6677888	-15.228615	131.54416	-474.6252	0.9993	0.0050
	F_K	0.01	4.7198851	26.092445	-76.364389	193.84426	0.9982	0.0254
		0.05	3.0977204	22.073034	-68.871248	99.894313	0.9994	0.0112
		0.10	2.3932811	20.125496	-59.807989	47.881951	0.9997	0.0079
	F_{SEAS}	0.01	3.8908411	32.625569	-95.725256	261.93441	0.9993	0.0195
		0.05	2.7031125	27.81624	-96.678618	197.87319	0.9998	0.0090
		0.10	2.1750663	25.43119	-90.365416	151.3075	0.9998	0.0066
	F_{ALL}	0.01	3.4294484	34.78914	-100.53054	300.41199	0.9996	0.0166
		0.05	2.4707113	29.916552	-103.762	231.20254	0.9998	0.0078
		0.10	2.0364976	27.593827	-100.52671	190.05771	0.9999	0.0060

(Continued)

Table 3. *(Continued)*

Deter	Type	Nominal size	θ_∞^p	θ_1^p	θ_2^p	θ_3^p	R^2	$\hat{\sigma}_\varepsilon$
CASE 4	t_0	0.01	-3.4240594	-10.555798	66.839623	-252.60934	0.9981	0.0078
		0.05	-2.8694267	-10.939314	87.891982	-332.34765	0.9991	0.0044
		0.10	-2.5859015	-11.344668	99.533099	-378.36706	0.9992	0.0039
	$t_{S/2}$	0.01	-2.6002007	-12.738076	87.668208	-326.35103	0.9984	0.0081
		0.05	-1.9843278	-14.491218	117.00689	-429.04365	0.9993	0.0052
		0.10	-1.6677164	-15.740811	133.48634	-483.72754	0.9993	0.0050
	F_K	0.01	4.7204106	26.102737	-70.902319	196.40945	0.9983	0.0255
		0.05	3.0925571	22.430856	-70.177911	106.56658	0.9995	0.0109
		0.10	2.387891	20.555888	-63.13051	60.869493	0.9997	0.0076
	F_{SEAS}	0.01	3.8906697	33.381043	-90.309043	273.66357	0.9994	0.0199
		0.05	2.7004043	28.52364	-94.29993	194.36907	0.9998	0.0091
		0.10	2.1718384	26.098291	-89.188768	145.03976	0.9998	0.0068
	F_{ALL}	0.01	4.3652945	41.380478	-148.50333	602.59639	0.9996	0.0189
		0.05	3.3081417	35.649826	-161.41505	543.9192	0.9999	0.0087
		0.10	2.8154926	33.24967	-166.76296	535.00317	0.9999	0.0063
CASE 5	t_0	0.01	-3.4217626	-13.179493	71.56037	-301.72588	0.9991	0.0079
		0.05	-2.8684513	-13.120393	91.33276	-366.27019	0.9995	0.0047
		0.10	-2.5849263	-13.352406	103.2619	-410.92894	0.9996	0.0041
	$t_{S/2}$	0.01	-3.4250965	-13.019092	69.285867	-291.10874	0.9991	0.0076
		0.05	-2.8703909	-13.034822	90.663643	-366.62196	0.9995	0.0046
		0.10	-2.5865534	-13.274299	102.48787	-409.96937	0.9996	0.0040
	F_K	0.01	8.658319	54.666891	-108.73738	537.09139	0.9994	0.0351
		0.05	6.5914446	47.231376	-173.72113	638.06151	0.9998	0.0161
		0.10	5.6307484	44.497686	-199.79752	691.47889	0.9998	0.0115
	F_{SEAS}	0.01	7.518367	64.231225	-183.85832	1080.1022	0.9997	0.0285
		0.05	5.955003	56.09077	-246.30303	1090.4206	0.9999	0.0143
		0.10	5.2153158	52.660717	-268.92965	1100.1364	0.9999	0.0113
	F_{ALL}	0.01	6.8682745	67.762763	-215.1383	1312.4729	0.9998	0.0252
		0.05	5.5787523	59.621558	-273.76368	1277.0575	0.9999	0.0140
		0.10	4.9624761	56.003649	-294.43467	1265.369	0.9999	0.0115

Note: CASE 1 to 5 refers to deterministic terms, which are described in the methodological part.

Table 4. Quantile coefficients for monthly data, p = 1%, 5% and 10%, OLS detrending case

Deter	Type	Nominal size	θ_{∞}^{p}	θ_{1}^{p}	θ_{2}^{p}	θ_{3}^{p}	R^2	$\hat{\sigma}_{\varepsilon}$
None	t_0	0.01	-2.5675401	1.3094475	-2.3015686	9.8562761	0.9554	0.0075
		0.05	-1.9417622	1.0719636	-0.5122449	0.5805563	0.9822	0.0041
		0.10	-1.6175985	0.9626317	-1.0662905	5.0677654	0.9846	0.0033
	$t_{S/2}$	0.01	-2.5664128	1.2292061	-1.2555395	6.2523839	0.9513	0.0078
		0.05	-1.9407732	0.9938746	0.3436014	-1.5542122	0.9807	0.0042
		0.10	-1.6170914	0.9163365	-0.5437617	4.0409648	0.9842	0.0034
	F_K	0.01	4.7322746	-2.5629962	-1.1271589	13.311456	0.9209	0.0221
		0.05	3.1095037	-2.0146195	-1.7868307	12.504621	0.9764	0.0096
		0.10	2.407046	-1.6520329	-2.0280066	12.957738	0.9842	0.0065
	F_{SEAS}	0.01	2.345647	0.2849185	-1.0546447	7.188119	0.5709	0.0063
		0.05	1.8775094	-0.1383075	-1.2240437	5.5672143	0.8075	0.0031
		0.10	1.6550286	-0.2935594	-1.0243531	4.1683709	0.9553	0.0023
	F_{ALL}	0.01	2.2890432	0.4002036	-0.6281681	5.9506023	0.7910	0.0060
		0.05	1.8481484	-0.0809114	-0.2333239	0.6058961	0.5071	0.0029
		0.10	1.6369129	-0.2188253	-0.7216096	2.8142843	0.9287	0.0022
CASE 1	t_0	0.01	-3.4307657	1.2029985	-2.1075018	4.2503217	0.9507	0.0069
		0.05	-2.8610683	1.1901521	-0.8464146	1.6992724	0.9873	0.0038
		0.10	-2.567425	1.2274372	-1.8215069	7.6229947	0.9918	0.0030
	$t_{S/2}$	0.01	-2.5658514	1.1654154	0.7221767	-3.4849549	0.9534	0.0079
		0.05	-1.9400205	0.9659154	1.2559339	-5.975413	0.9815	0.0042
		0.10	-1.6158989	0.8510861	0.946713	-3.8560344	0.9846	0.0034
	F_K	0.01	4.7347231	-3.1429516	3.0630371	-6.3132977	0.9329	0.0229
		0.05	3.1113755	-2.4513097	1.5869998	-2.506456	0.9782	0.0102
		0.10	2.4088408	-2.025763	1.2244484	-1.9689176	0.9850	0.0070
	F_{SEAS}	0.01	2.3447725	0.0251582	0.9828024	-3.6657225	0.1652	0.0062
		0.05	1.8780538	-0.3471399	-0.2138461	1.8730848	0.9181	0.0031
		0.10	1.6550581	-0.4510989	-0.610946	3.5495146	0.9742	0.0023
	F_{ALL}	0.01	2.5342473	0.5701665	-0.9048796	10.131331	0.8738	0.0065
		0.05	2.0695665	-0.0199388	-0.6262699	3.7168939	0.1746	0.0031
		0.10	1.8454335	-0.2328933	-0.4231408	1.5754821	0.9167	0.0023
CASE 2	t_0	0.01	-3.9549846	1.0122235	3.4984414	-30.675131	0.9563	0.0068
		0.05	-3.4083525	1.2863159	0.5118806	-6.9046369	0.9904	0.0037
		0.10	-3.125559	1.3222249	0.3796422	-4.9865777	0.9943	0.0029
	$t_{S/2}$	0.01	-2.566256	1.1300491	0.4033942	-3.0012104	0.9506	0.0077
		0.05	-1.9411086	0.9804559	-0.1545628	2.9056916	0.9802	0.0042
		0.10	-1.6165843	0.8409908	0.0914484	1.7285454	0.9838	0.0033
	F_K	0.01	4.731919	-3.4930734	1.4612942	13.170863	0.9211	0.0277
		0.05	3.1100357	-2.6862001	0.7078775	7.710322	0.9680	0.0136
		0.10	2.4080743	-2.2397875	1.1979622	1.7845587	0.9766	0.0096
	F_{SEAS}	0.01	2.3445369	-0.1178548	-0.6309749	10.633739	0.1253	0.0064
		0.05	1.8777836	-0.5155937	-0.0678793	3.9232476	0.9546	0.0031
		0.10	1.6550991	-0.6149477	-0.3324111	5.1974024	0.9837	0.0023
	F_{ALL}	0.01	2.7887716	0.9817603	-5.6804247	39.999899	0.9258	0.0066
		0.05	2.3078427	0.0578631	-1.382916	11.691416	0.1020	0.0032
		0.10	2.073475	-0.2250406	-0.4429236	4.0147672	0.8833	0.0025

(Continued)

Table 4. *(Continued)*

Deter	Type	Nominal size	θ_{∞}^{p}	θ_{1}^{p}	θ_{2}^{p}	θ_{3}^{p}	R^2	$\hat{\sigma}_{\varepsilon}$
CASE 3	t_0	0.01	-3.4305843	2.3483579	-3.6773595	5.3848125	0.9861	0.0071
		0.05	-2.8622944	2.2297365	-2.8249245	8.4082381	0.9963	0.0036
		0.10	-2.5677525	2.0958616	-2.4516818	10.745195	0.9976	0.0029
	$t_{S/2}$	0.01	-3.4305505	2.3500403	-3.6558281	4.4179047	0.9856	0.0072
		0.05	-2.8606026	2.0657875	0.1588679	-6.045421	0.9959	0.0038
		0.10	-2.5655316	1.9159787	0.8751589	-6.1630405	0.9973	0.0030
	F_K	0.01	8.8059579	-10.372729	13.962657	-0.0263542	0.9888	0.0280
		0.05	6.6439349	-8.9382092	7.0052185	0.8799325	0.9971	0.0131
		0.10	5.6291142	-8.0390517	5.5872678	-5.0031789	0.9980	0.0098
	F_{SEAS}	0.01	5.1879385	1.8400227	0.8305577	28.431909	0.9761	0.0104
		0.05	4.4703393	0.1298514	-1.454135	15.043602	0.3688	0.0052
		0.10	4.1108956	-0.554905	-2.3373856	11.801312	0.9615	0.0041
	F_{ALL}	0.01	5.0832624	2.4113652	2.7749309	15.9166	0.9866	0.0100
		0.05	4.40506	0.5778606	-0.4386137	8.9351938	0.9332	0.0050
		0.10	4.0639292	-0.1534111	-1.9719314	10.047546	0.8035	0.0038
CASE 4	t_0	0.01	-3.9559888	2.3216257	-2.0198937	-4.5424273	0.9876	0.0068
		0.05	-3.4088741	2.3214439	-0.852058	0.8878269	0.9969	0.0037
		0.10	-3.125523	2.2260245	0.3617691	-1.8467394	0.9981	0.0029
	$t_{S/2}$	0.01	-3.4282036	2.1868232	-1.0145889	-6.0207827	0.9855	0.0072
		0.05	-2.8612384	2.1208895	-0.9676025	1.747159	0.9957	0.0040
		0.10	-2.5661124	1.9492966	0.1919263	-1.208689	0.9972	0.0031
	F_K	0.01	8.8084506	-11.014045	17.130188	-7.8454988	0.9886	0.0294
		0.05	6.6445821	-9.2838275	7.5795355	0.4373092	0.9969	0.0139
		0.10	5.6294078	-8.3078736	5.8984249	-5.4548148	0.9980	0.0102
	F_{SEAS}	0.01	5.1872176	1.6501231	1.3986009	22.779383	0.9717	0.0103
		0.05	4.4699058	0.0051321	-2.0261104	15.795071	0.0822	0.0052
		0.10	4.1118809	-0.771061	-0.9547951	1.6726229	0.9751	0.0041
	F_{ALL}	0.01	5.3142769	2.698423	2.4181515	19.104695	0.9878	0.0104
		0.05	4.6223044	0.7932148	-2.1911885	17.976482	0.9464	0.0053
		0.10	4.2759089	-0.0996844	-1.6781388	5.9888628	0.7270	0.0040
CASE 5	t_0	0.01	-3.9559128	2.1481761	-5.6798976	6.2843492	0.9781	0.0071
		0.05	-3.4087024	2.1353073	-2.8526172	10.533851	0.9959	0.0037
		0.10	-3.1262605	2.0826855	-1.8548174	11.965616	0.9977	0.0029
	$t_{S/2}$	0.01	-3.9600056	2.3310207	-8.2349019	17.322549	0.9810	0.0067
		0.05	-3.4120999	2.2749461	-4.6162299	17.309801	0.9958	0.0038
		0.10	-3.1281091	2.1448211	-2.4136137	12.862881	0.9976	0.0029
	F_K	0.01	12.21408	-14.953704	37.803806	-35.469902	0.9904	0.0330
		0.05	9.7501738	-13.736069	21.923671	-48.117166	0.9979	0.0160
		0.10	8.5753322	-12.6811	14.044172	-38.983239	0.9988	0.0118
	F_{SEAS}	0.01	7.9955643	5.5364164	9.6733272	66.803089	0.9964	0.0130
		0.05	7.1481837	2.2480373	3.1483006	14.525138	0.9936	0.0066
		0.10	6.7192513	0.8071479	0.5972514	-1.3134692	0.9643	0.0049
	F_{ALL}	0.01	7.8673893	6.576094	10.458514	64.373974	0.9976	0.0123
		0.05	7.0632075	3.0625	4.7586321	7.0607786	0.9966	0.0063
		0.10	6.6546712	1.5835817	0.2754597	3.4407029	0.9907	0.0047

Note: CASE 1 to 5 refers to deterministic terms, which are described in the methodological part; *none* stands for no deterministic terms.

Table 5. Quantile coefficients for quarterly data, p = 1%, 5% and 10%, OLS detrending case

Deter	Type	Nominal size	θ_{∞}^{p}	θ_{1}^{p}	θ_{2}^{p}	θ_{3}^{p}	R^2	$\hat{\sigma}_{\varepsilon}$
None	t_0	0.01	-2.5676678	0.4751678	-2.0866101	11.057007	0.6419	0.0081
		0.05	-1.9411654	0.6589852	-1.3763645	11.727207	0.9512	0.0043
		0.10	-1.6168177	0.6567624	-0.679632	6.1568001	0.9711	0.0033
	$t_{S/2}$	0.01	-2.5677929	0.3946104	-0.5761377	3.9353396	0.6459	0.0082
		0.05	-1.9410209	0.5667288	1.151301	-5.4928701	0.9501	0.0043
		0.10	-1.6167719	0.5975269	0.6846232	-1.8546284	0.9712	0.0033
	F_K	0.01	4.729882	1.2719035	-1.246289	15.707069	0.7255	0.0238
		0.05	3.1105441	-0.4746164	-0.5826716	7.0978537	0.6564	0.0099
		0.10	2.4094103	-0.8841471	1.7175107	-9.3520629	0.9248	0.0067
	F_{SEAS}	0.01	3.9360076	2.1138295	1.5101301	2.2413873	0.9369	0.0177
		0.05	2.7444305	0.3096088	-0.9951215	7.6669009	0.5392	0.0078
		0.10	2.2154677	-0.2696407	0.6770328	-2.6692922	0.5989	0.0055
	F_{ALL}	0.01	3.4810704	2.844462	-0.7330297	23.059449	0.9749	0.0143
		0.05	2.5212908	0.7868214	-0.1738385	3.630294	0.9314	0.0065
		0.10	2.0866174	0.1436252	0.666834	-3.181357	0.5985	0.0044
CASE 1	t_0	0.01	-3.427993	-0.5812201	2.5521031	-28.067062	0.8580	0.0073
		0.05	-2.8602236	0.2317073	1.0513884	-9.7978441	0.7477	0.0042
		0.10	-2.5660487	0.4984623	0.5006179	-4.2384347	0.9572	0.0032
	$t_{S/2}$	0.01	-2.5654843	0.3327896	2.0609683	-11.813474	0.7229	0.0079
		0.05	-1.9409957	0.6440714	0.1991937	1.7642012	0.9562	0.0043
		0.10	-1.6169878	0.6647643	-0.0583101	3.1426989	0.9737	0.0034
	F_K	0.01	4.7324106	-0.1066813	1.2958689	28.570727	0.1815	0.0235
		0.05	3.1101654	-1.3008103	-0.0784826	18.377518	0.9177	0.0099
		0.10	2.4091106	-1.4663395	0.2107927	13.804404	0.9706	0.0067
	F_{SEAS}	0.01	3.9340198	1.3414927	-1.2242778	37.889254	0.8836	0.0174
		0.05	2.7441775	-0.3138404	-0.85485	16.809288	0.4560	0.0077
		0.10	2.2145428	-0.6840994	-0.8512198	13.143561	0.9292	0.0052
	F_{ALL}	0.01	4.3786638	4.8517259	-6.4152528	81.284739	0.9883	0.0162
		0.05	3.3069139	1.6576716	-3.9239019	37.87116	0.9777	0.0073
		0.10	2.8079755	0.6079089	-1.7231322	15.977384	0.9177	0.0052
CASE 2	t_0	0.01	-3.9578302	-0.9506303	0.0452801	-18.612543	0.9534	0.0075
		0.05	-3.4096333	0.0572423	0.6236054	-9.3269809	0.0788	0.0039
		0.10	-3.1271451	0.4822495	-0.5244749	0.7931118	0.9475	0.0030
	$t_{S/2}$	0.01	-2.5667639	0.1769365	0.6995052	-5.30954	0.3393	0.0081
		0.05	-1.9411076	0.4432638	0.8543688	-3.217485	0.9196	0.0044
		0.10	-1.6167623	0.4695933	0.6407396	-1.1257551	0.9534	0.0035
	F_K	0.01	4.731581	-1.5950089	6.5630488	32.858942	0.3827	0.0240
		0.05	3.1104266	-2.4251299	6.5757176	-2.626365	0.9670	0.0096
		0.10	2.4090477	-2.3224461	5.2016427	-3.3865231	0.9859	0.0064
	F_{SEAS}	0.01	3.9358326	0.4744963	0.5497029	59.866301	0.8154	0.0169
		0.05	2.745062	-1.0033942	3.0899005	9.5584463	0.8420	0.0075
		0.10	2.2156122	-1.2923094	3.4962851	-1.1475224	0.9666	0.0051
	F_{ALL}	0.01	5.2504668	6.3605264	0.5489734	83.53286	0.9927	0.0186
		0.05	4.093603	2.4357024	-1.195847	36.041727	0.9892	0.0083
		0.10	3.5482854	1.0664806	-0.6092601	17.885635	0.9723	0.0060

(Continued)

Table 5. *(Continued)*

Deter	Type	Nominal size	θ_{∞}^{p}	θ_{1}^{p}	θ_{2}^{p}	θ_{3}^{p}	R^2	$\hat{\sigma}_{\varepsilon}$
CASE 3	t_0	0.01	-3.4297763	0.3600138	0.0481116	-26.143721	0.2619	0.0076
		0.05	-2.860673	0.9722348	0.2604524	-9.29901	0.9774	0.0041
		0.10	-2.5665713	1.1718978	-0.1708337	-1.9030244	0.9911	0.0032
	$t_{S/2}$	0.01	-3.4286971	0.3977295	-1.8735404	-12.559216	0.2061	0.0074
		0.05	-2.8616439	1.0286748	-0.8145006	-3.3678362	0.9792	0.0039
		0.10	-2.5669902	1.177482	-0.1806585	-2.3249076	0.9913	0.0031
	F_K	0.01	8.8019274	3.3538848	14.277935	70.72439	0.9656	0.0318
		0.05	6.6424614	-0.926679	2.4713914	34.417913	0.3673	0.0147
		0.10	5.6266552	-2.138732	-0.0650036	22.338578	0.9650	0.0108
	F_{SEAS}	0.01	7.5396048	7.5991821	8.7130426	104.98413	0.9924	0.0250
		0.05	5.9104902	1.9392848	5.3296319	18.793816	0.9796	0.0117
		0.10	5.1271615	0.3200734	-0.9572912	25.507557	0.7386	0.0087
	F_{ALL}	0.01	6.8331686	10.088421	11.745679	108.79019	0.9962	0.0228
		0.05	5.4859552	4.2840335	1.1083491	45.083524	0.9949	0.0104
		0.10	4.835543	2.0705299	-0.8664969	26.985332	0.9872	0.0077
CASE 4	t_0	0.01	-3.9591996	0.0092665	-3.2784932	-20.098089	0.8393	0.0076
		0.05	-3.4091581	0.7622415	0.7557244	-14.45085	0.9643	0.0040
		0.10	-3.1264299	1.1007609	0.4040567	-3.7484708	0.9916	0.0030
	$t_{S/2}$	0.01	-3.4288406	0.4397071	-2.9928732	-7.7696786	0.1881	0.0076
		0.05	-2.8613813	1.0097287	-0.8642377	-1.1243402	0.9784	0.0040
		0.10	-2.5670798	1.1690247	-0.4872639	1.8289946	0.9917	0.0031
	F_K	0.01	8.8079433	1.3768422	27.927537	43.673534	0.9563	0.0313
		0.05	6.6484906	-2.4078609	13.518526	-6.3163026	0.8210	0.0149
		0.10	5.6313541	-3.1794977	5.3193418	5.0928481	0.9812	0.0105
	F_{SEAS}	0.01	7.54305	6.4424174	17.852255	94.686737	0.9913	0.0261
		0.05	5.9137395	1.1626669	11.496638	-3.2857448	0.9724	0.0119
		0.10	5.1309255	-0.3861874	4.8934131	2.3557045	0.3586	0.0088
	F_{ALL}	0.01	7.6191556	12.07151	16.260612	157.81323	0.9974	0.0237
		0.05	6.2116903	5.0771543	9.0521913	24.511814	0.9966	0.0110
		0.10	5.5243556	2.7015249	1.4485982	23.006445	0.9928	0.0079
CASE 5	t_0	0.01	-3.9588101	-0.2609943	-6.8041457	-28.5541	0.9646	0.0076
		0.05	-3.4103017	0.607464	-2.5274287	-11.745063	0.7679	0.0040
		0.10	-3.1270704	0.9216108	-1.8381578	0.6206047	0.9802	0.0031
	$t_{S/2}$	0.01	-3.9592924	-0.2064835	-6.981935	-31.230843	0.9638	0.0076
		0.05	-3.410415	0.5905968	-2.1206633	-13.207976	0.7949	0.0038
		0.10	-3.1275135	0.9040293	-1.2230855	-2.8750693	0.9818	0.0030
	F_K	0.01	12.207936	9.5227497	22.455666	358.7622	0.9925	0.0408
		0.05	9.7445908	1.5427798	9.9435242	98.777146	0.9742	0.0180
		0.10	8.5712092	-0.9858562	2.469288	52.913858	0.3884	0.0133
	F_{SEAS}	0.01	10.750947	16.028405	21.898973	446.20257	0.9977	0.0330
		0.05	8.8662471	6.7413723	10.22886	135.52841	0.9972	0.0146
		0.10	7.9494832	3.3764254	3.6260652	67.391514	0.9934	0.0109
	F_{ALL}	0.01	9.9188917	20.406505	17.886062	501.64223	0.9988	0.0280
		0.05	8.3530514	10.128553	9.8759863	159.10682	0.9989	0.0129
		0.10	7.5807963	6.4076414	-0.0392215	101.85158	0.9981	0.0098

Note: CASE 1 to 5 refers to deterministic terms, which are described in the methodological part; *none* stands for no deterministic terms.

Table 6. Critical values obtained from the response surface with low R^2 compared to the table values

Smith and Taylor (1998), quarterly frequency		
	CASE 5, F_K, 10%	
Sample	Table	Surface
12	8.59	8.537
25	8.49	8.539
34	8.51	8.546
50	8.51	8.553
100	8.51	8.562
R^2		0.388

Franses and Hobijn (1997), quarterly frequency								
	CASE 2, t_0, 5%		CASE 1, F_K, 1%		CASE 4, $t_{S/2}$, 1%		CASE 2, $t_{S/2}$, 1%	
Sample	Table	Surface	Table	Surface	Table	Surface	Table	Surface
10	-3.34	-3.407	4.95	4.763	-3.40	-3.423	-2.49	-2.547
20	-3.38	-3.406	4.83	4.734	-3.41	-3.415	-2.52	-2.557
30	-3.41	-3.407	4.61	4.731	-3.40	-3.418	-2.52	-2.560
40	-3.40	-3.408	4.83	4.731	-3.41	-3.420	-2.53	-2.562
R^2		0.079		0.182		0.188		0.339

Franses and Hobijn (1997), monthly frequency								
	CASE 4, F_{SEAS}, 5%		CASE 2, F_{ALL}, 5%		CASE 2, F_{SEAS}, 1%		CASE 1, F_{SEAS}, 1%	
Sample	Table	Surface	Table	Surface	Table	Surface	Table	Surface
10	4.50	4.466	2.35	2.311	2.38	2.337	2.40	2.353
20	4.46	4.467	2.32	2.309	2.35	2.338	2.38	2.348
30	4.47	4.468	2.33	2.309	2.33	2.340	2.34	2.347
40	4.44	4.469	2.30	2.309	2.36	2.341	2.36	2.346
R^2		0.082		0.102		0.125		0.165

Note: CASE 1 to 5 refers to deterministic terms, which are described in the methodological part; Sample size is in years.

Table 7. Critical values that differ from previous studies

Sample size	Type of statistics	Deterministic terms	Nominal size	Published CV	Calculated *p*-value	*p*-value difference	Suggested CV
HEGY (1990), quarterly frequency, OLS case							
48	t_0	CASE 4	0.10	**-3.37**	0.0476	0.0524	**-3.03**
48	$t_{S/2}$	CASE 4	0.10	**-2.73**	0.0567	0.0433	**-2.47**
48	t_0	CASE 3	0.10	**-2.72**	0.0579	0.0421	**-2.47**
48	$t_{S/2}$	CASE 3	0.10	**-2.69**	0.0620	0.0380	**-2.47**
48	t_0	CASE 4	0.05	**-3.71**	0.0209	0.0291	**-3.35**
100	t_0	CASE 4	0.10	**-3.22**	0.0732	0.0268	**-3.08**
48	t_0	CASE 3	0.05	**-3.08**	0.0242	0.0258	**-2.78**
48	$t_{S/2}$	CASE 4	0.05	**-3.08**	0.0243	0.0257	**-2.78**
136	t_0	CASE 4	0.10	**-3.21**	0.0767	0.0233	**-3.09**
48	$t_{S/2}$	CASE 3	0.05	**-3.04**	0.0268	0.0232	**-2.78**
48	t_0	CASE 2	0.10	**-3.21**	0.0776	0.0224	**-3.09**
100	t_0	CASE 3	0.10	**-2.63**	0.0785	0.0215	**-2.52**
100	$t_{S/2}$	CASE 3	0.10	**-2.63**	0.0785	0.0215	**-2.52**
100	$t_{S/2}$	CASE 4	0.10	**-2.63**	0.0786	0.0214	**-2.52**
Rodrigues and Taylor (2007), quarterly frequency, GLS case							
48	F_{SEAS}	CASE 4	0.10	**3.97**	0.9140	-0.0140	**3.81**
48	F_{ALL}	CASE 3	0.10	**3.88**	0.9134	-0.0134	**3.75**
48	F_{ALL}	CASE 4	0.10	**4.89**	0.9132	-0.0132	**4.74**
48	F_{SEAS}	CASE 3	0.10	**3.89**	0.9123	-0.0123	**3.75**
48	F_K	CASE 4	0.10	**3.85**	0.9118	-0.0118	**3.70**
48	F_{ALL}	CASE 5	0.10	**8.50**	0.9110	-0.0110	**8.32**
48	F_{SEAS}	CASE 5	0.10	**8.56**	0.9103	-0.0103	**8.37**
48	F_K	CASE 3	0.10	**3.81**	0.9100	-0.0100	**3.68**
Franses and Hobijn (1997), quarterly frequency, OLS case							
40	t_0	CASE 2	0.10	**-3.03**	0.1113	-0.0113	**-3.08**
40	F_K	none	0.10	**2.44**	0.9101	-0.0101	**2.33**
Franses and Hobijn (1997), monthly frequency, OLS case							
120	F_{ALL}	none	0.10	**1.65**	0.9110	-0.0110	**1.61**
120	F_{SEAS}	none	0.10	**1.66**	0.9109	-0.0109	**1.62**
360	F_{ALL}	CASE 1	0.01	**2.34**	0.9793	0.0107	**2.55**
120	t_0	CASE 2	0.10	**-2.95**	0.1105	-0.0105	**2.99**

Note: CASE 1 to 5 refers to deterministic terms, which are described in the methodological part; *none* stands for no deterministic terms.

Table 8. Cross verification results

Sample size	Type of statistics	Deterministic terms	CV reported by Franses and Hobijn (1997)	Nominal size reported by Franses and Hobijn (1997)	*p*-value, by Diaz-Emparanza (2014)	Our *p*-value
Monthly frequency						
480	F_{SEAS}	CASE 1	1.66	0.10	0.202	0.095
480	F_{ALL}	CASE1	1.84	0.10	0.169	0.100
480	F_{SEAS}	CASE2	1.66	0.10	0.201	0.094
480	F_{ALL}	CASE2	2.07	0.10	0.134	0.099
480	F_{SEAS}	CASE3	4.08	0.10	0.232	0.103
480	F_{ALL}	CASE3	4.04	0.10	0.238	0.104
480	F_{SEAS}	CASE4	4.07	0.10	0.233	0.104
480	F_{ALL}	CASE4	4.26	0.10	0.207	0.102
Quarterly frequency						
160	F_{SEAS}	CASE 1	2.20	0.10	0.121	0.100
160	F_{ALL}	CASE1	2.83	0.10	0.066	0.099
160	F_{SEAS}	CASE2	2.18	0.10	0.121	0.101
160	F_{ALL}	CASE2	3.59	0.10	0.031	0.098
160	F_{SEAS}	CASE3	5.09	0.10	0.122	0.104
160	F_{ALL}	CASE3	4.86	0.10	0.141	0.103
160	F_{SEAS}	CASE4	5.09	0.10	0.118	0.103
160	F_{AL}	CASE4	5.55	0.10	0.087	0.104

Note: CASE 1 to 5 refers to deterministic terms, which are described in the methodological part.

Chapter 3: The lag length selection and detrending methods for HEGY seasonal unit root tests using Stata

The lag length selection and detrending methods for HEGY seasonal unit root tests using Stata

Abstract

The paper extends the previous (Baum and Sperling (2001) and Depalo (2009)) HEGY seasonal unit root test commands in Stata which allow for the use of both quarterly and monthly data. It is also possible to choose between OLS and GLS detrending (Rodrigues and Taylor (2007)) procedures to deal with the deterministic part of the process. The command allows for the use of the sequential method proposed by Hall (1994) and Ng and Perron (1995), the adaptation of the MAIC criteria to the case of seasonal unit root tests (del Barrio Castro, Osborn and Taylor (2016)) as well as the the inclusion of AIC and BIC information criteria to determine the order of augmentation of the serial correlation in the augmented HEGY regression. Finally, the use of the command is illustrated with an empirical application to the case of monthly passenger airport arrivals to Palma de Mallorca.

Key words: HEGY test, GLS detrending, optimal augmentation lag

1.- Introduction

Seasonality is an important feature of economic time series. Despite the fact that adjustments such as X-11 and ARIMA-Based methods (SEATS) are widely used by practitioners in order to get rid of seasonal fluctuations in economic time series, the use of seasonally adjusted data in applied work, instead of simplifying the data, may actually produce undesirable features in the filtered data such as non-invertible Moving Average processes and complicated dynamics, see Maravall (1993) for details. Hence, it would be recommendable to use raw (non-seasonally adjusted) data when employing econometric tools based on fitting Autoregressive processes or Vector Autoregressive Models.

Over the last three decades, a debate has been conducted in the literature as to whether seasonality is deterministic or attributable to one or multiple unit roots at seasonal frequencies. Over the years, this situation has led to the development of a large number of seasonal unit root testing procedures; see for example Dickey, Hasza and Fuller (1984), Osborn, Chui, Smith and Birchenhall (1988), Hylleberg, Engle, Granger and Yoo (1990) (henceforth HEGY) and Rodrigues and Taylor (2004, 2007), among others. The HEGY approach has become the most popular one to test for the presence of seasonal unit roots.

Recently, Rodrigues and Taylor (2007) have extended the GLS detrending procedure proposed by Elliot et al (1996) for the zero frequency Augmented Dickey-Fuller test to the case of the Augmented-HEGY tests. Rodrigues and Taylor (2007) show that GLS detrended Augmented-HEGY tests have asymptotic local power functions which lie arbitrarily close to the Gaussian power envelopes. Also, they find power gains over the standard OLS detrended HEGY tests when using GLS detrended HEGY tests.

Dealing with serial correlation and determining the order of augmentation in the augmented HEGY regression has been an important issue in seasonal unit root testing procedures as their performance depends critically on handling this feature of the data. In the context of the standard unit root tests, the most popular way of determining the order of augmentation in the Augmented Dickey-Fuller tests is with the Modified AIC (MAIC) criteria proposed by Ng and Perron (2001); recently the MAIC criteria has been extended by del Barrio Castro, Osborn and Taylor (2016) to the case of Augmented HEGY tests.

So far, the HEGY seasonal unit root testing routines were developed for Stata to consider quarterly data and OLS detrending. In particular, the command *hegy4* developed by Baum and Sperling (2001) performs the HEGY procedure to test for seasonal unit roots with quarterly data. It provides the t-statistics and F-statistics to test the presence of unit roots at the zero, semi-annual, and annual frequencies. The statistics are reported jointly with the critical values. The program can automatically conduct a sequential t-test to determine the optimal lag length to be included in the auxiliary regression which is in line with the proposal made by Hall (1994) and Ng and Perron (1995). Additionally, Depalo (2009), in his article, continues to develop the seasonal unit root tests under Stata. The command *sroot* has a more diverse output: the *generate()* option creates time series of HEGY auxiliary variables and residuals. This information is important according to Engle at al. (1993), and seasonal integration can be studied from the transformed variables.

In this paper, we expand on the previous HEGY test commands in Stata in various aspects. First, our *hegy* command allows for the use of quarterly and monthly data. Also, the user can choose between OLS detrending and GLS detrending to deal with the specified deterministic part of the process. For all of the possible specifications of the deterministic part (see section 2 for details) and for OLS and GLS detrending, the command provides critical values at the 1%, 5% and 10% levels of significance obtained from response surfaces according to the proposal in Mackinnon (1996) (see del Barrio Castro, Bodnar and Sansó (2015) for details). In the case of our *hegy* command, a wider range of possibilities for determining the order of augmentation in the HEGY regression is available. As in the case of the *hegy4* command, the sequential method proposed by Hall (1994) and Ng and Perron (1995) is available, but it is also possible to use this method jointly with the conventional method based on the AIC and the BIC criteria to determine the lag length in the augmented HEGY regression and also the recently extended seasonal version of the MAIC criteria by del Barrio Castro, Osborn and Taylor (2016). Finally, the *hegy* command reports the autocorrelation function (ACF) and partial ACF (PACF), as well as the Ljung-Box Q-statistics of the residuals. In developing the *hegy* command we use the base structure of the *hegy4* command from Baum and Sperling (2001).

This paper is organized as follows: the next section discusses all the methodological details of the *hegy* command, Section 3 describes the implementation of the *hegy* command, Section 4 is devoted to an empirical application of the command using the monthly time series of tourism arrivals to Palma de Mallorca from the UK, and the last section provides a conclusion.

2.- Methodology

2.1.- The seasonal model.

Here we employ the conventional HEGY methodology to test for the presence of seasonal unit roots. The augmentation lag selection and detrending methods suggested by del Barrio Castro et al. (2014) and Rodrigues and Taylor (2007) are used.

The data generating process (DGP) of a univariate seasonal time series is assumed to be as follows:

$$y_{St+s} = \mu_{St+s} + x_{St+s}$$

$$\alpha(L)x_{St+s} = u_{St+s},\ s = 1-S,...,0,\ \ t = 1,2,...,N \tag{1}$$

S denotes the number of seasons. As the Stata commands deal with monthly (*S*=12) and quarterly (*S*=4) data, in the rest of the paper we are going to assume that *S* is even. *N* represents the number of years. It is assumed that the observed time series y_{St+s} can be decomposed into two parts: the deterministic part μ_{St+s} and the stochastic part x_{St+s}.

$\alpha(L)$ is an AR(S) polynomial $\alpha(L) = \left(1-\hat{\alpha}_1 L - \hat{\alpha}_2 L^2 - \cdots - -\hat{\alpha}_S L^S\right)$ where L is the usual lag operator. This polynomial can be factorized as:

$$\alpha(L) = (1-\alpha_0 L)(1+\alpha_{S/2}L)\prod_{j=1}^{S^*}\left(1-2\left[\alpha_j\cos\left(\frac{2\pi j}{S}\right)-\beta_j\sin\left(\frac{2\pi j}{S}\right)\right]L+\left(\alpha_j^2+\beta_j^2\right)L^2\right), \tag{2}$$

with $S^* = S/2-1$. Our focus is to test for the presence of unit roots in the polynomial $\alpha(L)$. Note that the parameter α_0 of $(1-\alpha_0 L)$ is associated with the zero frequency, the parameter $\alpha_{S/2}$ of $(1+\alpha_{S/2}L)$ is associated with the Nyquist frequency (π) and the parameters α_j and β_j of $\left(1-2\left[\alpha_j\cos\left(\frac{2\pi j}{S}\right)-\beta_j\sin\left(\frac{2\pi j}{S}\right)\right]L+\left(\alpha_j^2+\beta_j^2\right)L^2\right)$ are associated with the conjugate (harmonic) seasonal frequencies $\frac{2\pi j}{S}$ and 2π - $\frac{2\pi j}{S}$ for $j = 1,\ldots,S^* = S/2-1$.

Following Smith and Taylor (1998), Rodriguez and Taylor (2007) and Smith, Taylor and del Barrio Castro (2009), it is possible to define six scenarios for the deterministic part μ_{St+s} : no deterministic terms, zero frequency intercept (one intercept), zero frequency intercept with trend, seasonal intercepts, seasonal intercepts with zero frequency trend and seasonal intercepts with trends. Hence in terms of $\mu_{St+s} = \delta' z_{St+s,\xi}$ we have:

Case 0: No deterministics:

μ_{St+s}=0.

Case 1: Only a constant:

$z_{St+s,1} = [1]$ with $\delta = (\delta_0)$.

Case 2: Constant and zero frequency trend:

$z_{St+s,2} = [1, St+s]'$ with $\delta = (\delta_0, \bar{\delta}_0)$'.

Case 3: Seasonal intercepts:

$$z_{St+s,3} = [1, \cos(2\pi(St+s)/S), \sin(2\pi(St+s)/S), ...$$
$$..., \cos(2\pi S^*(St+s)/S), \sin(2\pi S^*(St+s)/S), (-1)^{St+s}]'$$

with $\delta = (\delta_0, \delta'_1, ..., \delta'_{S*}, \delta_{S/2})$', $\delta_k = (\delta_{k,1}, \delta_{k,2})$', $k = 1, \ldots, S^*$ =S.

Case 4: Seasonal intercepts and zero frequency trend:

$z_{St+s,4} = [z'_{St+s,3}, St+s]'$ $\delta = (\delta_0, \delta'_1, ..., \delta'_{S*}, \delta_{S/2}, \bar{\delta}_0)$', $\delta_k = (\delta_{k,1}, \delta_{k,2})$', $k = 1, \ldots, S^*$.

Case 5: Seasonal intercepts and trends:

$z_{St+s,5} = [z'_{St+s,3}, (St+s)z'_{St+s,3}]'$

with $\delta = (\delta_0, \delta'_1, ..., \delta'_{S*}, \delta'_{S/2}, \bar{\delta}_0, \bar{\delta}'_1, ..., \bar{\delta}'_{S*}, \bar{\delta}_{S/2})$', $\delta_k = (\delta_{k,1}, \delta_{k,2})$', $k = 1, \ldots, S^*$.

As shown by Smith and Taylor (1998) and Smith, Taylor and del Barrio Castro (2009), the inclusion of seasonal intercepts allows for tests invariant to the presence of non-zero initial conditions to be obtained under the null hypothesis of seasonal integration, and the inclusion of both seasonal intercepts and trends allows tests invariant to both the presence of non-zero initial values and seasonal drifts to be obtained. As will be discussed later, the deterministic part considered in the seasonal unit root procedures plays an important role in the distribution of the tests.

The overall null hypothesis of the presence of all unit roots is $H_0 : \alpha(L) = 1 - L^S = \Delta_S$; hence, the time series y_{St+s} is seasonally integrated. This can be partitioned into the following null hypotheses:

$$\begin{aligned} &H_{0,0} : \alpha_0 = 1, \qquad H_{0,S/2} : \alpha_{S/2} = 1 \\ &H_{0,k} : \alpha_k = 1, \beta_k = 0 \quad k = 1, \cdots, S/2-1 \end{aligned} \tag{3}$$

Under $H_{0,0}$ we have a unit root associated with the zero frequency, under $H_{0,S/2}$ we have a unit root associated with the Nyquist Frequency (π). And under $H_{0,k}$ we have a pair of complex conjugate roots associated with the seasonal harmonic frequencies $\frac{2\pi k}{S}$ for $k = 1, \ldots, S^* = S/2-1$. The alternative hypothesis is of stationarity at one or more of the zero or seasonal frequencies; that is, $H_1 = U_{j=0}^{S/2} H_{1,j}$, where:

$$\begin{aligned} &H_{1,0} : \alpha_0 < 1, \qquad H_{1,S/2} : \alpha_{S/2} < 1 \\ &H_{1,k} : \alpha_k^2 + \beta_k^2 < 1 \quad k = 1, \cdots, S/2-1 \end{aligned} \tag{4}$$

The filters that remove the possible presence of unit roots at the zero ($\Delta_0^0(L)$), Nyquist ($\Delta_{S/2}^0(L)$) and seasonal harmonic ($\Delta_k^0(L)$) frequencies $\frac{2\pi k}{S}$ for $k = 1, \ldots, S^* = S/2-1$, are defined as follows:

$$\Delta_0^0(L) = \frac{1-L^S}{1-L} = (1 + L + L^2 + \ldots + L^{S-1})$$

$$\Delta_{S/2}^0(L) = -\frac{1-L^S}{1+L} = -(1 - L + L^2 - \ldots - L^{S-1}) \tag{5}$$

$$\Delta_k^0(L) = -\frac{1-L^S}{(1 - 2\cos[\omega_k]L + L^2)} = -\frac{\sum_{j=0}^{S-1} \sin[(j+1)\omega_k]L^j}{\sin[\omega_k]} =$$

$$= -(1-L^2) \sum_{j \neq k, j=1}^{S^*} (1 - 2\cos[\omega_j]L + L^2)$$

for k=1,…, (S–1)/2.

As it will be seen in the next section, these filters are closely related to the auxiliary variables used in the HEGY procedure.

2.2.- The HEGY tests.

Following HEGY (1990) and Smith et al (2009), the regression-based approach for testing for unit roots in $\alpha(L)$ can be carried out in two steps. The first step is detrending the data in order to obtain tests that will be invariant to the parameters that characterize the deterministic part μ_{St+s}. The most popular methods use OLS detrending (see for example HEGY (1990) and Smith et al (2009)) or GLS detrending (see Rodrigues and Taylor (2007)). In the case of OLS detrending, the resulting detrended time series is obtained from $y^{\xi}_{St+s} = y_{St+s} - \hat{\delta}' z_{St+s,\xi}$, where $\hat{\delta}'$ is obtained from the OLS regression of y on z_{ξ}, with y being a vector with the generic element y_{St+s}, and z_{ξ} is a matrix with generic row element $z_{St+s,\xi}$. Then, ξ corresponds to the deterministic part being considered. In the case of GLS detrending, the resulting detrended time series is defined as $y^{\xi}_{St+s} = y_{St+s} - \hat{\delta}' z_{St+s,\xi}$, and in this case $\hat{\delta}'$ is obtained from the OLS regression of y_c on $z_{c,\xi}$, with:

$$y_c = (y_{1-S}, y_{2-S} - \alpha_1^c y_{1-S}, y_{3-S} - \alpha_1^c y_{2-S} - \alpha_2^c y_{1-S}, ..., y_0 - \alpha_1^c y_{-1} - ... - \alpha_S^c y_{1-S}, \Delta_c y_1, ..., \Delta_c y_T)'$$

$$\begin{aligned} z_{c,\xi} = (&z_{1-S,\xi}, z_{2-S,\xi} - \alpha_1^c z_{1-S,\xi}, z_{3-S,\xi} - \alpha_1^c z_{2-S,\xi} - \alpha_2^c z_{1-S,\xi}, ..., z_{0,\xi} - \alpha_1^c z_{1,\xi} - ... - \alpha_S^c z_{1-S,\xi}, \\ &\Delta_c z_{1,\xi}, ..., \Delta_c z_{T,\xi})' \end{aligned} \qquad (6)$$

and:

$$\begin{aligned} &\Delta_c = (1-\overline{\alpha}_0 L)(1-\overline{\alpha}_{S/2} L)\prod_{j=1}^{S/2-1}\left(1-2\left[\alpha_j \cos\left(\frac{2\pi j}{S}\right)\right]L + \overline{\alpha}_j^2 L^2\right) = \left(1-\sum_{j=1}^{S}\alpha_j^c L^j\right) \\ &where: \\ &\overline{\alpha}_0 = 1+\frac{c_0}{ST}, \quad \overline{\alpha}_{S/2} = 1+\frac{c_{S/2}}{ST}, \quad \overline{\alpha}_j = 1+\frac{c_j}{ST} \quad j = 1,2,\cdots,S/2-1 \end{aligned} \qquad (7)$$

Table 1 presents the values for the parameters c_j $j = 0,1,2,\cdots,S/2$ suggested by Elliot, Rothenberg and Stock (1996), Gregoir (2006) and Rodrigues and Taylor (2007) and which are used in our Stata command.

Table 1. The QD detrending parameters

Parameter	(Case 1) Only a constant	(Case 2) Constant and zero frequency trend	(Case3) Seasonal intercepts	(Case 4) Seasonal intercepts and zero frequency trend	(Case 5) Seasonal intercepts and trends
c_0 [(a)]	-7.00	-13.5	-7.00	-13.5	-13.5
$c_j \ \ j = 1,2,\cdots,S/2-1$ [(b)]	0.00	0.00	-3.75	-3.75	-8.65
$c_{S/2}$ [(a)]	0.00	0.00	-7.00	-7.00	-13.5

Source: (a) Elliot, Rothenberg and Stock (1996) and (b) Gregoir (2006)

Then, using the detrended data obtained by OLS or GLS detrending, the HEGY (1990) approach is based on expanding $\alpha(L)$ around the zero and seasonal frequency unit roots ($\exp(\pm i 2\pi j / S)$, $j = 0,\ldots,S/2$); hence, the testing equation of the augmented HEGY approach can be written as:

$$\Delta_S y^{\xi}_{St+s} = \pi_0 y^{\xi}_{0,St+s} + \pi_{S/2} y^{\xi}_{S/2,St+s} + \sum_{j=1}^{S/2-1} (\pi_{1j} y^{\xi}_{1j,St+s} + \pi_{2j} y^{\xi}_{2j,St+s}) + \sum_{j=1}^{k} d_j \Delta_S y^{\xi}_{St+s-j} + e^{\xi}_{St+s,k} \tag{8}$$

where

$$y^{\xi}_{0,St+s} = \Delta^0_0(L) y^{\xi}_{St+s-1} = \sum_{i=0}^{S-1} y^{\xi}_{St+s-i-1} \tag{9}$$

$$y^{\xi}_{S/2,St+s} = \Delta^0_{S/2}(L) y^{\xi}_{Sn+s-1} = \sum_{i=0}^{S-1} \cos[(i+1)\pi] y^{\xi}_{St+s-i-1}$$

$$y^{\xi}_{1j,St+s} = -[\cos(\omega_j) - L]\Delta^0_j(L) y^{\xi}_{Sn+s-i-1} = \sum_{q=0}^{S-1} \cos[(q+1)\omega_j] y^{\xi}_{St+s-q-1}$$

$$y^{\xi}_{2j,St+s} = \sin(\omega_j)\Delta^0_j(L) y^{\xi}_{St+s-1} = -\sum_{q=0}^{S-1} \sin[(q+1)\omega_j] y^{\xi}_{St+s-q-1}$$

$j = 1,\ldots,S/2-1$.

Under the HEGY approach, the possible presence of serial correlation in the innovation u_{St+s} in equation (1) is handled by augmenting regression (9) by adding lags of $\Delta_S y^{\xi}_{St+s}$, which approximates the possible serial correlation in u_{St+s} by a finite AR(k) process. As shown by del Barrio Castro et al. (2014), this approach is valid for innovations that are allowed to follow a general linear process, and hence u_{St+s} allows for causal and invertible ARMA(p,q) representation. See del Barrio Castro et al. (2014) for details regarding assumptions.

As shown in HEGY (1990) and Smith et al (2009), testing $H_{0,0} : \alpha_0 = 1$ and $H_{0,S/2} : \alpha_{S/2} = 1$ is equivalent to testing $H_{0,0} : \pi_0 = 0$ and $H_{0,S/2} : \pi_{S/2} = 0$, respectively. Note that the coefficients π_0 and $\pi_{S/2}$ in equation (8) are associated with the auxiliary variables $y^{\xi}_{0,St+s}$ and $y^{\xi}_{S/2,St+s}$ respectively, and these variables refer to the unit roots at zero and Nyquist frequencies, respectively. In both cases, the test is carried out using lower tailed regression t-test statistics.

When testing for the pairs of complex conjugate unit roots ($H_{0,k} : \alpha_k = 1, \beta_k = 0$, $k = 1, \cdots, S/2-1$), it is equivalent to test $H_{0,k} : \pi_{1k} = 0, \pi_{2k} = 0$ associated with the auxiliary variables $y^{\xi}_{1j,St+s}$ and $y^{\xi}_{2j,St+s}$. For this purpose, a lower tailed regression t-test statistic for $\pi_{1k} = 0$ and a two tailed regression t-test statistic for $\pi_{2k} = 0$ are proposed in the original HEGY paper. Also, an upper-tailed regression F-type test is suggested to test the joint null $H_{0,k} : \pi_{1k} = 0, \pi_{2k} = 0$.

Further, Ghysels et al. (1994) and Smith et al. (2009) consider joint frequency tests, in particular, the F-type test for controlling for the presence of any seasonal unit roots by checking the hypotheses $H_{0,S/2} : \pi_{S/2} = 0$ and $H_{0,k} : \pi_{1k} = 0, \pi_{2k} = 0$. Finally, the presence of any unit root is tested jointly by the hypotheses $H_{0,0} : \pi_0 = 0$, $H_{0,S/2} : \pi_{S/2} = 0$ and $H_{0,k} : \pi_{1k} = 0, \pi_{2k} = 0$.

Burridge and Taylor (2001) and Smith, Taylor and del Barrio Castro (2009) in the case of autoregressive (AR) innovations, del Barrio Castro and Osborn (2011) for moving average (MA) innovations and del Barrio Castro, Osborn and Taylor (2012) in the case of general linear processes, show that if regression (8) is properly augmented, the limiting null distributions of the t-statistics for unit roots at the zero and Nyquist frequencies and joint F-type statistics are pivotal, while those of the t-statistics at the harmonic seasonal frequencies depend on nuisance parameters which are functions of the parameters associated with the process followed by the innovation. So in practice, it is recommended to use only the t-statistics for unit roots at the zero and Nyquist frequencies and joint F-type statistics for all other unit roots. Hence, the *hegy* command reports results from the left tail t-ratio tests associated with the zero and Nyquist frequencies and all the F-type statistics described above.

As shown in Smith and Taylor (1998) and Smith, Taylor and del Barrio Castro (2009) in the case where there is no deterministic part (μ_{St+s}=0) the distribution of the tests is a function of standard Brownian motions. In the case of OLS detrending, when seasonal intercepts are considered (Case 1) the distribution of the tests is a function of demeaned Brownian motions. In the case of a zero frequency intercept, only the distribution of the tests associated with the zero frequency is a function of demeaned Brownian motion. When seasonal intercepts with trends are included, the distribution of the tests is a function of demeaned and detrended Brownian motions. And finally, if seasonal intercepts with a zero frequency trend are considered, the distribution of all the tests is a function of demeaned Brownian motions, expect for the zero frequency test which is function of demeaned and detrended Brownian motion. Finally, with GLS detrending when considering the inclusion of seasonal intercepts and seasonal intercepts with trends, the limit distribution of the statistics can be obtained by replacing the detrended Brownian motions with their relevant local GLS detrended analogues; see Theorem 5.1 of Rodrigues and Taylor (2007, pp. 559–560).

2.3.- Lag length selection methods.

In our *hegy* command, the user can choose between five different methods to determine the order of augmentation of equation (8). The user can provide a desired order of augmentation for the augmented HEGY regression, or use information criteria (such as the AIC, BIC and MAIC) or the Hall (1994) and Ng and Perron (1995) sequential method to determine the lag length to be used in equation (8). In the latter case, the user can specify a maximum lag length (k_{max}) or use the following default rule $k_{\max} = Int\left[12(T/100)^{1/4}\right]$, where $Int[.]$ is the integer part and T is the total sample size (T=SN). In the case of the sequential method, the procedure starts by fitting model (8) with k= $k_{\max}$ and tests sequentially for the significance of the coefficient associated with Δy^{ξ}_{St+s-k} until the null is rejected. In order to determine the individual significance of the lags of Δy^{ξ}_{St+s} the standard normal distribution critical values are used, and a 10% level of significance is used as the default value following the suggestion of Ng and Perron (1995). When using the sequential method the user is allow to change the default 10% for another significance level (say 5% or 1%), but the results reported in Ng and Perron (1995) show that with the 10% level, the sequential method achieves better results. In the case of the method based on the information criteria, the process chooses the order of augmentation from k_{max} to 0, which obtains the lowest value for the AIC, BIC or the MAIC criteria. In the case of the MAIC criteria we must first consider the usual AIC:

$$AIC = \ln(\hat{\sigma}^2_k) + \frac{2k}{T} \tag{10}$$

where $\hat{\sigma}^2_k = RSS_k/(T-k)$ and in which RSS_k is the sum of squared residuals obtained from testing the regression with k lags. Then the MAIC has an additional penalization term, $\tau_T(k)$, added where T is the number of observations. The optimal augmentation lag corresponds to the lowest AIC value:

$$MAIC = \ln(\hat{\sigma}^2_k) + \frac{2(\tau_T(k)+k)}{T-k_{\max}} \tag{11}$$

$$\tau_T(k) = (\hat{\sigma}_k^2)^{-1}\left(\hat{\pi}_0^2 \sum_t \sum_s (y_{0,St+s}^{\xi})^2 + \hat{\pi}_{S/2}^2 \sum_t \sum_s (y_{S/2,St+s}^{\xi})^2 + \right.$$
$$\left. + \sum_{j=1}^{S/2-1} \hat{\pi}_{1j}^2 \sum_t \sum_s (y_{1j,St+s}^{\xi})^2 + \hat{\pi}_{2j}^2 \sum_t \sum_s (y_{2j,St+s}^{\xi})^2 \right) \quad (12)$$

Where $\tau_T(k)$, as part of the penalty function, takes into account the possible non-stationarity of the regressors ($y_{0,St+s}^{\xi}, y_{S/2,St+s}^{\xi}, y_{1j,St+s}^{\xi}, y_{2j,St+s}^{\xi}$) in (8) (see Ng and Perron (2001) and del Barrio Castro, Osborn and Taylor (2015) for details). Following Perron and Qu (2007) and del Barrio Castro, Osborn and Taylor (2015), the MAIC is computed based on equations (8) and (9) with OLS detrending to determine the order of augmentation of the augmented HEGY regression even if GLS detrending is considered.

2.4.- Critical values.

In our *hegy* command, we utilize the results of del Barrio Castro, Bodnar and Sansó (2015). The critical values at 1%, 5% and 10% significance level are calculated from the quantile functions, which are estimated as follows:

$$q^p(T_i) = \theta_\infty^p + \theta_1^p T_i^{-1} + \theta_2^p T_i^{-2} + \theta_3^p T_i^{-3} + \varepsilon_i \quad (13)$$

where q^p is a quantile value at p significance level and T_i is a corresponding sample size. The quantile coefficients are computed based on 9,6 million Monte Carlo simulations for each of the 27 sample sizes, six deterministic terms for the OLS case and three for the GLS case. Monthly and quarterly frequencies are considered separately.

2.5.- The hegy command

hegy *varname* [*if*] [*in*] [, maxlag(*integer*) det(*string*) level(*integer*) mode(*string*) residuals(*string*) noac noreg gls]

Options

det(string) controls for the deterministic terms present in the time series. The string option may take on values **none, const**, **seas**, **trend**, **strend** or **mult**, in order to specify the deterministic part of the process to be tested. The default value is **seas**, as suggested by

HEGY (1990) and Ghysels et al. (1994). It indicates that a set of seasonal intercepts is to be included in the regression; **none** specifies that no deterministic variables are to be included; **const** specifies only a constant; **trend** specifies that a linear trend is to be included along with a constant term; **strend** specifies that a linear trend is to be included along with seasonal intercepts; **mult** specifies that seasonal intercepts along with seasonal trends are to be included (the case of multiplicative seasonality recommended by Smith and Taylor (1998)). If the **gls** option is selected, **det()** cannot be **none**.

maxlag(integer) specifies the maximum lag order to be included when augmenting the model with AR terms. Its default value is obtained from the following expression $Int\left[12(T/100)^{1/4}\right]$, where $Int[.]$ is the integer part, and T is the total sample size (T=SN). This option is used for all methods: AIC, MAIC, BIC, sequential *t*-test and fixed lag.

gls introduces the GLS detrending procedure proposed by Rodrigues and Taylor (2007) before applying the HEGY test. This option requires deterministic terms to be specified as **const**, **trend**, **seas**, **strend** or **mult**.

mode(string) specifies the method for selecting the augmentation lag. The string option may take the values **aic**, **maic**, **bic**, **seq** or **fix**. **aic** corresponds to the Akaike Information Criteria, **maic** to the Modified AIC, **bic** to the Bayesian Information Criteria, **seq** to the sequential t-test method and **fix** to a user specified lag. The default method is **maic**.

level(integer) indicates the significance level for the sequential t-test as a percentage. This option must and can only be used with **mode(***seq***)** option. The default value is 10, which corresponds to a 10% significance level.

residuals(string) generates a variable containing the residual terms.

noreg suppress the corresponding regression table, which is reported by default..

noac suppress the autocorrelation function (ACF) and partial ACF, as well as the Ljung-Box Q-statistics of the residuals. In case the option is not specified, the default value of ACF lags is equal to **maxlag**.

3.- Execution of the command

When we use monthly data, expressions (8) and (9) become:

$$\Delta_{12} y^{\xi}_{12t+s} = \pi_0 y^{\xi}_{0,12t+s} + \pi_6 y^{\xi}_{6,12t+s} + \sum_{j=1}^{5} (\pi_{1j} y^{\xi}_{1j,12t+s} + \pi_{2j} y^{\xi}_{2j,12t+s})$$
$$+ \sum_{j=1}^{k} d_j \Delta_S y^{\xi}_{12t+s-j} + e^{\xi}_{12t+s,k}$$

where

$$y^{\xi}_{0,12t+s} = y^{\xi}_{12t+s-1} + y^{\xi}_{12t+s-2} + y^{\xi}_{12t+s-3} + y^{\xi}_{12t+s-4} + y^{\xi}_{12t+s-5} + y^{\xi}_{12t+s-6} + y^{\xi}_{12t+s-7} + y^{\xi}_{12t+s-8} + y^{\xi}_{12t+s-9} + y^{\xi}_{12t+s-10}$$
$$+ y^{\xi}_{12t+s-11} + y^{\xi}_{12(t-1)+s}$$

$$y^{\xi}_{6,12t+s} = -y^{\xi}_{12t+s-1} + y^{\xi}_{12t+s-2} - y^{\xi}_{12t+s-3} + y^{\xi}_{12t+s-4} - y^{\xi}_{12t+s-5} + y^{\xi}_{12t+s-6} - y^{\xi}_{12t+s-7} + y^{\xi}_{12t+s-8} - y^{\xi}_{12t+s-9} + y^{\xi}_{12t+s-10}$$
$$- y^{\xi}_{12t+s-11} + y^{\xi}_{12(t-1)+s}$$

$$x^{\xi}_{1j,12t+s} = \sum_{q=0}^{11} \left[\cos(q+1)\frac{2\pi j}{S} \right] y^{\xi}_{12t+s-q-1} \qquad j = 1,2,3,4,5$$

$$x^{\xi}_{2j,12t+s} = -\sum_{q=0}^{11} \sin\left[(q+1)\frac{2\pi j}{S} \right] y^{\xi}_{12t+s-q-1} \qquad j = 1,2,3,4,5$$

$$t = 1,2,\cdots,N \quad s = -11,-10,-9,-8,\cdots,-1,0$$

In this case, the *hegy* command reports the following tests:

H_0	Roots	Frequency ω_j	$2\pi/\omega_j$ [a]	test	tail
$\pi_0 = 0$	One real root	0	∞	t[0]	Left tail
$\pi_6 = 0$	One real root	π	2	t[Pi]	Left tail
$\pi_{11} = \pi_{21} = 0$	Two complex conjugate roots	$\frac{\pi}{6}$	12	F[Pi/6]	Upper tail
$\pi_{12} = \pi_{22} = 0$	Two complex conjugate roots	$\frac{\pi}{3}$	6	F[Pi/3]	Upper tail

H_0	Roots	Frequency ω_j	$2\pi / \omega_j$ [a]	test	tail
$\pi_{13} = \pi_{23} = 0$	Two complex conjugate roots	$\frac{\pi}{2}$	4	F[Pi/2]	Upper tail
$\pi_{14} = \pi_{24} = 0$	Two complex conjugate roots	$\frac{2\pi}{3}$	3	F[2Pi/3]	Upper tail
$\pi_{15} = \pi_{25} = 0$	Two complex conjugate roots	$\frac{5\pi}{6}$	$\frac{12}{5}$	F[5Pi/6]	Upper tail

[a] Number of periods to complete a full Cycle

Additionally, the command reports the following joint tests:

H_0	Roots	Frequencies ω_j	test	Tail
$\pi_{11} = \pi_{21} = \pi_{12} = \pi_{22} = \pi_{13} = \pi_{23} = \pi_{14} = \pi_{24} = \pi_{15} = \pi_{25} = \pi_6 = 0$	All seasonal roots	$\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi$	F[seas]	Upper tail
$\pi_0 = \pi_{11} = \pi_{21} = \pi_{12} = \pi_{22} = \pi_{13} = \pi_{23} = \pi_{14} = \pi_{24} = \pi_{15} = \pi_{25} = \pi_6 = 0$	All roots in $\left(1 - L^{12}\right)$	$0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi$	F[All]	Upper tail

Finally, when we use quarterly data, expressions (8) and (9) become:

$$\Delta_4 y^{\xi}_{4t+s} = \pi_0 y^{\xi}_{0,4t+s} + \pi_2 y^{\xi}_{2,4t+s} + \pi_{11} y^{\xi}_{1j,12t+s} + \pi_{21} y^{\xi}_{2j,12t+s} + \sum_{j=1}^{k} d_j \Delta_S y^{\xi}_{12t+s-j} + e^{\xi}_{12t+s,k}$$

where

$$y^{\xi}_{0,4t+s} = y^{\xi}_{4t+s-1} + y^{\xi}_{4t+s-2} + y^{\xi}_{4t+s-3} + y^{\xi}_{4(t-1)+s}$$

$$y^{\xi}_{2,4t+s} = -y^{\xi}_{4t+s-1} + y^{\xi}_{4t+s-2} - y^{\xi}_{4t+s-3} + y^{\xi}_{4(t-1)+s}$$

$$x^{\xi}_{11,4t+s} = -y^{\xi}_{4t+s-2} + y^{\xi}_{4(t-1)+s}$$

$$x^{\xi}_{2j,12t+s} = -y^{\xi}_{4t+s-1} + y^{\xi}_{4t+s-3}$$

$$t = 1,2,\cdots,N \quad s = -3-2,-1,0$$

And the tests reported are:

H_0	Roots	Frequency ω_j	$2\pi / \omega_j$ [a]	Test	Tail
$\pi_0 = 0$	One real root	0	∞	t[0]	Left tail
$\pi_2 = 0$	One real root	π	2	t[Pi]	Left tail
$\pi_{11} = \pi_{21} = 0$	Two complex conjugate roots	$\frac{\pi}{2}$	4	F[Pi/2]	Upper tail

[a] Number of periods to complete a full Cycle

H_0	Roots	Frequencies ω_j	test	Tail
$\pi_{11} = \pi_{21} = \pi_2 = 0$	All seasonal roots	$\frac{\pi}{2}, \pi$	F[seas]	Upper tail
$\pi_0 = \pi_{11} = \pi_{21} = \pi_2 = 0$	All roots in $\left(1 - L^4\right)$	$0, \frac{\pi}{2}, \pi$	F[All]	Upper tail

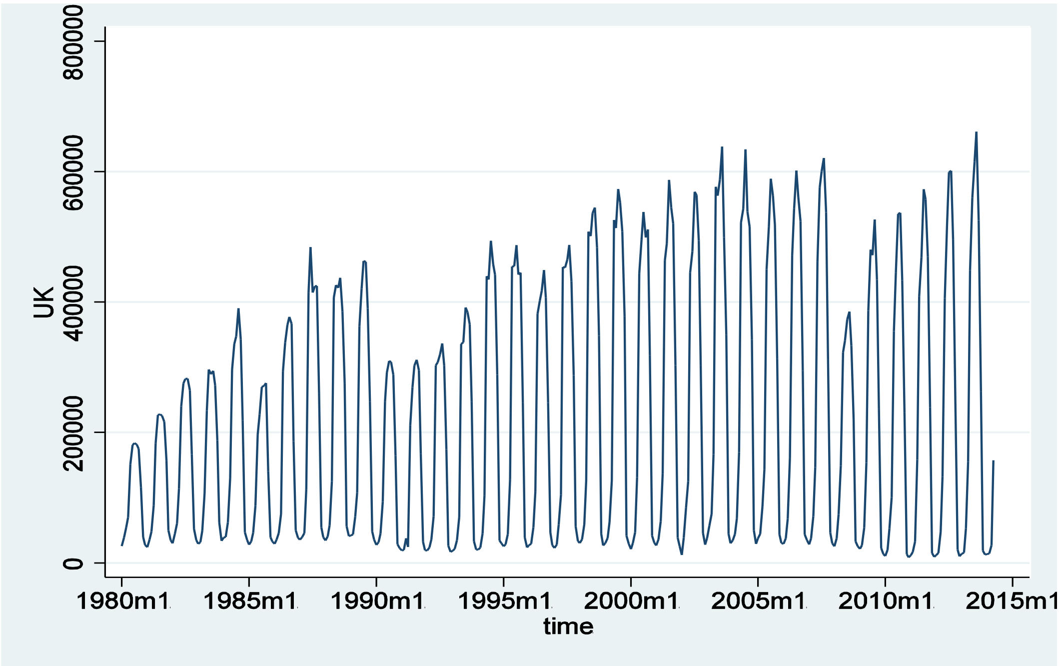
4.- Empirical Application

In this section, we use the monthly data of airport passenger arrivals to Mallorca from the United Kingdom as an illustrative example. The island of Mallorca is a summer tourist destination with peak tourist activity occurring in July and August. As such, the data in hand shows strong seasonal behavior. The results are based on a sample size that covers the period 1980M1 to 2014M4, which is 412 observations.

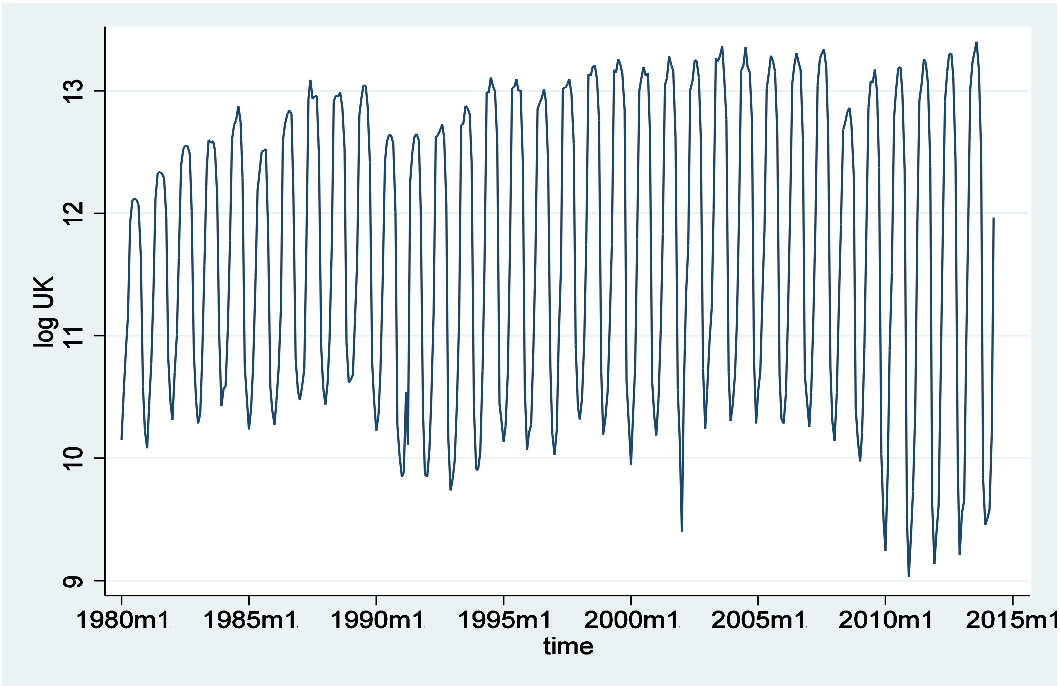
Tourism activity is highly influenced by seasonality. The reasons are obvious, and arrivals to most "sun and sea" destinations demonstrate high peaks during the summer season and low troughs in the winter. Graph 1 bellow depicts the evolution of the analyzed time series in levels (panel (a)) and natural logs (panel (b)). It is interesting to try to determine if the seasonality observed in this time series shows only a deterministic nature or if it is also caused by the presence of seasonal unit roots. That is, is the clear seasonal pattern observed in Graph 1 only deterministic in nature or is it also stochastic?

Graph 1. Levels and Natural log of UK arrivals to Palma de Mallorca, Spain

(a) Levels



(b) Natural Logs



Based on Graph 1, we decided to perform the analysis for our time series using natural logs of the data. Note that the evolution of panel (b) shows a less volatile evolution that the one depicted in panel (a), hence in order to avoid negative effects in the performance of the seasonal unit roots tests caused by a changing variance in our sample we decide to use natural logs. Table 2 reports the results obtained from our *hegy* command using OLS and GLS detrending considering the case of seasonal intercepts with seasonal trends for the deterministic part of the process. We report the results that were obtained when the order of augmentation was based on the MAIC criteria and then with the sequential method. In both cases, we use the default option to determine the maximum lag length with which to start, that is, ($k_{max} = Int\left[12(T/100)^{1/4}\right]$). In our case, it is 17 lags.

Graph 2 depict the evolution of the periodogram of the our time series in natural logs, note that we observe important peaks associated to 0, 1/12, 1/6, 1/4, 1/3, 5/12 and 1/2 in the frequency axis of the graph, that correspond to frequencies (ω_j) 0, π/6, π/3, π/2, 2π /3, 5π/6 and π that complete a full cycle after ∞, 12, 6, 4, 3, 12/5 and 2 months (periods $2\pi/\omega_j$) respectively. Based on graphs 1 and 2 we decide to use seasonal intercepts and trends (case 5) as we observe a clear seasonal behavior in our data and important peaks in the periodogram at seasonal frequencies that could be caused by seasonal trends.

Graph 2. Periodogram of Natural log of UK arrivals

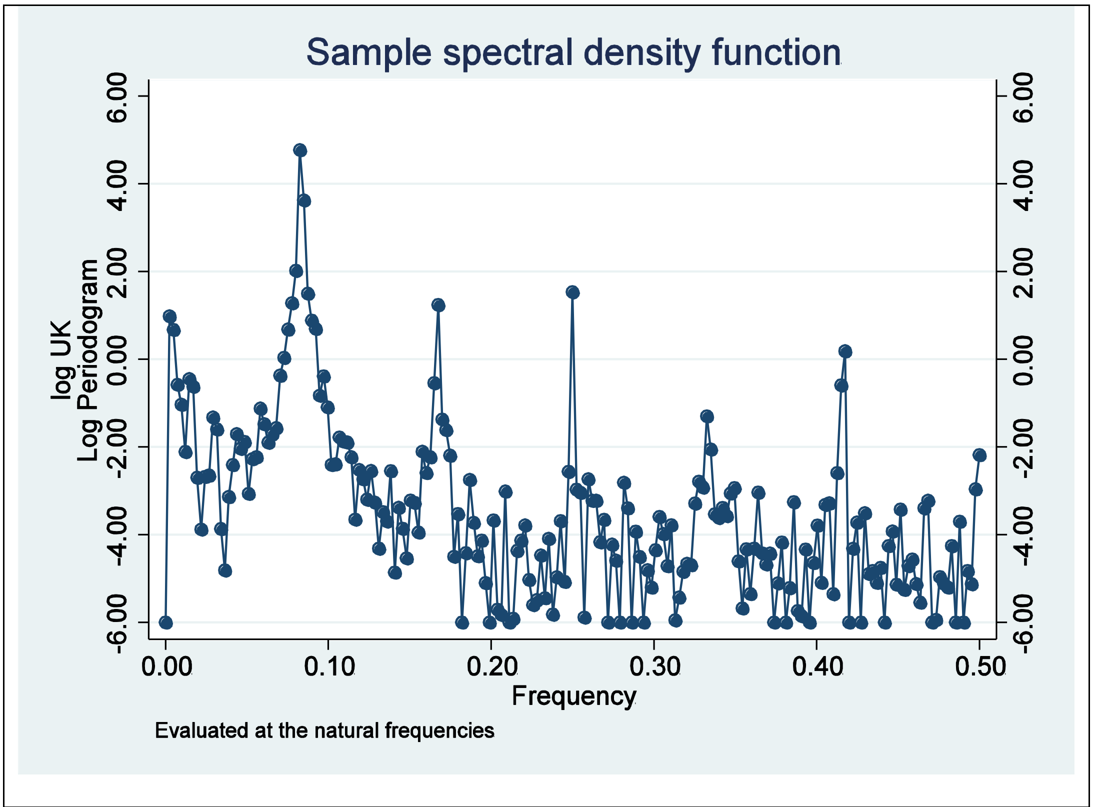


Table 2 is organized in 4 panels. Panel (a) collects the results obtained with OLS detrending and with the order of augmentation determined with the MAIC criteria, panel (b) collect the results with GLS detrending and the MAIC criteria and finally panels (c) and (d) collect the results where the order of augmentation it is determined with the sequential method and with OLS and GLS detrending respectively.

Table 2. Results of HEGY seasonal unit root tests

OLS, MAIC

. hegy luk, mode(maic) det(mult) noreg noac

HEGY Monthly seasonal unit root test for luk

Number of observations: 399
Deterministic variables: Seasonal intercepts and seasonal trends
Optimal lag selection method: Modified AIC
Lags tested: 17
Augmented by lags: 1

	Stat	1% critical	5% critical	10% critical
t[0]	-2.263	-3.896	-3.347	-3.065
t[Pi]	-4.643	-3.897	-3.347	-3.065
F[Pi/6]	9.696	11.798	9.356	8.206
F[Pi/3]	25.254	11.798	9.356	8.206
F[Pi/2]	15.996	11.798	9.356	8.206
F[2*Pi/3]	23.226	11.798	9.356	8.206
F[5*Pi/6]	19.944	11.798	9.356	8.206
F[All seas]	21.444	8.173	7.219	6.744
F[All]	20.136	8.076	7.160	6.703

GLS, MAIC

. hegy luk, mode(maic) det(mult) noreg noac gls

HEGY Monthly seasonal unit root test with GLS detrending for luk

Number of observations: 399
Deterministic variables: Seasonal intercepts and seasonal trends
Optimal lag selection method: Modified AIC
Lags tested: 17
Augmented by lags: 1

	Stat	1% critical	5% critical	10% critical
t[0]	-1.914	-3.691	-3.143	-2.865
t[Pi]	-4.672	-3.691	-3.143	-2.866
F[Pi/6]	8.858	9.740	7.578	6.583
F[Pi/3]	21.138	9.740	7.578	6.583
F[Pi/2]	13.275	9.740	7.578	6.583
F[2*Pi/3]	23.759	9.740	7.578	6.583
F[5*Pi/6]	19.775	9.740	7.578	6.583
F[All seas]	19.097	6.507	5.734	5.353
F[All]	17.836	6.455	5.714	5.348

OLS, SEQUENTIAL

. hegy luk, mode(seq) det(mult) noreg noac

HEGY Monthly seasonal unit root test for luk

Number of observations: 387
Deterministic variables: Seasonal intercepts and seasonal trends
Optimal lag selection method: Sequential at 10% level
Lags tested: 17
Augmented by lags: 13

	Stat	1% critical	5% critical	10% critical
t[0]	-2.159	-3.895	-3.345	-3.063
t[Pi]	-3.299	-3.895	-3.345	-3.064
F[Pi/6]	3.921	11.786	9.344	8.194
F[Pi/3]	10.335	11.786	9.344	8.194
F[Pi/2]	8.166	11.786	9.344	8.194
F[2*Pi/3]	10.895	11.786	9.344	8.194
F[5*Pi/6]	5.804	11.786	9.344	8.194
F[All seas]	8.763	8.179	7.221	6.745
F[All]	8.414	8.083	7.163	6.704

GLS, SEQUENTIAL

. hegy luk, mode(seq) det(mult) noreg noac gls

HEGY Monthly seasonal unit root test with GLS detrending for luk

Number of observations: 387
Deterministic variables: Seasonal intercepts and seasonal trends
Optimal lag selection method: Sequential at 10% level
Lags tested: 17
Augmented by lags: 13

	Stat	1% critical	5% critical	10% critical
t[0]	-1.950	-3.697	-3.149	-2.872
t[Pi]	-3.313	-3.697	-3.149	-2.872
F[Pi/6]	3.441	9.769	7.603	6.606
F[Pi/3]	7.289	9.769	7.603	6.606
F[Pi/2]	5.913	9.769	7.603	6.606
F[2*Pi/3]	11.046	9.769	7.603	6.606
F[5*Pi/6]	5.885	9.769	7.603	6.606
F[All seas]	7.333	6.541	5.764	5.382
F[All]	7.045	6.490	5.745	5.378

As mentioned in section 2, based on the proposal of Perron and Qu (2007) and del Barrio Castro, Osborn and Taylor (2015), the order of augmentation for the OLS and GLS detrending results is based on the same OLS detrended augmented HEGY regression. So, the order of augmentation in Table 2 with the MAIC criteria is the same as it is for both OLS and GLS detrending.

Note that the order of augmentation obtained with the MAIC criteria is one lag, but in the case of the sequential method, the order of augmentation is set to 13 lags. In order to decide if one lag is enough to purge the presence of serial correlation in the residuals of the HEGY regression, we do not use the *hegy* command's *noac()* option, hence the command reports the autocorrelation function (AC), partial autocorrelation function (PAC) and Ljung-Box Q-statistic associated to the residuals of the HEGY augmented regression. These results can be found in Table 3, the sample AC and PAC coefficients do not show evidence in favor of the presence of serial correlation in the residuals. Note also that with the Ljung-Box Q-statistic we clearly do not reject the null hypothesis of a white noise behavior, as the p-values reported in the last column of table 3 clearly show that the null hypothesis is not rejected at the 1%, 5% and 10% levels. Note for example, that in the case of say 12 lags the p-value associated to the Q-statistic is 0.9601, hence we need to work with a significance level of the 96% to reject the null that the AC coefficients from lag 1 to 12 are equal to zero. So we could conclude that one lag in the augmented-HEGY regression is enough to deal with serial correlation. In the part that follows we focus on the results obtained when using the MAIC criteria results (panels (a) and (b)).

Table 3. Correlogram of residuals

LAG	AC	PAC	Q	Prob.>Q
1	-0.0248	-0.0252	0.2474	0.6189
2	-0.0067	-0.0072	0.2657	0.8756
3	0.0246	0.0254	0.5110	0.9165
4	0.0513	0.0543	1.5775	0.8128
5	0.0228	0.0263	1.7887	0.8775
6	0.0448	0.0476	2.6051	0.8565
7	0.0492	0.0511	3.5917	0.8254
8	0.0176	0.0177	3.7180	0.8816
9	0.0156	0.0130	3.8180	0.9230
10	0.0124	0.0063	3.8817	0.9525
11	0.0462	0.0411	4.7627	0.9421
12	-0.0204	-0.0253	4.9345	0.9601
13	0.0543	0.0500	6.1560	0.9403
14	0.0083	0.0033	6.1844	0.9616
15	0.0019	-0.0035	6.1858	0.9765
16	-0.0846	-0.0948	9.1731	0.9061
17	0.0183	0.0057	9.3140	0.9299

Based on panels (a) and (b) of table 2, it is possible to check that the null hypothesis of the presence of a unit root at frequency zero is not rejected at all significance levels, note that the t[0] statistic is a left tail test and that in both cases (OLS and GLS detrending) the reported statistics are less negative than the critical values associated 1%, 5% and 10% levels, hence the statistics do not fail in the rejection area of the decision rule. And clearly, the null hypothesis of a unit root at frequency zero is not rejected. In the case of the t[Pi] test (the other left tail test) we are in the opposite situation. The test statistics with OLS and GLS detrending are clearly smaller than the critical values, hence we clearly reject the null of the presence of a unit root in frequency π. All the remaining tests (F[Pi/6] to F[All]) are upper tail tests, hence the null hypothesis is rejected when the test statistics is bigger than the critical value. In the remaining frequencies, the null hypothesis is rejected at all significance levels except in the case of the two complex roots associated with frequency $\pi/6$ (test F[Pi/6]) where the null is rejected at the 5% and 10% levels. Hence, it is possible to say that we only find evidence of one unit root at frequency zero, associated with pure trend behavior (oscillations that need infinity periods to complete a cycle), and the two complex conjugate unit roots associated with frequency $\pi/6$, which is associated with annual seasonality (oscillations that need 12 months to complete a full cycle). Therefore, based on the augmented HEGY procedure, when the lag length is determined by the MAIC criteria, it is possible to say that the UK tourist arrivals to Mallorca have non-stationary behavior associated with the zero frequency, and with the $\pi/6$ frequency (associated with seasonal oscillations that complete a full cycle after 12 months).

5.- Conclusions

In this paper we present the *hegy* command for Stata. This command allows for the HEGY procedure to be used in order to test for the presence of seasonal unit roots in quarterly and monthly data. The order of augmentation of the HEGY regression can be determined using the MAIC criteria, as was recently applied to the case of seasonal unit roots by del Barrio Castro, Osborn and Taylor (2016), the sequential method proposed by Hall (1994) and Ng and Perron (1995) or the use of the AIC and BIC criteria. In terms of the deterministic part of the process, the relevant cases of seasonal intercepts, seasonal intercepts with zero frequency trend and seasonal intercepts with trends are implemented. In addition to the usual OLS detrending, it is also possible to use GLS detrending as was recently proposed by Rodrigues and Taylor (2007). Finally, 1%, 5% and 10% critical values are reported by means of the response surfaces implemented in del Barrio Castro, Bodnar and Sansó (2015). The use of the command is illustrated with an empirical application using monthly UK airport arrivals to Mallorca.

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Chapter 4: Causality between outbound holiday and business tourism in EU countries

Causality between outbound holiday and business tourism in EU countries

Abstract

This research is aimed to find interdependency between business and holiday tourism activity using outbound tourism data. It will be useful for improving forecasts and might be considered while creating an effective tourism policy. Vector auto regression (VAR) models are estimated in order to measure the impact of one kind of tourism on another one. The interrelations are checked with the help of Granger causality test and validated with out of sample forecasting.The results are represented with impulse response function graphs. We have found that most of the considered time series are stationary and long run behavior of outbound tourism in the selected countries is deterministic.

Keywords: outbound tourism, vector autoregression (VAR) model, Granger causality test, ADF unit root test, impulse response.

1.- Introduction

Forecasting of tourism activity has become a significant tool for tourism economics. Forecasts on macroeconomic level help to create an effective tourism policy and allocate resources in an efficient way. The quality of forecast has a direct impact on the outcomes of the policy. Overestimation of demand may cause excess of capital investment and resulting inefficient input. Underestimation is not desirable because of congestions and lost opportunities. On the other hand, understanding of tourism behavior helps policymakers to choose the most efficient set of tools. Promotion of one kind of tourism might have positive indirect impact on other one. The opposite thesis also holds: restrictions in one activity might have a negative effect on other activity.

The purpose of our research is to establish the link between outbound holiday and business tourism for a large set of European countries. This link will imply that both of them are the part of one bigger process of tourism and that both have to be considered together for forecasting and designing policy actions. Vector Auto Regressive (VAR) modeling is applied in this study. It is a very flexible general approach in econometrics and adapts to the data in hand (Lütkepohl, 2006). That is why we consider the VAR modeling as an appropriate tool to explore the link between holiday and business tourism time series.

Tourism activity on macroeconomic level is usually measured with the help of aggregate numbers of tourist arrivals/departures or their expenditure. We will analyze the outbound tourism departure from EU countries through the time. Outbound tourism activity is more rarely analyzed than inbound activity. We consider that investigation of tourists' behavior in their origin might have interesting implications in promotion of destinations. If some considerable causality relations are observed, then it can be used in advertising strategy and more efficient budget allocation.

The aggregated tourism departures from EU countries are separated by the purpose of travel on business and holiday tourists. These two time series will be used as endogenous variables of VAR model. Instead of forecasting aggregated data, we will check causality relations between these two types of tourism and try to improve forecasts using these interdependencies. Understanding of causality relations between business tourism activity and holiday tourism activity will also help us to understand better tourism phenomena as a whole.

The structure of the paper is the following. First, we make an overview of existing studies about modeling tourism activity. This is important in order to allocate our investigations within the context of current research. More attention is paid to those papers that consider VAR modeling or contrast it with other techniques. Our study is an empirical one, that is why the next two sections describe the data sets and methodology used. Coherence and comparability of primary data is very important for comparability of results in our case. Next, the section of results presents the empirical findings of the study. It specifies deterministic and stochastic components of considered data as well as identifies and describes the causality between business and holiday tourism. Finally, the last section describes possible implementations of empirical findings in tourism policy and considers future investigations.

2.- Literature Review

VAR models are some times used in tourism literature in order to provide forecasts and estimate dependencies between tourist time series (De Mello et al, 2005; Dritsakis, 2004; Brierley, 2011; Baker et al 2011). However, the most frequently used data is inbound tourism arrivals. For instance, Santos (2009), in his article, tries to improve forecasts of tourism arrivals to Spain by disaggregating total tourism arrivals by 12 origins. These 12 time series are analyzed separately and forecasts are showing slight improvement comparing to aggregated forecasts. The author does not consider interdependency between these data, which can be captured with help of VAR model.

The causality relations are analyzed with help of VAR model tools in paper of Dritsakis (2004). The author concentrates his attention on dependencies between tourism arrivals and other economic indicators. VAR tools and techniques that estimate long term relations were successfully applied to tourism data.

Forecasting accuracy is a topic for debate in the papers of Brierley (2011) and Backer et al (2011). The authors describe the best techniques of economic forecasting that realized to be superior in online forecasting competition of tourism time series.

All these papers deal with inbound tourism using the time series of arrivals. Outbound tourism is rarely analyzed. Among the studies that deal with departures is the paper of Shen et al (2009). The authors try different techniques in forecasting of outbound tourism from UK. The paper identify that none of the econometric models is superior. It means that every time series should be checked and estimated separately in order to provide the best possible forecast. Another finding of the research is that seasonal treatment can improve forecast accuracy.

Some works compare the accuracy of VAR modeling with other techniques. De Mello et al (2005) compare it with Almost Ideal Demand System (AIDS) and establish long run dependencies between integrated variables: tourism shares, tourism prices and UK tourism budget. This study is dealing with outbound tourism from UK.

Our paper extends the data field of investigation to 27 European countries. Due to consolidated statistics of EU the results obtained from analyzing outbound tourism from one country can be compared with the results of others. Previous studies were mostly considering one country or destination and analyzed it with variety of techniques. Our study is aimed to test one particular issue of causality on big number of countries. Consistence of European statistics plays a crucial role in this case. The goal of the study is an improvement of tourism time series forecasts and technical analysis of factors that drive tourism phenomena as well.

3.- Data description

Primary data of outbound tourist departures is obtained from Eurostat (2012a, 2012b), the statistical department of European Commission. This collection covers outbound tourism from all members of EU and some bordering countries on the quarterly basis. "Outbound tourism" means residents of one country traveling to another country. The information on tourism concern trips of the population aged at least 15 years and the main purpose of which is holiday or business. The trip should involve at least one or more consecutive nights spent away from the usual place of residence. Trip characteristics are asked for each trip separately. Each trip has one main purpose but it may involve secondary motivations, several visits with possibly different purposes and several activities.

The quarterly data is available from 1994 onwards and refers to all member states of EU, as well as for Iceland, Liechtenstein, Norway, Switzerland, Albania, Bosnia and Herzegovina, Croatia and the former Yugoslav Republic of Macedonia. The most recent available data is referred to the fourth quarter of 2011. That is why the largest possible number of observations is 64. However, many of the countries do not have such long data sets. In order to estimate the model we defined the lower bound of available observations as equal to 30. The countries that did not fulfill this criterion for both series of holidays and business (Croatia, Estonia, Iceland, Malta, Netherlads and Sweden) are not considered in this study.

Tourism statistics are compiled by the competent national statistics authorities. Data are collected and compiled according to the Council Directive 95/57/EC (EUR-Lex, 2012) and with the Code of Practice applicable to all processes for collecting and compiling European statistics. After reception of the data, thorough quality control and validation checks are performed by Eurostat before releasing the data. Comparability of the data is reported as “very good” (Eurostat, 2012b). It refers both to geographical comparability (between countries) and over time comparability (between different years). The quality checks and validation rules include checks on the internal coherence across tables and across reference periods. Provisional quarterly data are revised 6 months after reference quarter. The advantage of the data provided by Eurostat consists in the same methodology of collecting it through different countries. This condition creates internal coherence and comparability, which is very important in investigating the common patterns of tourist behavior

4.- Methodology

For achieving our goal in estimating relations between holiday and business tourism we use an established methodology of VAR modeling.[2] First of all, the primary data are checked on stationarity with help of Augmented Dickey-Fuller test (ADF) with seasonal dummies, trend and allowing for structural breaks. The ADF tests the hypothesis of unit root presence in current time series, which is usually labeled as an I(1) variable. If the null hypothesis is rejected, the time series is stationary (I(0) variable). The testing equation reads:

$$\Delta y_t = \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \ldots + \delta_{p-1} \Delta y_{t-p+1} + \mu + \varepsilon_t \tag{1}$$

where μ denotes deterministic terms, seasonal dummies and dummies that represent possible structural breaks. The null hypothesis of non-stationary is $\gamma=0$, and the alternative of stationary is given by $\gamma<0$. If the null hypothesis is rejected for both holiday and business time series at the 5% significance level, then VAR model can be directly applied. The critical values are calculated according to MacKinnon (1996).

Given we are using quarterly data and that it would be possible to find seasonal integration the lowest level of augmentation in ADF unit root test is 4 (see Ghysels et al, 1994). A more robust way to deal with this problem would be to perform a HEGY seasonal unit root test (Hylleberg et al, 1990), but given the small sample we do not follow this approach. It is well known the low power of the HEGY test for short spawns of data.

Structural breaks may be present in some time series. For this reason we use dummy variable that takes value of one after the period of structural break and zero otherwise. Critical values of t-statistics for these data are obtained from paper Carrion-i-Silvestre et al (2004). We should notice that it is harder to reject the hypothesis of unit root if a structural break is present provided that critical values are much higher in this case.

The data is not considered as appropriate for VAR modeling if one of the time series is stationary and the other one is integrated. In this case the long-run behavior of both time series is so different that the simultaneous study of both time series has little interest.

We will apply Johansen co-integration test if the unit root is not rejected and both time series are I(1). This test prevents us from “spurious” regressions. In case of co-integration we will estimate Error Correction Model (ECM).

If both time series are stationary we are able to estimate VAR. The first step is to check the optimal lag length. This could be done with the lag exclusion Wald test or using information criteria such as Akaike (AIC) or Schwartz (BIC). Once the optimal lag of the VAR is established, it is possible to check the causality between holiday tourism and business tourism using Granger

[2] See Lütkepohl (2006) for a good introduction to this topic

causality test. The null hypothesis of this test is performed by testing the nullity of coefficients. The testing equation is given:

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \dots + a_m y_{t-m} + b_1 x_{t-1} + b_2 x_{t-2} + \dots + b_m x_{t-m} + \mu + \varepsilon_t \qquad (2)$$

were the “X” variable may cause (or not) “Y”, and the null hypothesis is $b_1=b_2=\dots=b_m=0$ that is, “X” does not (Granger) cause “Y”. If X does not (Granger) cause Y, then introducing X in the equation that explains Y will not lead to any improvements in forecasting. Rejection of this hypothesis will bring us to the idea that history of X contains some relevant information about current value of Y. In this case, we can say that “X” Granger-cause “Y” and validate the results with help of out of sample forecasting.

This methodology is applied to outbound tourism of 27 countries. Stating that datasets are coherent and comparable (Eurostat, 2012b), we can claim that test results are also comparable.

5.- Results

The primary data is tested for being stationary (I(0)) or integrated (I(1)) before estimating the VAR model. Table 1 in the appendix summarizes the results concerning the stationarity test. According to the results of Augmented Dickey-Fuller (ADF) test none of the pairs of time series realized to be I(1) at the same time. Consequently, ECM models were not used, and we may apply VAR modeling for level values.

Almost all selected time series are stationary. It implies that our data have a short memory. In this case, long term behavior of outbound tourism in selected countries is driven by deterministic or broken trends. Stochastic component is present only in short run. In other words, after external shocks, the value of tourism departures turns back to the deterministic path.

However, structural breaks occurred during observed period. They are introduced into the model with help of dummy variables. Critical value of t-statistics in this case is equal to -4.76 for 5% significance level (Carrion-i-Silvestre et al, 2004). According to the results reported in Table 1, Denmark, Estonia, Greece and Romania are affected by structural break in holiday tourism; Spain, France and Norway in business tourism; Ireland and UK in both.

In most of the countries except Austria, Belgium, Greece and Hungary outbound holiday and business tourism show the same trend behaviour. Departures from Germany, Spain, France, Luxembourg, Norway and United Kingdom are driven by ascending trend; Czech Republic, Denmark, Finland, Ireland, Italy, Latvia, Poland, Portugal, Romania, Slovenia and Slovakia are stationary around average level. Holiday tourism is driven by trend in Belgium and Greece; and business tourism follows a trend in Austria and Hungary.

The deterministic seasonal behavior is present more often in holiday tourism than in business tourism. It is represented with significance of seasonal dummies in ADF unit root test. 22 of 23 holiday tourism time series show deterministic seasonal behavior (the exception is Ireland). And only 7 of 21 cases for business tourism results that deterministic seasonality is significant (Belgium, Spain, Finland, France, Italy, Norway and Slovakia).

According to the results of ADF unit root test the countries that simultaneously have stationary time series are: Austria, Belgium, Germany, Denmark, Greece, Spain, Finland, France, Hungary, Ireland, Italy, Luxembourg, Latvia, Norway, Poland, Portugal, Romania, Slovenia and United Kingdom.

Having the unit root test results, we proceed to finding the optimal lag and estimating the VAR model. The optimal lag for different cases varies from one to four, which is essential for quarterly data (Table 2). Next, Granger causality is tested between pairs of tourism time series. The direction of causality for most of the countries (seven of ten) is: holiday outbound tourism Granger-causes business outbound tourism. These countries are Germany, France, Luxembourg, Norway, Poland, Romania and Slovenia. The Granger causality is established at 5% significance level.

Table 2. Granger causality tests results

Causality direction	Country	Optimal lag length	Causality direction (P-values of Granger causality test)	
			holiday	bussiness
Holiday tourism Granger causes Business tourism	Germany	4	0,170	**0,001***
	France	4	0,089	**0,002***
	Luxembourg	2	0,206	**0,018****
	Norway	1	0,654	**0,027****
	Poland	1	0,451	**0,029****
	Romania	4	0,233	**0,000***
	Slovenia	3	0,597	**0,000***
Business tourism Granger causes Holiday tourism	Greece	3	**0,000***	0,210
	United Kingdom	2	**0,017****	0,368
Causality in both directions	Hungary	4	**0,084*****	**0,000***
Causality is not proven	Austria	2	0,236	0,219
	Belgium	1	0,766	0,790
	Denmark	1	0,509	0,416
	Spain	1	0,942	0,738
	Finland	2	0,746	0,442
	Italy	2	0,679	0,463
	Portugal	1	0,133	0,485

Note: The null hypothesis of the test is: A (independent variable) does not Granger cause B (dependent variable). * - denotes rejection of the null hypothesis at 1% significance level, ** - denotes rejection of the null hypothesis at 5% significance level; *** - denotes rejection of the null hypothesis at 10% significance level; Optimal lag length according to AIC.

Greece and UK show an opposite causality to the majority. In their case, business tourism activity has an impact on holiday tourism. This result is interesting because of scale issue. Holiday tourism is significantly bigger than business. Business activity is on average 14,2% of holiday activity in absolute values. The intuitive assumption is that a smaller component will be driven by a bigger component in case that there is a relation between these kinds of tourism. But Greece and UK shows the opposite dynamics. It might have interesting consequences for tourism promotion policy that will be discussed in the section of policy implications.

Hungary shows causality in both directions. The results are more robust for holiday causing business (1% significance level) than for business causing holidays (10% significance level). That is why we refer this country to the major group.

Table 2 also shows the results of Granger causality tests for Austria, Belgium, Denmark, Spain, Finland, Italy and Portugal. These countries cope with the data requirements of VAR modeling (sufficient number of observation, the same order of integration), but did not succeed to reject null hypothesis of the Granger causality test. This implies that there is no link between holyday and business outbound tourism in these countries. Thus, this means that VAR modeling is not appropriate for the pairs of series of these countries and that univariate techniques can be considered for each of the time series.

After having considered the causality between pairs of tourism time series, we analyze the Impulse Respond Function of selected countries. The middle lines of Graph 1 in the appendix represent the impulse response of business tourism activity to a shock in holiday tourism. Shocks are standardized and equal to one standard deviation. Upper and lower lines show the bounds of two standard errors. The common effect of positive shock in holiday tourism during current quarter is positive response in business tourism during the next quarter. This response converges to zero during the future periods of time, a fact which agrees with the stationarity of the time series. Consequently, the negative shock has a negative response during the next period and converges with zero through time.

Results of response test for Germany and Romania are not so robust. The shocks have direct impact on the next quarter, but do not converge with zero later. It might indicate the integration of time series. However, the ADF test shows a certain stationarity of inputted data. These two cases should be investigated more in future studies.

The impact of business tourism on holiday tourism in UK and Greece show common converging behavior (Graph 2). The only difference is that the peak of response takes place not during the next quarter, but after six and nine month for UK. For Greece the lag is one year.

After testing outbound tourism of 27 EU countries, we can summarize that in seventeen cases Granger causality was not observed or data was not sufficient for this kind of analysis (Graph 4). For those countries where causality was estimated and according to its direction we may separate two groups. The major group contains the countries with holiday driven tourism and minor group with business driven tourism.

Having the causality relations estimated, we proceed to validation of the results. We use the accuracy of out of sample forecasts to check if this causality holds for future periods. The last four quarters are used as a validation period. The Table 3 represents Mean Absolute Percent Error (MAPE) statistics and smaller values are preferred. The forecasts of bivariate models are contrasted to the univariate model where deterministic terms and autoregressive lag are remained the same. Forecasting improvements are observed in such countries as France, Luxembourg, Norway, Poland, Slovenia, Hungary, Greece and UK. However, Romania and Germany are not validating the causality relation. These two countries are performed better with univariate models.

6.- Policy implications and conclusions

The hypothesis about causality between holiday and business tourism processes holds in approximately one third of the cases. It implies important consequences for these countries while forecasting tourism activity and creating tourism policy.

Causality links between outbound holiday activity and outbound business activity brings new information for building forecast models. Disaggregating of the data may lead to improvements in forecasting and policy simulation. These results are consistent with the investigation of Santos (2009) that uses disaggregating technique applied to tourism in Spain.

Endogeneity of the processes also implies lessons for policy makers. Outbound holiday and business tourism are the part of one bigger process. The shocks in one kind of tourism will have a corresponding response in another one. This relation may be successfully used in promotion policy. Moreover, most of the time series are stationary. As a result, they perform a deterministic behavior in the long run. Such behavior implies that shocks resulting from policy actions will have the effect limited in time. That is this response will fade out during the future periods of time. In order to keep permanent impact, the promotion policy should be continuous. The only way to shift permanently these figures is to introduce very noticeable event and provoke a structural break.

Future investigations might be spread to inbound tourism. Inbound and outbound tourism are the two sides of the same process. That is why there is certain chance that results of our study might also hold for inbound tourism respectively. The consolidated and coherent data of outbound tourism provided by Eurostat is like a pool of recourses for making first selection. The result of primary filtering is a list of eigth countries with proven causality between kinds of tourism. Now inbound tourism from these countries may be analyzed more precisely. Current results make the data selection for future investigations

If further studies will prove that the same causality holds for inbound tourism, it might be an effective instrument for promotion of destinations. For instance, holiday tourists from United Kingdom hold a significant share of tourism market in the Balearic Islands. Taking into account that business tourism has a positive impact on holiday tourism, promotion of Mallorca's conference halls, particularly for UK residents, will lead to growing arrivals of holiday tourists to the destination.

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Appendix

Table 1. ADF unit root test results.

Country	Type of tourism	t-statistic	Critical values, 5% level	Presence of trend	Seasonal dummies significance	Structural break presence	Order of integration
Austria	Holiday	-6,96	-2,92	No	Yes	No	I(0)
	Business	-5,87	-3,51	Yes	No	No	I(0)
Belgium	Holiday	-3,58	-3,51	Yes	Yes	No	I(0)
	Business	-3,66	-2,92	No	Yes	No	I(0)
Czech Republic	Holiday	-3,26	-4,76	No	Yes	2008Q1 - 2011Q4	I(1)
	Business	-3,89	-2,95	No	No	No	I(0)
Germany	Holiday	-7,75	-3,49	Yes	Yes	No	I(0)
	Business	-5,29	-3,50	Yes	No	No	I(0)
Denmark	Holiday	-7,07	-4,76	No	Yes	2005Q1-2011Q4	I(0)
	Business	-5,53	-2,91	No	No	No	I(0)
Greece	Holiday	-5,19	-4,76	Yes	Yes	2004Q1 - 2011Q4	I(0)
	Business	-4,70	-2,91	No	No	No	I(0)
Spain	Holiday	-20,55	-3,52	Yes	Yes	No	I(0)
	Business	-9,13	-4,76	Yes	Yes	2005Q1-2011Q4	I(0)
Finland	Holiday	-5,79	-2,91	No	Yes	No	I(0)
	Business	-3,46	-2,91	No	Yes	No	I(0)
France	Holiday	-14,50	-3,54	Yes	Yes	No	I(0)
	Business	-9,01	-4,76	Yes	Yes	2008Q4 - 2011q4	I(0)
Hungary	Holiday	-5,81	-2,96	No	Yes	No	I(0)
	Business	-4,44	-3,56	Yes	No	No	I(0)
Ireland	Holiday	-7,44	-4,76	No	No	2004Q1 2011Q4	I(0)
	Business	-17,44	-4,76	No	No	2004Q1 2011Q4	I(0)
Italy	Holiday	-21,20	-2,92	No	Yes	No	I(0)
	Business	-6,23	-2,92	No	Yes	No	I(0)
Luxembourg	Holiday	-4,28	-3,49	Yes	Yes	No	I(0)
	Business	-7,21	-3,50	Yes	No	No	I(0)
Latvia	Holiday	-10,87	-2,97	No	Yes	No	I(0)
	Business	-3,58	-2,97	No	No	No	I(0)
Norway	Holiday	-14,17	-3,53	Yes	Yes	No	I(0)
	Business	-5,53	-4,76	Yes	Yes	2009Q1 - 2011Q4	I(0)
Poland	Holiday	-12,39	-2,97	No	Yes	No	I(0)
	Business	-5,43	-2,97	No	No	No	I(0)
Portugal	Holiday	-12,23	-2,93	No	Yes	No	I(0)
	Business	-5,69	-2,94	No	No	No	I(0)
Romania	Holiday	-5,81	-4,76	No	Yes	2007Q2 - 2011q4	I(0)
	Business	-4,76	-2,97	No	No	No	I(0)
Slovenia	Holiday	-17,81	-2,95	No	Yes	No	I(0)
	Business	-7,88	-2,95	No	No	No	I(0)
Slovakia	Holiday	-14,03	-2,95	No	Yes	No	I(0)
	Business	-3,11	-4,76	No	Yes	2008Q4 - 2011q4	I(1)
United Kingdom	Holiday	-7,05	-4,76	Yes	Yes	2000Q1 - 2005Q1	I(0)
	Business	-7,63	-4,76	Yes	No	2000Q1 - 2005Q1	I(0)

Note: t-statistics is calculated according to ADF unit root test with seasonal dummies; critical values at 5% level are calculated according to MacKinnon (1996), for time series with structural break critical values are taken from Carrion-i-Silvestre et al (2004); Trend: “yes” represents the significance of deterministic trend at 5% level; Seasonal dummies: “yes” represents the statistical significance of season dummies at 5% level, Q4 is a reference quarter and is included in the intercept; Structural break presence represents a statistical duration of structural breaks.

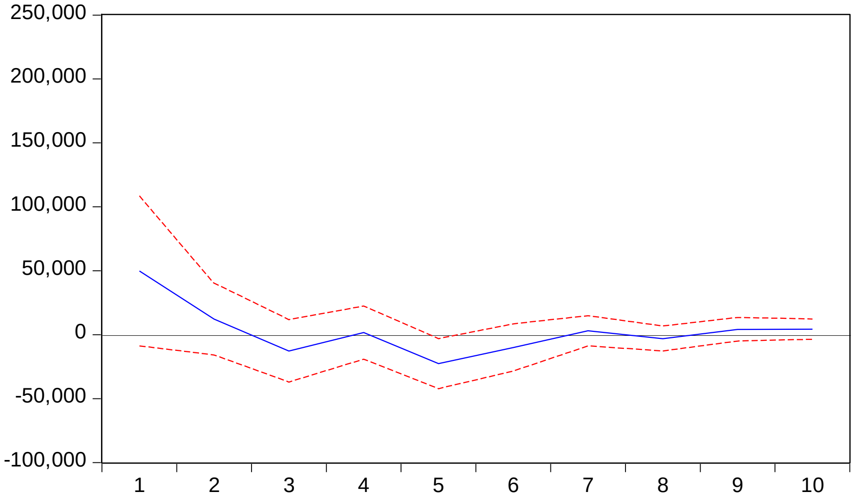
Table 3: Out of sample forecasts statistics, MAPE

Business tourism		
	bivariate	univariate
France	5,59	8,22
Luxembourg	8,83	10,42
Norway	6,08	6,55
Poland	8,33	12,65
Romania	12,46	11,9
Slovenia	12,98	23,34
Hungary	68,5	68,74
Germany	21,68	18,05
Holiday tourism		
	bivariate	univariate
Greece	18,52	22,85
UK	5,03	7,29

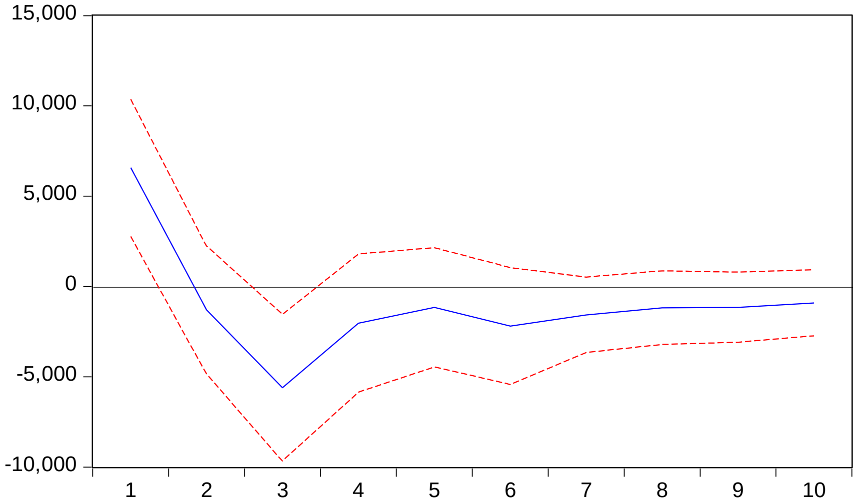
Note: Mean Absolute Percentage Error (MAPE) is calculated for bivariate model and contrasted to univariate model. The deterministic terms and autoregressive lag are remained the same for both models

Graph 1. Impact of Holiday outbound tourism on Business outbound tourism: impulse response graphs

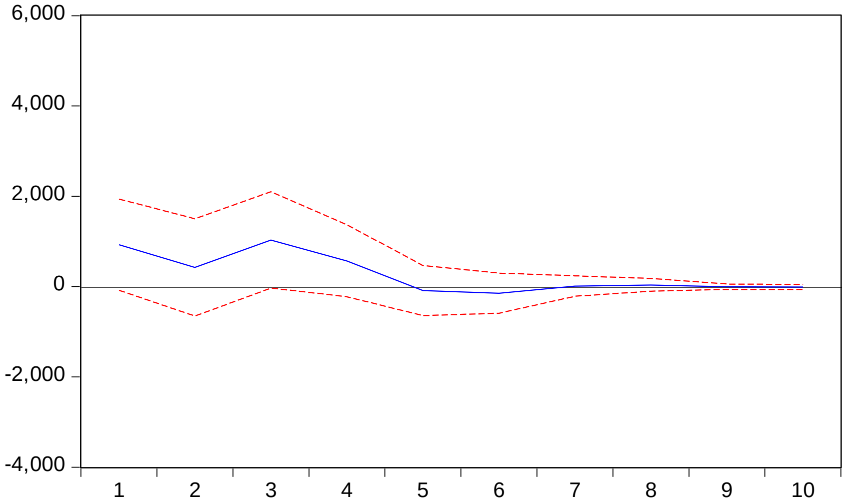
Response of FR_B to FR



Response of SI_B to SI

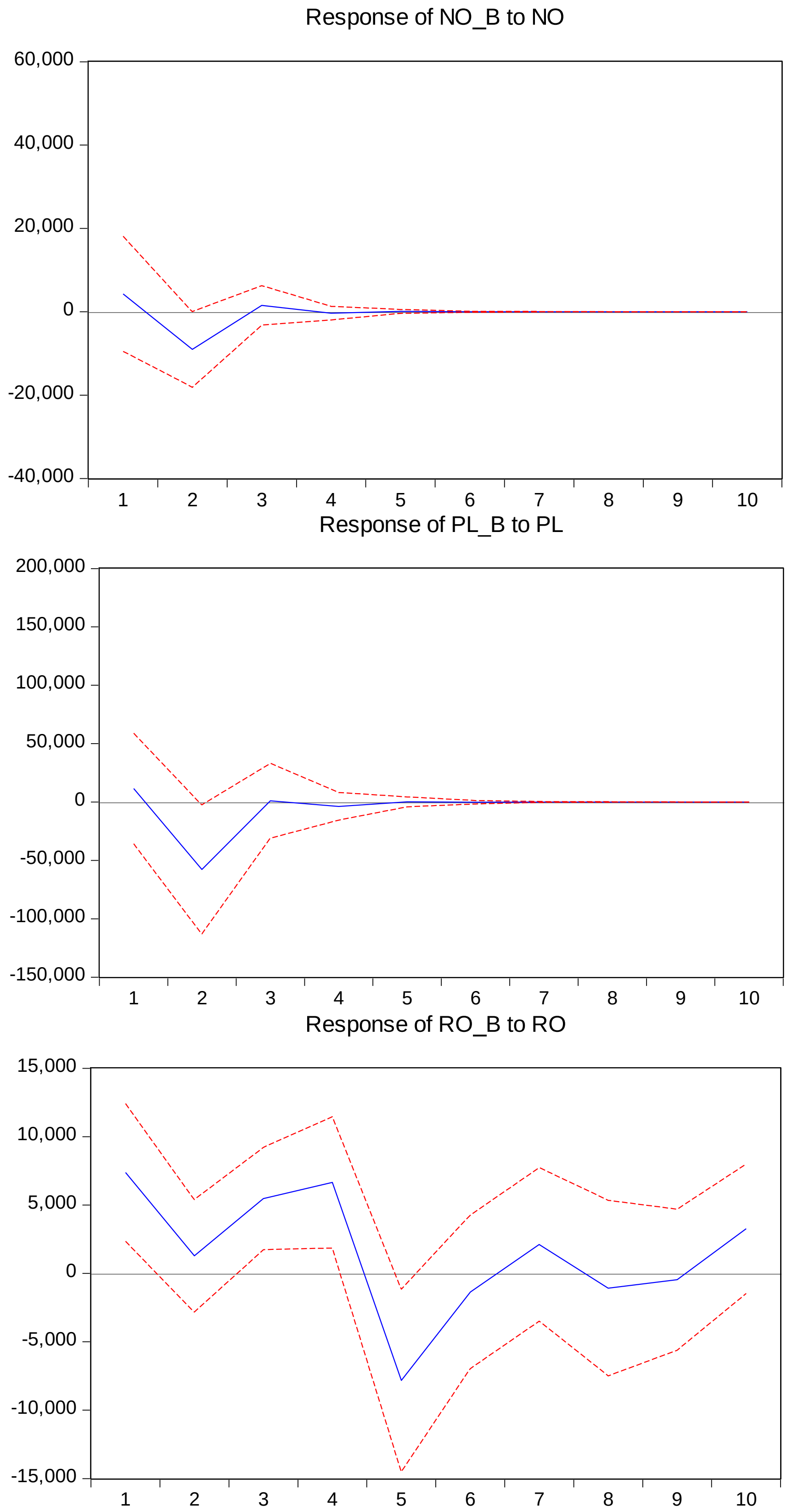


Response of LU_B to LU



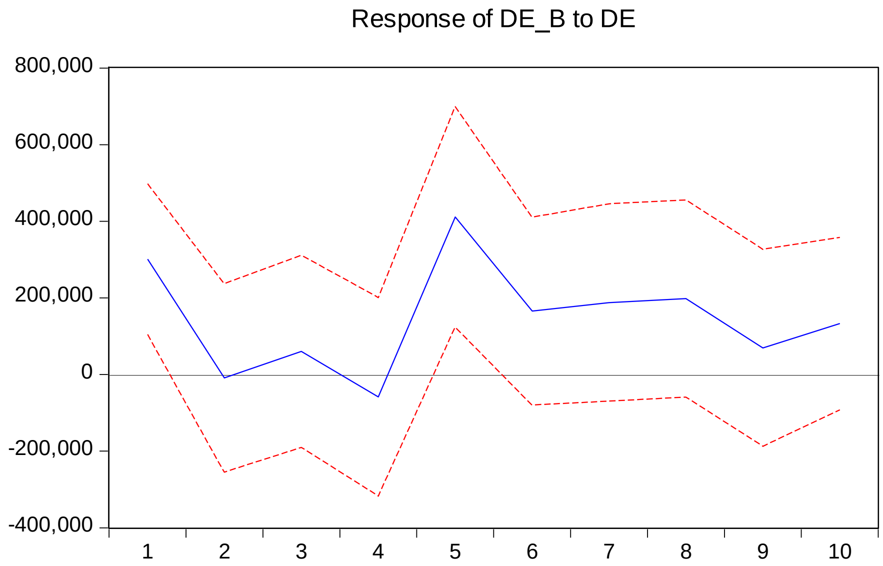
(Continued)

Graph 1. *(Continued)*



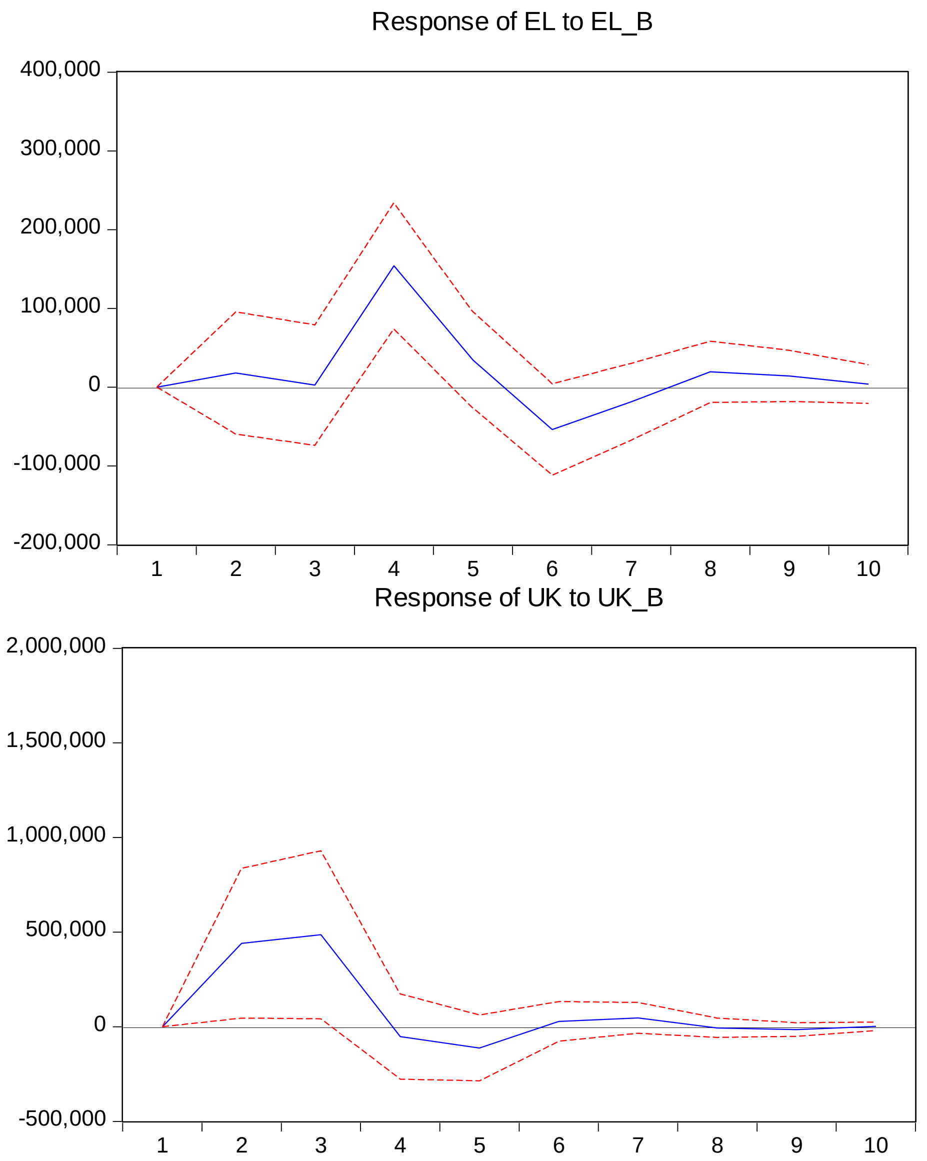
(Continued)

Graph 1. *(Continued)*



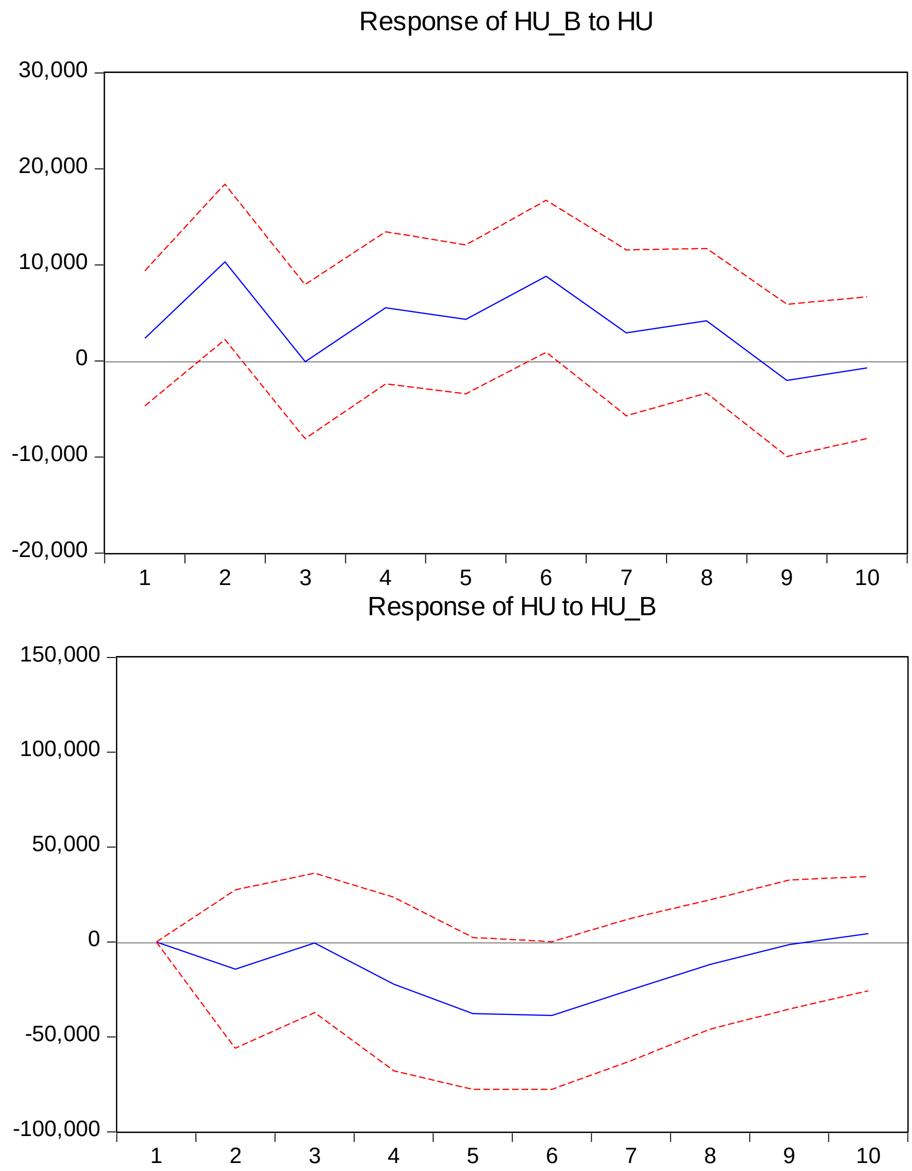
Note: Country abbreviation for holiday tourism: FR – France, SI – Slovenia, LU – Luxembourg, NO – Norway, Pl – Poland, RO – Romania, DE – Germany; suffix "_B" denotes "business" tourism

Graph 2. Impact of Business outbound tourism on Holiday outbound tourism: impulse response graphs



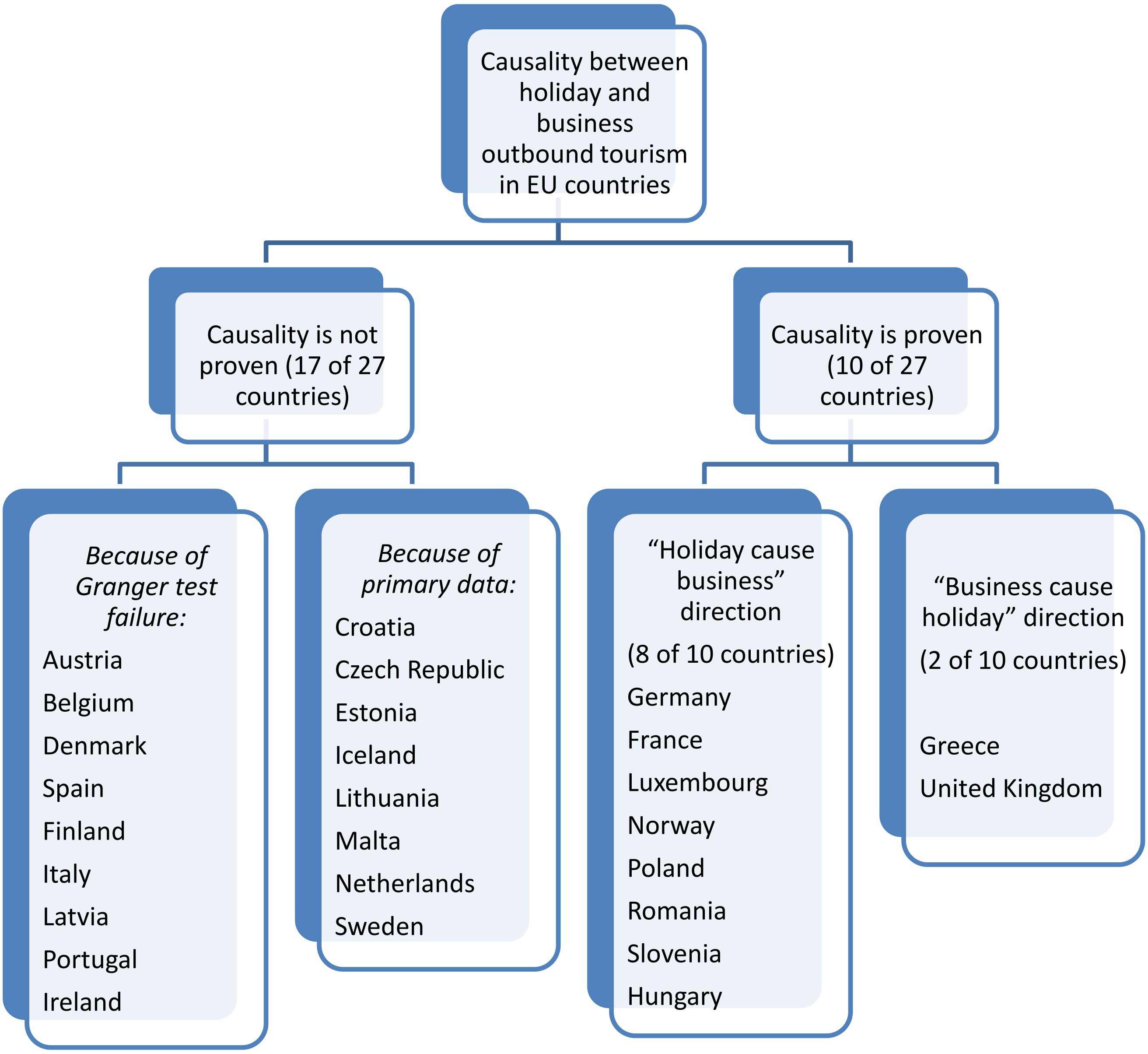
Note: Country abbreviation for holiday tourism: EL – Greece, UK – United Kingdom; suffix "_B" denotes "business" tourism

Graph 3. Impulse response graphs for Hungary.



Note: Country abbreviation for holiday tourism: HU – Hungary; suffix "_B" denotes "business" tourism.

Graph 4. Groups of countries according to the causality presence.



Conclusions

The thesis deals with seasonal tourism time series and develops econometric tools for modeling and forecasting. The study follows the context of HEGY seasonal unit root test with OSL and GLS detrending. In the first essay the *p*-value calculator is developed under Gauss programming language using extensive Monte Carlo simulations. The second essay transfers the results from the previous paper into Stata. In the third essay we investigate the causality relations between holiday and business tourism activity in the bunch of European countries with help of VAR modeling.

The data used in the study is collected at monthly and quarterly frequencies, which are the most common for tourism time series. The sample sizes of considered data vary from ten to forty years. The inbound tourism activity is represented with airport passenger arrivals to Palma de Mallorca and the outbound tourism is studied with help of airport departures from 27 European countries.

The first paper implements the approach of MacKinnon (1996) in order to obtain the response surface of test statistics for seasonal unit root test with OLS and GLS detrending. As the result of the study we have obtained a matrix of coefficients and a subroutine, which return the *p*-value for *t* and F-type test statistics. The subroutine has six options of deterministic terms, two frequencies of the data and two types of detrending procedure. The empirical part of the paper tests the time series of airport passenger arrivals from Germany to Palma de Mallorca. We found out that seasonal unit roots are present at zero and π/6 frequencies, which corresponds to long-run and one-year seasonality.

Since the *p*-value calculator is available as a subroutine for Gauss, we have accomplished the verification of our results with the previous studies. There are two comparable types of previous research. The first type is the papers that report the tables with critical values (Hylleberg et al., 1990; Franses and Hobijn, 1997; Smith and Taylor, 1998; Rodrigues and Taylor, 2007). Such critical values have been inputted directly into the Gauss function and the resulting *p*-values are compared with the nominal sizes. For such kind of verification we consider the difference on the second decimal to be significant and report it in the comparative tables. The second type of research reports the *p*-value calculating functions (Diaz-Emparanza, 2014). In this case we have inputed the same value of the test statistics into both functions and compare the resulting *p*-values.

The match of results with tabled values is high across all the types of the tests and deterministic terms. The original paper of Hylleberg et al. (1990) report the statistic for quarterly data and the match of results is 88.3%. The research of Franses and Hobijn (1997) covers both monthly and quarterly frequency. The match of results is 99.0% and 99.5% correspondingly. Smith and Taylor (1998) deal with quarterly data and our results perfectly match (100%). The last study that deals with the tables of critical values is the paper of Rodrigues and Taylor (2007). The authors report the statistics for the HEGY seasonal unit root test with GLS detrending at the quarterly frequency. The match of results is 97.8%. The critical values of HEGY test with GLS detrending for monthly data have not been reported in previous studies, therefore cannot be justified.

The critical value comparison verifies both our results and previous research. Even though that the match of results is high across the tables, there are some points where the difference is notable. The comparative tables of the first paper report these cases.

The response surface of HEGY seasonal unit root test statistics is constructed following the methodology of MacKinnon (1996), which does not consider the augmenting lags. The possibility for future investigations is the inclusion of autoregressive lags into the model. It will be more time consuming, while every additional lag will be associated with four weeks increase in computing time. However, there are several ways to deal with this problem like using greater number of computers or discarding the smallest nominal sizes together with the number of simulations.

The inclusion of autoregressive lags is directly associated with the form of the response surface. If the augmentation order has to be included, then the original equation will not be valid any more. The asymptotic properties of the formula are consistent with the asymptotic properties of the numerical distribution in current methodology. If we include the new elements into the formula like in Diaz-Emparanza (2014), then the edges of the surface beyond the tested intervals may become unstable. However, for practical purposes we are mostly interested in the fitted intervals. Within these bounds the new formula may perform well.

Current methodology suggests a three dimensional area for the response surface, where the *p*-value is a function of test statistics and sample size. The response surface is represented by 221 two dimensional functions. Every function consists of 4 elements, which in total gives us 884 coefficients that describe the response surface for the particular deterministic terms on the defined interval. The subject for future investigation might be a functional form in general. We can try to fit other kind of equation with number of elements up to 884. There is a certain chance that there is a function that can describe the response surface with enough accuracy and smaller number of coefficients. It will open a possibility for easy one step calculation of *p*-value instead of four step interpolation procedure. However, the asymptotic properties of such model most probably will be lost.

In this research we are working with the frequency of four and twelve. It corresponds to quarterly and monthly time series. Sometimes tourism time series are observed at other time frames like weeks, days, hours etc. Diaz-Emparanza (2014) has provided a response surface regression approach for original HEGY seasonal unit root test for any periodicity. In the future investigations we can extend the GLS detrending case to any periodicity as well.

There is a certain interest in the seasonal unit root tests in other programming environments. The *p*-value calculator can be transferred to other statistical languages like R and Gretl in the future.

The second paper develops the HEGY seasonal unit root test with OLS and GLS detrending in Stata. It follows the general context of the first paper. The subroutine includes a number of useful options, like two types of detrending procedures, four different information criteria for determining the optimal augmentation lag, a choice of six possible deterministic terms and a test for serial correlation in residuals.

The test statistics is reported together with critical values, which are calculated from the corresponding response surfaces. The subroutine reports the critical values at 1%, 5% and 10% significance level. We tried to introduce the *p*-value calculator into the test, but it was not possible to do it with the means of *.ado* command. The methodology of *p*-value calculation involves almost 80 thousand coefficients. The matrix of this size is far beyond the limits of *.ado* command in the academic version of Stata/IC 11, which we are currently using. However, this feature is a very handy tool for the users and it is a subject for improvements in the next versions of the command. The possible way of solving this issue is writing the *p*-value calculator in the matrix programming language of Stata (Mata). Another option is to shrink the reported *p*-value to the interval from 1% to 10% significance level, which is the most interesting for the researchers. The *p*-values bigger than 10% or smaller than 1% may be dropped from the output of the command.

The HEGY seasonal unit root test with GLS detrending might also be extended to any periodicity. Currently, the command works only with monthly and quarterly data. Other frequencies might be considered. However, the problem of critical values will arise, while there is no study at the moment that covers GLS detrending case for the frequencies other than four or twelve. The extension of the test with additional time frames should either include response surface study or follow after the existing one. Another improvement might be the inclusion of user defined dummy variables that will take into account the possible structural breaks. However, in this case we will also face the problem of critical values.

The third paper deals with a data set of 27 European countries. This research tests the hypothesis about causality relations between holiday and business tourism. The time series of airport departures are split into two categories by the purpose of travel. The Granger causality test reveals the relation and the impulse response graph demonstrates its dynamics.

The first result of the study is that the most of the time series are stationary. It implies the "short memory" of considered data. The long time behavior is driven by deterministic or broken trend and the impact of external shock is limited in time. The holiday tourism is more influenced by seasonality than business tourism. This pattern is represented by the statistical significance of seasonal dummies in the unit root tests.

The Granger causality is observed in ten of 27 countries. The prevailing direction is that holiday tourism causes business tourism (eight of ten cases). Such tendency seems to be natural, while the weight of business tourism is about 14,2% of holiday tourism on average.

Impulse response graphs demonstrate the effect of one standard deviation change in the related time series. Mostly, the impact is positive during the following quarter and fades out with time. Stability of the results is verified with out of sample forecasting. Mean average percentage error is calculated for bivariate and univariate models, where deterministic terms and augmentation lag remain the same. The improvement of bivariate forecast is observed in nine of ten cases. Therefore, we conclude that the relations between holiday and business tourism can be used for forecast improvements.

The next step for investigations would be a deeper exploration of tourism activity from the countries with revealed causality to the particular destination. If the arrivals demonstrate similar relations, then it might be used in tourism policy with a high degree of certainty.

Disaggregation of the data gives an additional insight into the tourism phenomena. Time series of arrivals may also be studied using this methodology. Moreover, it is possible to change the criteria for disaggregation like the length of stay, mode of transport or gender.

The important finding of this study is that the outbound tourism activity is a stationary process with a very few exceptions. This feature has consequences for the tourist destinations. The size of the market is constant in the short run and the destinations have to compete for the tourist’s loyalty.

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