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GADE

ESTIMATION OF THE TEMPORARY STRUCTURE OF INTEREST RATES

Based on public debt information provided by Bank of Spain

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TABLE OF CONTENTS

1 Summary

The aim of this work is to estimate the temporary structure of interest rate (ETTI), based on the information on public debt provided by the Bank of Spain, applying the Nelson and Siegel (1987) model. Moreover, as additional targets, it will be discussed the different theoretical macroeconomic and financial methodologies and approaches used to estimate the ETTI over the years. Nelson and Siegel methodology will be explained as the reasons why it is the chosen method of estimation.

In addition, this work goes beyond the knowledge acquired with the degree in Business Administration, deepening in complex concepts like the temporary structure of interest rates itself.

Furthermore, due to actual financial crisis, it has been decided to choose two periods of time (2006 and 2011) in order to be able to compare the changes happened in ETTI between pre-crisis period and in-crisis one.

Resumen

El objetivo principal de este trabajo es estimar la estructura temporal de los tipos de interés (ETTI), basada en la información de deuda pública facilitada por el Banco de España, utilizando el modelo de Nelson y Siegel (1987). Además, como objetivos adicionales, se comentarán los diferentes teorías macroeconómicas y metodologías financieras y los enfoques utilizados a lo largo de los años para estimar la ETTI.

Este trabajo va más allá de los conocimientos adquiridos en el grado de Administración y Dirección de Empresas, profundizando en conceptos complejos como la estructure temporal de los tipos de interés en sí misma.

Además, debido a la actual crisis, se ha decidido seleccionar dos periodos de tiempo (2006 y 2011) con el objeto de ser capaces de comparar los cambios ocurridos en la ETTI entre los periodos pre-crisis y durante-crisis.

2 Introduction

Currently investors use financial assets as risk hedging instruments and it is, therefore, important to determine the process by which these are valued in the capital market. A way to get this information is through the zero coupon yield curve that is build from the temporary structure of interest rates (ETTI, in Spanish).

The Temporary Structure of Interest Rates (ETTI) provides economic information to the agents involved in the stock market to serve as an instrument of analysis between the performance and the maturity for assets that have the same risk. Usually it is used assets issued by the state because they have no risk of insolvency or liquidity problems. The question arises: Why is it important the ETTI for economic agents and for analysts of financial markets? In building the ETTI, we get the yield curve, which is associated with real economic activity, inflation and interest rates that serve as a guide to agents on what is the most appropriate market strategy that allows greater profits. Similarly, estimating a temporary structure of interest rates allows analyzing the impact of monetary policy and its transmission mechanisms, from a macroeconomic perspective, and determines the valuation of assets and the assessment of risk associated with investment decisions providing guidance on the design of hedging strategies, from a financial perspective.

In reviewing the literature there is a wide range of theoretical models with macroeconomic and financial approaches that calculate the term structure and try to infer on how we value those assets in the capital market, of these we will pay special attention to the model of Nelson and Siegel (1987), which is known for its ability to predict the price of financial assets under a mathematical formulation composed of variables associated with the formation of expectations on the part of the agents.

The main objective of the work, as it has been said, is to simulate the model of Nelson and Siegel (1987). As additional targets, we will take a look on the different theoretical macroeconomic and financial methodologies used to estimate the ETTI, summarizing the different models that have been developed around the yield curve.

3 International methodological development

Over time, the methodology development for building the temporary structure has a variety of models. These have arisen from different methodologies and theoretical approaches that support the building of the temporary structure.
From the theoretical point of view there are different developments that can be classified as follows:

- The financial approach, to determine the valuation of assets expressed according to maturity, found in Vetzal (1994), Campbell, Lo and McKinlay (1997), Moreno (2000).

- The macroeconomic approach, based on the ratio of the spot interest rates, forward interest rates and economic variables as found in Shiller (1990) who also supported with results such econometric models.

- And finally the approach linking the financial and macroeconomic ones, found in Pegan, Hall and Martin (1996).

In relation to estimation methods of ETTI are found, on one hand, the econometric estimation methods or parametric which require an estimation process, and on the other hand, methods that are not econometric or nonparametric that not require a process of estimation to obtain spot rates that generate a discrete temporary structure and can be continuously transformed by interpolation methods.

Within the literature of econometric estimation methods for ETTI are the recursive or bootstrapping method as presented in Caks (1977) and the approximation of spot rates for the internal rate of return (IRR) by Buse (1970). It is found that these methods are suitable for estimating the short term ETTI, as for long term estimation can be biased in the approximation of the yield curve.
Meanwhile, for non econometric models are found the Splines models and the discount function with constraints. Splines models generate simple functional

forms but without offering soft shapes for longer periods and therefore does not guarantee positive forward rates, these models are found in McCulloch (1971, 1975b), Vasicek and Fong (1982) and Contreras and Navarro (1993). The discount function models with constraints assume that the forward rate converges asymptotically as found in Nelson and Siegel (1987) and Svenson (1994); in these models the level and rate of convergence are parameters to be estimated. It should be noted that the Svenson model (1994) is an extension to the model of Nelson and Siegel (1987) with an additional term that offers greater flexibility to the yield curve.

Then, there is huge variety of methodologies and theoretical approaches associated with ETTI, despite this, in Nuñez (1995) it is concluded that the most appropriate methods for estimating the temporary structure in the market of public debt are those proposed by Nelson and Siegel (1987) and Svensson (1994). According to this, our interest is to focus this work towards the model of Nelson and Siegel (1987) since through this model we found soft and flexible temporary structures.

4 Methodologies applied for the ETTI's estimation by the Central Bank

In this chapter, we present a summary table of the models used to estimate the ETTI by central banks of countries around the world:

Central Bank	Methodology[1]	Estimation available	Frequence	Error minimization	Width
Belgium	SV NS	01.09.1997	Daily	Weighted price	From 2 days to 6 years
Canada	SV	23.06.1998	Daily	Weighted price	From 1 to 30 years
Finland	NS	03.11.1998	Daily	Weighted price	From 1 to 12 years
France	SV NS	03.01.1992	Weekly	Weighted price	Up to 10 years
Germany	SV	07.08.1997 01.01.1973	Daily Weekly	Yield	From 1 to 10 years
Italy	NS	01.01.1996	Daily	Weighted price	Up to 10 years
Japan	SS	29.07.1998	Weekly	Prices	From 1 to 10 years
Norway	SV	21.01.1998	Monthly	Yield	Up to 10 years
Spain	SV NS	01.01.1995 01.01.1991	Daily Monthly	Weighted price	Up to 10 years
Sweden	SV VRP	09.12.1992	Weekly	Yield	Up to 10 years
Switzerland	SV VRP	04.01.1998	Daily	Yield	From 1 to 30 years
Britain	SV VRP	04.01.1998	Daily Monthly	Weighted price	From 1 week to 30 years
United States	SS	14.06.1961	Daily	Prices	From 1 year to 10 years

[1] SV: Svenson Method. NS: Nelson and Siegel Method. SS: Splines Method. VRP: variable of penalty for thoroughness

Source: BIS Data Bank

It is found that the central banks of several countries show preference for parsimonious models based on features such as the model of Nelson and Siegel (1987) and Svensson (1994) (Bank for International Settlements, 1999). These methodologies are highlighted due to its great ability to predict and because in the mathematical formulation of the model does not include variables of expectations building by the agents. We conclude the importance of considering the model of Nelson and Siegel (1987) to obtain the yield curve, because although the model of Svensson (1994) is often more used, this is an extension of the model of Nelson and Siegel (1987).

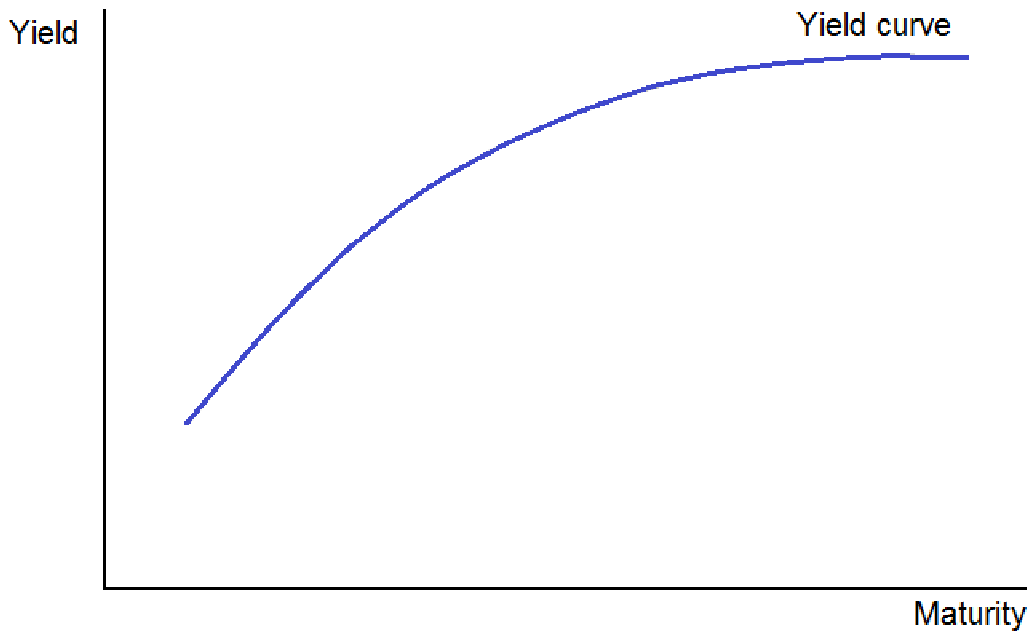
5 Theoretical frame

5.1 Temporary Structure of Interest Rates

The role of interest rates in the allocation of funds in the financial markets is analogous to the role of prices in the allocation of resources in markets for goods and services, it means, they are signs used for economic agents' decisions on consumption, investment and financing. When there is a relatively high price of a particular good, it tends to allocate resources to their production, in the same way, when there is a relatively high interest rate of a particular instrument, funds are allocated to activities that it finances.

Economic theory suggests that an important factor explaining the discrepancy in interest rates between two financial instruments with similar characteristics such as risk, taxation, issuer and market, it is due to the difference between the maturity dates of each of these. This relationship between the maturity of the instruments and their market interest rates is known as Temporary Structure of Interest Rates (since now, ETTI as it is known in spanish).

The ETTI, to a point in time, can be represented using a diagram that relates the performance of these instruments with expiration date, known as yield curve. The yield curves can present a wide variety of shapes and movements, which provides an explanation in economic and financial sense. Traders try to prognostic these movements and forms in order to anticipate the market looking to get the most profits.



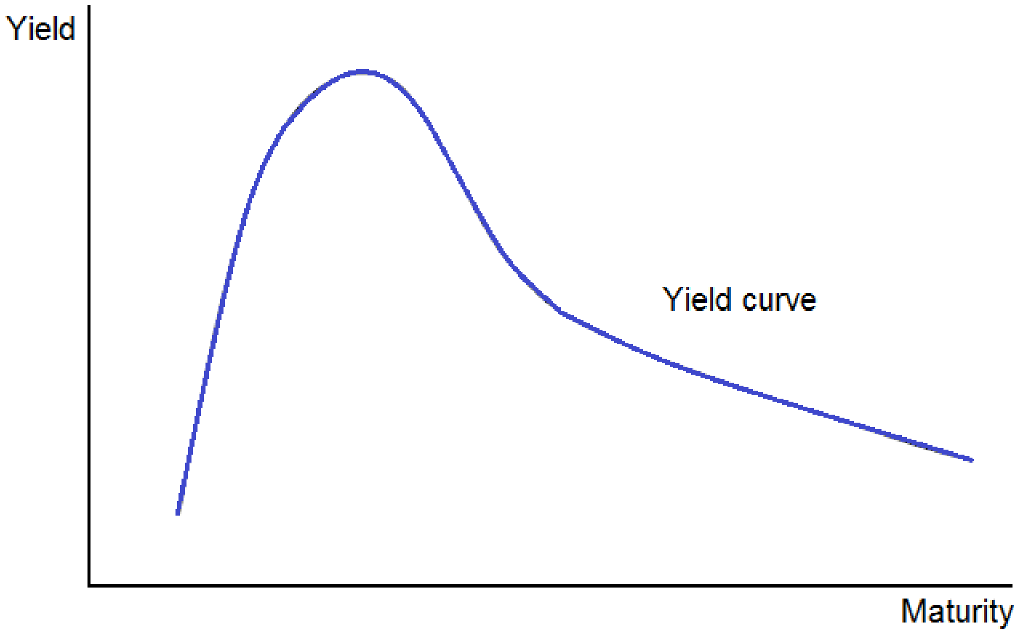
Source: Own creation

The yield curves generally show a positive slope, the greater the time to maturity of an instrument, it is expected to be higher performance. This is mainly due to that the greater is the time to maturity of an instrument, the investor faces greater risks. In other words, it increases the chances of a catastrophic event occurs that impacts on investment, so it is necessary to pay off with a risk premium for the risk that the investor is exposed.

Moreover, the positive slope of the ETTI may also reflect investors' expectations about the growth of the economy in the future, or in opposition, may be associated with the risk of higher inflation. This expectation of higher future inflation generates both perspectives of future monetary policy of the central bank will contract in order to eliminate inflation pressures leading to an increase in interest rates (due to the reduction in the money supply) and the requirement of a risk premium associated with the uncertainty about the rate of inflation and its effect on the value of the cash flows. For this reason, investors will value these risks and will incorporate them in the yield curve, demanding higher returns for farthest investment horizons.

Similarly, it may be the case where interest rates are higher than short-term to long-term, which implies a negative slope of the ETTI. This abnormal and contradictory situation occurs when investors expect that future returns are lower than those of today, this is because they observe a possible future recession or otherwise, implies that the market believes inflation will remain low. However, these are not the only ways the yield curve can take. For example, there may be flat yield curves where the differences between the rates of the various terms to maturity of the instruments are small, which sends signals of uncertainty in the economy. In addition, other behaviour that has been observed in the yield curves are hump-forms that appear when yields short and long term are the same and medium-term yields are varying.

Also, there may be changes in the rates of the instruments that make the yield curve causing displacement of it. For example, when there is a change in the rates of equal magnitude in all the instruments and terms of the yield curve, it suffers a parallel displacement; in other words, the shape of the curve will not be changed even though the rates of interest have changed. However, rarely yield curve shows this type of movement, being more common non-parallel shifts caused by irregular changes in interest rates.



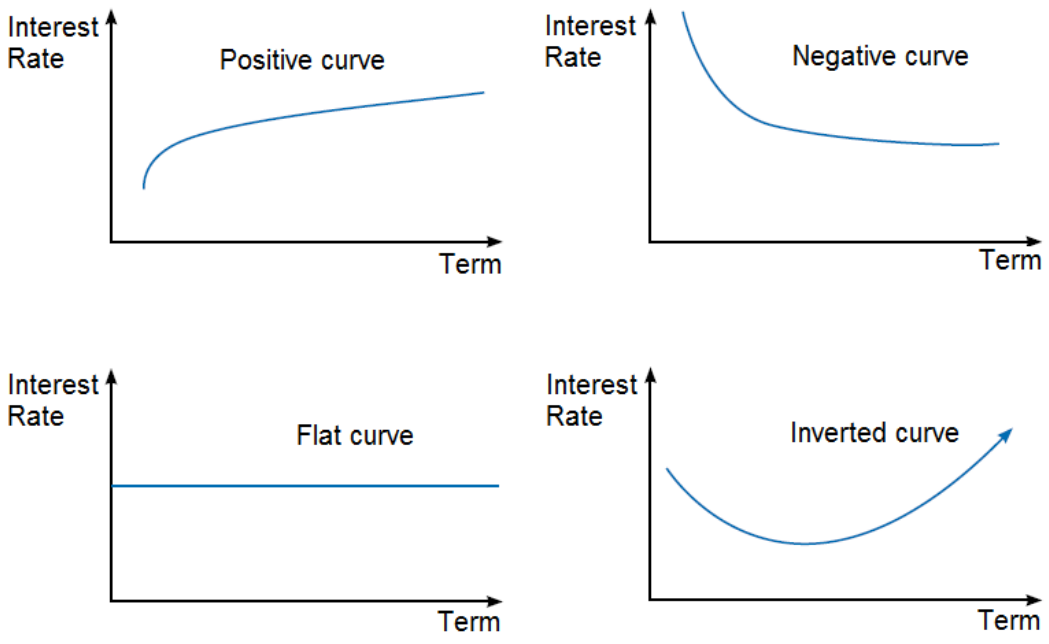
Source: Own creation

It can be thought then, that the ETTI, displays a set of prices for future consumption and investment at different maturities. It is evident that economic agents involved in the financial markets are meant to delay their present consumption decisions for future consumption, receiving an interest rate to compensate their wait. Moreover, some agents can require those funds to use in the development of investment projects that allow them to pay the interest rate that savers demand, so that different perceptions about the performance of the projects in the future should be reflected in different rates of interest according to the terms of each of these.

5.2 ETTI's explicative theories

One of the biggest challenges facing the development of a theory of the ETTI, undoubtedly, is to try to explain some empirical regularities or facts that appear in the market:

a) It has been observed that bonds with different maturity offer different interest rates,
b) Most of the time, the yield curve has a positive slope,
c) When short term interest rates are relatively high, the yield curve is fairly flat or negative slope, however, when short term interest rates are relatively low, the yield curve is steeper and positive.



Source: Own creation

From these facts have emerged various theories that have attempted to model the behaviour of the yield curve, as well as have been proposed seeking a full explanation of the ETTI in order to have valid and precise tools for obtaining monetary and financial information.

However, these attempts to find a theory that fits as much as possible to reality, it should be noted that none of these theories, by itself, is completely suitable for fully explain the shape of the ETTI. In contrast, using a combination of them, it is possible to explain some segments of the yield curve.

5.2.1 Expectations theory

The expectations theory of the ETTI, apparently dating from the work of Irving Fisher (1896) who believes that in a progressive economy it is assumed that the interest rate changes from period to period. Assuming that economic agents have perfect foresight and ignore the costs of the loans themselves (in terms of effort and money in respect of income), we can think of the interest rate of a long term loan as the average short term interest rate within the same time period. Thus, if the future short-term interest rates were known, it would be possible to calculate the long term interest rates, today; the determination of short-term rates for the future, in effect, simultaneously determine the prevailing ETTI this period.

This first approach on ETTI was retaken by Friedrich A. Lutz (1940) who deepens on this point of view, presenting a more formal way what is known as the pure theory of expectations. Lutz suggests three assumptions that allow us to find a relationship between short term interest rates and long term ones:

1) All market participants are well aware of what will be the future short-term rates,

2) There are no costs associated with the investment,

3) There is complete mobility for both investors and borrowers.

If these assumptions are met, proposals can be formulated for the relationship between short term interest rates and long term ones:

a) We can conceive long-term interest rate as the average of future short-term rates,

b) Long-term rate can not fluctuate more than the short-term interest rate, this is due to future changes in the short term rate are already reflected in the long term rate in the present, in addition to the time period in which these changes in the short-term rate materialize lesser extent affect the long-term rate because fades changing short-term rate to another,

c) It is possible that long-term rate may temporarily move in the opposite direction to the short term,

d) The current yield of a long-term bonus at the end of maturity will be above the short-term rate, provided that the average short-term rates increase to the bond's maturity date and is above the current short-term rate (and vice versa), and

e) The performance in all possible investments of equal periods will be the same, regardless of the form in which they are located.

f) In this sense, in the near future all financial assets of the same credit quality should offer the same return regardless of their due dates, because investors, looking for opportunities to make profits, eliminate the yield differentials between them.

If, for example, agents have expectations of higher future interest rates, will focus on acquiring short-term instruments, with which they may reinvest its resources to higher interest rates, while lenders hire long-term loans to ensure lower interest rate. As a result of this interaction, it will generate an excess demand for long-term funds and an excess supply of short-term funds. These imbalances will be corrected by changes in the interest rate: a drop in the short term interest rate, accompanied by an increase in the long term rate, thus, will generate an inclination of the yield curve. The shape of the yield curve is due solely to the expectations of the agents involved in the market on interest rates.

The expectations theory implies that financial markets are highly efficient. It is considered that a market is efficient when the price of an asset reflects all available information that affects the instrument's valuation. If there is the possibility of making profits, a sufficient number of market participants would recognize and exploit these opportunities. When looking to obtain these gains, the price of the instruments change, reflecting the new information. The expectations theory suggests that all relevant information is built into expectations about future interest rates.

There is a vast amount of empirical studies that seek to validate the expectations theory of the ETTI. A major study was conducted by David Meiselman, who through an error-learning model found consistent evidence with the expectations theory. However, many studies show conflicting results to what theory predicts of expectations. John Y. Campbell and Robert J. Shiller (1984) found that when the ETTI has a excessive positive slope, long term interest rates subsequently tend to fall, while the short-term increases, so that there is no consistency with the expectations theory due to it suggests that both should be increased.

On the other hand, there are in the literature some studies (Fama and Bliss, 1987; Jorion and Mishkin, 1991) where the evidence suggests that short-term rates of ETTI are useless for predicting future spot rates; however, increases as the forecast horizon temporal structure best predict performance on future interest rates.

The assumptions on which the expectations theory is built are very restrictive: perfect foresight, risk neutrality, indifference on the liquidity of financial assets, zero transaction costs and optimal and unbiased expectations. Therefore, this theory has undergone several reviews, as empirical facts have shown that these assumptions do not hold in reality, emerging new theories that try to correct these defects.

5.2.2 Preference for liquidity theory

The traditional theory of expectations can be extended by relaxing the assumption of perfect foresight and replacing it with the assumption that market participants form their expectations under uncertainty about future spot rates. The theory of liquidity preference was developed by Hicks (1946) that although it agrees with the importance of future spot interest rates expected, emphasizes the effects of risk attitudes of market participants.

The theory of liquidity preference states that risk aversion will make that the forward curves are found consistently above expected spot rates on average, even this difference should increase with maturity. This argument is based on the assumption that most of the loans are used for long term projects so that borrowers seek financing options for the same period, to hedge against possible fluctuations in interest rates.

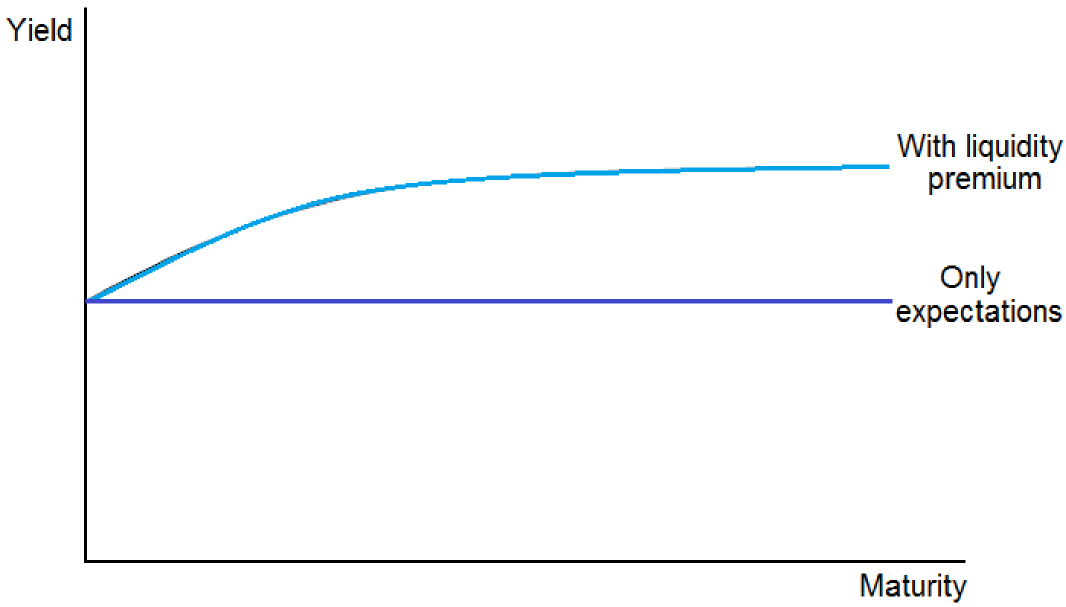
Moreover, following the theory proposed by Hicks, lenders prefer liquidity and minimal risk associated with fluctuations in the value of its portfolio, which that they will choose to keep short-term instruments.

These differences in the preference for instruments with different maturities cause differences between individuals that lend and the ones that borrow. To solve this situation, it requires an increase or premium to induce the investor to maintain “riskier” long-term instruments.

As Hicks said, there is a weakness in the forward market loans, which provides a chance for speculation. If it is not offered an "extra income" for long-term loans, investors will prefer the short-term ones, generating an excess of demand for long-term funds. Borrowers, therefore, will have to offer better terms to attract resources from investors to enter in the forward market. In this way, investors play the same role as a speculator since they would enter in these markets only if the expected gain is enough to compensate for the risk to which they are exposed.

Under this theory, the forward rates will be biased estimations of future interest rates, exceeding them in an equal amount to risk premium or maturity. The presence of rewards to maturity implies a bias towards yield curve positively sloped. In fact, the yield curve could present negative slope only when expected future short-term rates were under the current short-term rate including the respective reward to maturity.

To illustrate the effect that exerts the liquidity premium on ETTI, we will suppose that market participants expect future short-term rates are the same as the current rate. In this sense, expectations, alone themselves, would imply a horizontal yield curve; however, due to the presence of premiums provided in the forward rates, it would present a positively sloped yield curve as the following graph shows:



Source: Own creation

If there is a positive bias in forward rates, one of the most important propositions of the theory of expectations is not met: the performance in all possible investments with the same periods will not be the same. Investments in long-term instruments provide an expected return greater than the short-term reinvestment in each maturity.

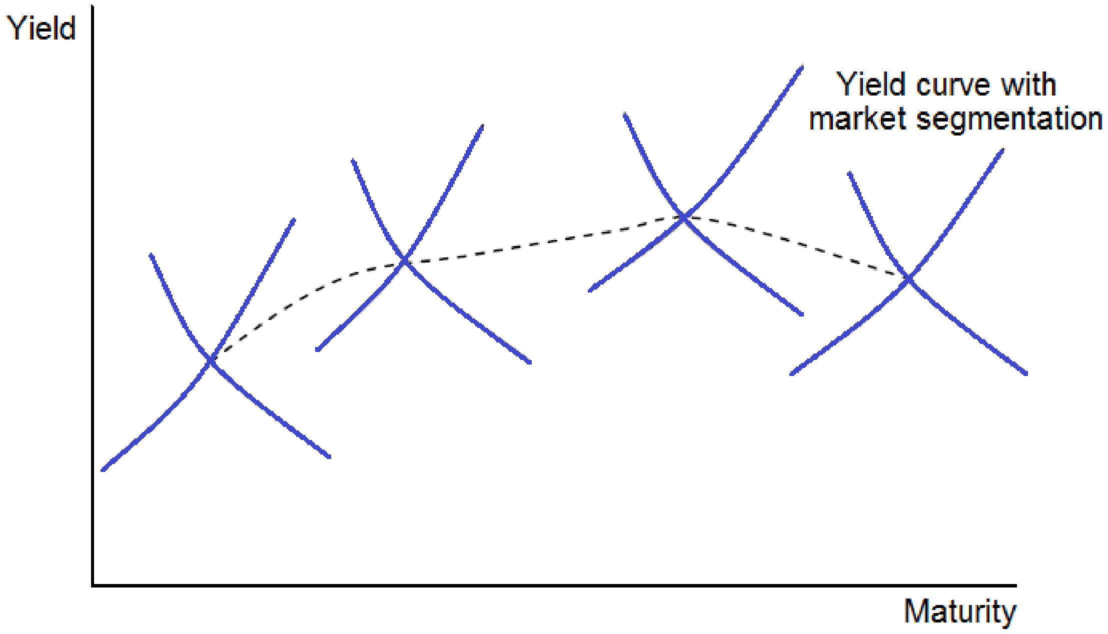
5.2.3 Market segmentation theory

Both expectancy as liquidity preference theories were seen by market participants as simple academic tricks, which could not give a satisfactory explanation of the temporary structure of interest rates. Culbertson (1957) formulated a third temporary structure theory suggesting that the behaviour of investors and borrowers in segmented markets determined the shape of the yield curve. Culbertson's basic idea was that financial markets, through the process of supply and demand, determined the yield of the market.

Due to legal and strategic restrictions, market participants show strong preferences for different maturity financial instruments, these being separated and traded in different markets. Most commercial banks are inclined for short and medium term instruments due to the nature of their liabilities and the emphasis on liquidity. However, insurance companies and other institutional investors contract long term liabilities therefore prefer longer maturities. Otherwise, borrowers relate the maturity of their debt with the need for funds.

The theory of market segmentation implies that the interest rate for a particular maturity is determined only by the conditions of supply and demand for that maturity, regardless of the conditions for instruments with different maturities. By this way, market participants have rigid preferences on certain maturity without being interested on how attractive could be the yield in other market with different term.

According to the theory of Culbertson, the loan market will be entirely segmented according to the maturity of the securities. Therefore, for each maturity there will be a market, where supply and demand for it will determine the interest rate. As an example, if there were four segmented markets, the temporary structure of interest rates could be observed as shown:



Source: Own creation

According to the theory of market segmentation, demand and supply for a given maturity instruments are supposedly few affected by instrument prices near to that maturity, likewise, arbitrage across different maturities is limited. In this sense, there is no reason that the rewards are positive or an increasing function of maturity, as stated in the theory of liquidity preference. This theory has been seriously questioned since the empirical implementation is difficult to carry out, and there should be a limit on how far one can go keeping close maturity instruments without being close substitutes.

5.2.4 Preferred habitat theory

Modigliani and Stutch (1966) used some similar arguments of the theory of market segmentation, recognizing its limitations and combining aspects of other theories, generating a more moderate version of it. As Culbertson theory, it recognizes the existence of heterogeneous groups of borrowers and lenders, which show preference for instruments of different maturities.

Each market participant has its preferred habitat of maturity, that is, every investor and borrower will enter the market of loans that best suits their preferences, legal restrictions, investment horizon and financing needs. Within each habitat or market segment, yields are determined in the same manner as in the above theory, by supply and demand.

While each of the participants is in their habitat, they can be induced to leave this market if there are other better returns in other ones; in other words, both investors and borrowers operate in the preferred environment of maturity but they will tend to move if an enough attractive differential in interest rates arises. When yields in other markets are not significantly attractive, so that participants from both sides of the market are not willing to move from habitat, they will remain in their favourite one, so that the loan market will be partially segmented. This point of view of Stutch and Modigliani contradicts the theory of market segmentation, since preferences on certain instruments are not that rigid, mattering other market conditions and the yields they offer. This is where the expectations and the rewards come into play to define the shape of the yield curve.

Furthermore, the authors of this theory argue that the rewards for maturity could be positive or negative, without implying the existence of a systematic pattern of maturity. Melino (1986) mentions that, although Stutch and Modigliani argue that the pattern of maturity reward depends on the changes in wealth, investor preferences and the distribution of the instruments offered, are unable to find empirical evidence that these factors generate substantial changes in the pattern of the rewards.

5.3 ETTI's applications

The ETTI is an important tool in economic and financial matters, as it is able to provide information on future macroeconomic conditions of a country. Within the information that the ETTI transmits, we found expectations on future interest rates, economic activity, inflation and monetary policy effectiveness.

Moreover, the knowledge of the ETTI contributes in evaluating investment projects, valuation of assets and financial instruments such as derivatives that imply the use of interest rate such as forwards, futures or swaps contracts.

5.3.1 Expectations about the economic activity of a country

As mentioned, the temporary structure of interest rates not only is useful for investors looking for the best option to invest their resources or to those seeking financing. It has been argued that behind the temporary structure of interest rates is possible to predict what the behaviour of a country's economic activity will be.

One of the first studies that try to find a relationship between interest rates and future economic activity is Mankiw (1982), who argued that interest rates or interest rate differentials are useful for predicting future consumption, improving the idea of Hall (1978), who had used the price level of stocks, but unfortunately the data did not support his hypothesis.

On the other hand, Hamilton and Kim (2002) discuss the positive relationship between interest rate differentials and future economic activity mentioning that can be due both expectations effect (the difference as a sign of the expected future interest rates), as a reward structure effect (changes in rewards risk over time). The effect of expectations described by these authors suggests that a monetary contraction increases the short term interest rate consequently decreasing the interest rate differential, the same way that reduces spending on sectors of the economy that are sensitive to interest rates resulting economic slowdown; on the opposite, monetary expansion will tend to reduce the short term interest rate rising rate differential increasing spending on sectors of the economy, which leads to an acceleration in economic activity. Moreover, the effect of reward structure suggests that changes in the rewards to risk affect investment decisions disrupting economic growth.

Another empirical study developed by Estrella and Hardouvelis (1991) finds a positive relationship between the interest rate structure and economic activity. Using U.S. data, they show that, even when controlling for the current position of monetary policy, the relationship persists. Also, they try to estimate the probability of an economic recession with the information of the ETTI, concluding that the interest rate structure of four quarters ago predicts relatively well the economic recessions in that country. Also prove that ETTI explains better the future economic activity than other alternative indicators.

Similarly, Estrella and Mishkin (1995) obtained similar conclusions concerning the information contained in the temporary structure of interest rates about future economic activity in Germany, France, England and Italy.

As it has been seen, the usefulness of spreads between long term and short term interest rates to predict the future economic activity has been concentrated on studies in the United States and other industrialized countries. In contrast, there is little research that has been done in emerging countries, this mainly due to the relatively recent development of financial and capital markets in these economies. Camero and Castellanos (2003) mention that the little evidence available about emerging economies is mixed with respect to what are the most appropriate interest rates, in other words, the problem is whether to use longer or shorter terms of interest rates, real rates or nominal ones, or qualitative variables instead of quantitative ones.

Some studies developed from emerging economies data are those by Fernandez (2000), in the case of Chile, and Camero and Castellanos (2003) for Mexico. Fernández finds that the interest rate differential has some power to predict fluctuations of the Monthly Economic Activity Indicator of the Central Bank of Chile; however, this relationship is stable only when considering a time horizon of 12 months. Moreover, also finds that the level of the real short term interest rate, which is associated with the current monetary policy position, is the one that explains a greater percentage of changes in this indicator instead of nominal and real rate differentials.

The study carried out by Camero and Castellanos, finds that interest rate differentials provide good results to predict economic activity (as measured by the Index of Industrial Activity) up to 18 months later. Also, they conclude that the results for Mexico are similar to those found in some industrialized countries where financial instruments reflect higher information content.

5.3.2 Effectiveness of monetary policy

All studies concerning the role of the interest rate structure and its use to predict future economic activity presented here, somehow consider the role of monetary policy in determining interest rates. It is clear that, at least in the short term rates, the central bank can affect the structure of interest rates when intervening in financial markets.

If the central bank wants to modify the amount of money indirectly has to be able to influence its demand. One of the ways in which it can achieve this is through interest rates. It is needed to understand the process of monetary policy transmission to appropriate instrumentation.

According to Schwartz (1998), the main instruments that have been used by central banks for liquidity management are reserve requirements (which includes the legal deposit and zero average deposit), the open market operations, swap operations with foreign currency, the windows for liquidity management and, in some cases, the transfer of government deposits between the central bank and credit institutions.

One of the channels through which the central bank acts to influence the demand for money by changing the interest rate, is via discretionary actions by varying the target of average balances of the current accounts of credit institutions.

The objective of average balances is designed to induce credit institutions to not maintain on average positive balances, or incurring overdrafts in their accounts, as well as to try to compensate with other banks their excess and shortages of resources to market interest rates. The temporary structure of interest rates provides information about the effectiveness of this and other monetary policies carried on by central banks.

There are studies that have used the temporary structure of interest rates of interest rates as a research tool in order to analyze the effectiveness of central bank intervention in the modification of the demand for money. Within these works highlights the one by Castellanos (2000) and Diaz de Leon and Greenham (2000).

In order to measure the impact of monetary policy on the interest rate structure, Castellanos proposes, firstly, to study the behaviour of the overnight interest rate. Castellanos mentions the existence of an inverse relationship between overnight interest rates and the objective of accumulated balances, adding a direct relationship between this rate and the exchange rate. Using a linear model for determining the interest rate measurable by ordinary least squares (OLS) method, using as explanatory variables the accumulated balance target, the logarithm of the exchange rate, the treasury bond to 30 years interest rate U.S., the logarithm of the index of the stock market of Russia and Brazil (these variables are considered by the infection effects on the Mexican economy) and the spread of sovereign Brady bond[2] rates in Mexico (all these data using the first difference due to non-stationary in levels). The results of this regression are consistent with previous studies: the overnight rate rises in response to a reduction in the target balance.

However, a more thorough analysis of these results reveals some problems with the OLS estimators. In order to correct in a certain way the instability of regression coefficients obtained with OLS, Castellanos makes a rolling regression with the same variables, where the coefficients are allowed to vary over time. The results from this mechanism are more reliable, in average is lower than the previous one but stable.

Furthermore, it is also analyzed the primary interest rates of CETES[3] 28, 91, 182 and 364 days by error correction model taking the short term interest rate as the overnight rate. The results obtained by these estimations according to economic theory: the hypothesis that the interest rate differential is stationary, so that no arbitrage opportunities persist to long-term, is not rejected by the data. It is also observed that the coefficient of the error correction term in the

[2] Brady bond is generally an Eurobond or domestic bond issued by the government of a developing country to refinance their debt to foreign commercial banks
[3] CETES stands for Certificates of the Treasury of the Federation that are just an investment vehicle offered by the federal government of Mexico

equation of CETES decreases in absolute value as it extends the period from 28 to 364 days; this means that the speed of adjustment to exogenous shocks is higher in shorter term interest rates.

For other part, Diaz de Leon and Greenham analyze the impact of change in the "short" on the ETTI, discounting and not discounting the effect "reaction" (For an explanation of the effect "reaction" see Diaz de Leon and Greenham (2000)) on these rates, through the technique of autoregressive vectors. In this study, the authors found that the effect of monetary restriction is higher on shorter term interest rates decreasing over time.

5.3.3 Inflation expectations

From the ETTI it is possible to detect what the expectations are about future inflation. Nominal rates are composed of real interest rates plus a premium that represents the market forecast for inflation. Market participants can predict future inflation changes which are reflected in the slope of the temporary structure of interest rates.

One of the first studies that addressed the issue of inflation expectations contained in the ETTI was Mishkin (1990), who takes the equation of Fisher for his study considering that agents' expectations are rational. It also assumes that the slope of the structure of real interest rates is constant over time, and therefore, the shape of the yield curve is determined by inflation expectations. Therefore, there is a positive relationship between the nominal temporary structure of interest rates and future inflation.

For a study developed in the United States, Mishkin finds that the spread of interest rates of the securities with maturity of up to 6 months does not provide information about future inflation, however, the spread of securities with maturities less than 9 months does contain relevant information about inflation expectations. As a result, concludes that for the case of the U.S. the long term yield curve is an important indicator of expectations about future inflationary pressures (when the slope becomes more positive, it is expected inflationary pressures and vice versa).

By contrast, Mishkin (1991) finds results that provide little evidence for the relationship of the temporary structure of interest rates and inflation expectations, when replicates the study expanding the sample to 10 countries. Only the United States, France and Britain agree their results with previous studies.

In the case of Mexico, Castellanos and Camero (2002) following the methodology proposed by Mishkin, found that the slope of the yield curve has some ability to forecast future inflation since 1996. However, the predictive capacity is limited and significantly lower compared with the use of an ARIMA[4] .

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[4] Autoregressive Integrated Moving Average model is an statistical model that uses variations and statistical regressions in order to find patterns for predicting the future

6 Functions, Methodology and Data

6.1 Discount function or Value function ($v(t)$)

Defining the real function of the real variable $v(t_0,t):\to R$ for $t \geq t_0$, naming the discount function in t_0 as

$$v(t_0,t) = e^{-\int_{t_0}^{t} r(h)dh} \text{ for } t \geq t_0,$$

Being $r(h)$ the instantaneous interest rate in $t = h$.

This function provides the real number that represents the discounted value in t_0 of one monetary unit available in $t(t \geq t_0)$.

In the particular case that $t_0 = 0$ (actual moment), we refer to the function $v(0,t)$ as just a discount function and it will be denoted by $v(t)$, that can be interpreted as that function that provides the real number that represents the actual value of one monetary unit available in t.
By the definition of discount function, we have that

$$v(t) = e^{-\int_{0}^{t} r(h)dh} \text{ para } t \geq 0$$

From here, we can deduce a number of properties that functions and discounting factors have to verify:

1) $v(t) = \dfrac{1}{A(0,t)}$ for $t \geq 0$.

From this property it is easy to deduce the following ones:

2) $v(0) = 1$

3) $0 < v(t) < 1$ for $t > 0$

This property is derived that $A(t_0,t) > 1$ for $t > t_0$.

4) If $t_1 < t_2$, then $v(t_1) > v(t_2)$, it means, $v(t)$ is a decreasing function in regard to t.

5) For all $t_1 < t_2$, $v(t_1) \cdot v(t_1,t_2) = v(t_2)$.

This property is derived from the consistency principle of the accumulative factors.

6) $r(t) = -\frac{d\ln\left[v(t)\right]}{dt} = -\frac{\frac{dv(t)}{dt}}{v(t)}$

This relation is obtained of the relation between the instantaneously capitalized interest rate and the accumulative function, since $\ln\left[v(t)\right] = -\ln\left[A(0,t)\right]$.

However, we will focus on the following properties and assumptions:

1. $v(0) = 1$
2. $\beta_0 = 1$

6.2 Relation between the Discount function (or value function) ($v(t)$) and ETTI ($r(h)$)

If we suppose continuous capitalization, one monetary unit valued in t, today it worth $e^{-rt} \rightarrow e^{-\int_0^t r(h)dh} = v(t)$ assuming non constant interest rate.

The objective is to obtain $r(h)$. For this, according to the properties of the exponentials, we will separate the e exponent:

$$-\int_0^t r(h)dh = \ln(v(t))$$

To make the job easier, we will change the sign of the equation, so it will be:

$$+\int_0^t r(h)dh = -\ln(v(t))$$

Then, we will derive the equation resulting in $r(t)$

$$+\int_0^t r(h)dh = -\ln(v(t)) \Rightarrow r(t) = -\frac{d(\ln(v(t))}{dt} \qquad (0.1)$$

By this way, knowing the discount function $v(t)$ we can calculate the value of $r(t)$. From this point, we will explain how to obtain $v(t)$ in order to, finally, represent the ETTI.

6.3 Nelson and Siegel Methodology

As we said in previous chapters, the study of Nelson and Siegel (1987) offers a simple model that is able to represent the yield curve with high level of reliability.

This method is widely used both in theoretical work as in the industry (even some central bank), so can be considered as a standard. It is a simple functional parametric model and, at the same time, flexible enough to capture the profiles which typically presents the curve.

From an initial combination of values of the four parameters to be estimated and applying nonlinear minimum least squares we obtain parameters that minimize the sum of squared residuals. In practice, the initial combination is matched to estimated parameters in the previous session. This ensures that the resulting combination will not differ too much from the previous day.

Another advantage of this model is that the three terms of the equation of interest rates can be interpreted as level, slope and curvature. However, it should be noted that many different combinations of the four parameters provide nearly identical curves.

One problem with this method is the setting for the short term. By minimizing errors in prices, more attention is paid to the assets more sensitive to these errors, it means, the ones with higher maturity and term. This is neglected asset prices in the very short term.

For our estimation, it is proposed a cubic polynomial as a functional form f the value function (or discount) $v(t)$, it means,

$$v(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 \quad (0.2)$$

In order to have this function well defined, it has to meet, among others, the following properties discussed above

$$\beta_0 = 1$$
$$v(0) = 1$$

Then, according to the prevoius result obtained (1.1), we replace $v(t)$ by its functional form (1.2) and proceed to derive the neperian logarithm according to its derivation properties

$$r(t) = -\frac{d\ln(v(t))}{dt} = -\frac{d\ln(\beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3)}{dt} - \frac{\beta_1 + 2\beta_2 t + 3\beta_3 t^2}{\beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3} \qquad (0.3)$$

This function is known as the temporary structure of interest rate.

6.3.1 Estimation Methodology

To adjust the discount polynomial function, we use the formula for the valuation of bonds:

$$P = \sum_{\tau=1}^{T} c_t v(\tau_t) - CC + N \cdot v(T) \qquad (0.4)$$

Where P is the average price of the bond or liability, c are the cash flows we will obtain and CC is the accrued interest (cupón corrido, in Spanish) for the next coupon.

$$P + CC = \sum_{\tau=1}^{T} c_t v(\tau_t) + N \cdot v(T)$$

Being a case of fixed income, we know that $C_\tau = C_T \forall_{\tau,T}$, then

$$P + CC = \sum_{\tau=1}^{T} c \cdot \left(\beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3\right) + N \cdot v(T)$$

We continue isolating until the summation of the discount function is to one side of the equation. Knowing that one of the properties is that $\beta_0 = 1$,

$$P + CC = \sum_{\tau=1}^{T} c \cdot \beta_0 \sum_{\tau=1}^{T} c \cdot \left(\beta_1 t + \beta_2 t^2 + \beta_3 t^3\right) + N \cdot v(T)$$

Isolating the summation of the constant coupons to be received in the future, we have that

$$P + CC - \sum_{\tau=1}^{T} c = \sum_{\tau=1}^{T} c \cdot \left(\beta_1 t + \beta_2 t^2 + \beta_3 t^3\right) + N \cdot v(T)$$

By this way, we obtain what have named as the dependent variable, Y_T

$$Y_T = P + CC - \sum_{\tau=1}^{T} c \qquad (0.5)$$

For example, taking the data of first bond we have chosen for the estimation of the temporary structure of interest rate (that will be explained in the following chapter), we have that the bond ES00000120H2 B EST provides the following data:

Due date	Coupon	Average price	Date of last coupon received	Days after	Days left
31.10.2008	2,90	98,692	31.10.2005	304	61

From this information, we can calculate the accrued interest (CC) and knowing this information we will continue calculating the dependent variable. First of all, we have to obtain the CC:

$CC = \frac{c}{365} \cdot t_{daysleft}$, so we have that $CC = \frac{2,90}{365} \cdot 61$

The result obtained is 0,484657534. Once we know the value for CC, we have to calculate the summation of the cash flows we will receive. In this case, this bond is waiting to receive three payments, it means,

	t0	t1	t2	Summation
ES00000120H2 B EST	2,90	2,90	102,90	108,70

We have to pay attention that the last year we will receive the coupon plus de nominal we invested initially.

Once we have all data we need to calculate the dependent variable, we can proceed calculating it (Y_T) replacing the values we have for P, CC and $\sum_{\tau=1}^{T} c$ in the function explained before (1.5):

$$Y_T = 98,692 + 0,484657534 - 108,70$$

The value of the dependent variable (Y_T) for the first bond is -9,523342466. The steps done before would be repeated for the next bonds and liabilities. In the next chapter, we will see the resulting data referred to these steps.

6.4 Data and Results

From the information provided by the Bank of Spain in relation to debt, we selected bonds and liabilities on two different dates given the current financial crisis. Taking the years 2007-2008 as the starting years of the financial crisis, the first selection of bonds and liabilities has been made to date 01.09.2006. It was considered appropriate to make an additional selection on date 01.09.2011, a period that corresponds with the current financial crisis; all these with the aim of comparing the results on both dates.

6.4.1 First period of time: pre-crisis (2006)

Bonds and liabilities available as of 01.09.2006 are the following:

E M I S I O N	NUMERO OPERACS	IMPORTE CONTRATADO	PRECIO (EX-CUPON)			RENDTO. INTERNO MEDIO	ANTERIOR PRECIO MEDIO (FECHA)
			MEDIO	MAXIMO	MINIMO		
ES0000012908 B EST 3.00 30.07.07	2	7,08	99,585	99,585	99,584	3,47	99,592 (30/08/2006)
ES0000012825 B EST 4.25 31.10.07	1	10,00	100,805	100,805	100,805	3,52	100,814 (31/08/2006)
ES00000120H2 B EST 2.90 31.10.08	35	372,37	98,692	98,705	98,551	3,54	98,674 (31/08/2006)
ES0000012882 B EST 3.60 31.01.09	5	65,18	100,072	100,080	100,066	3,55	100,045 (31/08/2006)
ES0000012064 O EST 5.15 30.07.09	27	1.229,00	104,296	104,310	104,280	3,56	104,264 (31/08/2006)
ES0000012239 O EST 4.00 31.01.10	66	754,80	101,358	101,390	101,320	3,56	101,280 (31/08/2006)
ES00000120E9 B EST 3.25 30.07.10	8	85,00	98,744	98,755	98,730	3,60	98,682 (31/08/2006)
ES0000012387 O EST 5.40 30.07.11	5	74,00	107,747	107,875	107,720	3,64	107,720 (31/08/2006)
ES0000012452 O EST 5.35 31.10.11	9	93,10	107,896	107,930	107,887	3,64	107,677 (31/08/2006)
ES0000012791 O EST 5.00 30.07.12	6	51,00	106,995	107,020	106,965	3,66	106,922 (31/08/2006)
ES0000011660 O EST 6.15 31.01.13	5	50,00	113,868	113,910	113,840	3,68	113,714 (31/08/2006)
ES0000012866 O EST 4.20 30.07.13	5	60,00	102,976	103,100	102,797	3,70	102,858 (31/08/2006)
ES0000012098 O EST 4.75 30.07.14	12	208,00	107,217	107,280	106,946	3,68	107,090 (31/08/2006)
ES0000012916 O EST 4.40 31.01.15	15	399,90	104,621	104,660	104,580	3,74	104,383 (31/08/2006)
ES00000120G4 O EST 3.15 31.01.16	17	160,17	95,228	95,280	95,190	3,76	94,970 (31/08/2006)

The selected bonds and liabilities are those with due date greater than 01.09.2007 or less than 01.09.2016. Within these have been selected those eight that have at least 10 trades and among them, those with more hiring. Therefore, the chosen ones are the following:

Bond	**Excoupon Price**	**Coupon**	**IRR**	**Due date**	**Days until first coupon**
ES00000120H2 B EST	98,692	2,90	3,54	31.10.2008	61
ES0000012064 O EST	104,296	5,15	3,56	30.07.2009	334
ES0000012239 O EST	101,358	4,00	3,56	31.01.2010	153
ES00000120E9 B EST	98,744	3,25	3,60	30.07.2010	334
ES0000012452 O EST	107,896	5,35	3,64	31.10.2011	61
ES0000012098 O EST	107,217	4,75	3,68	30.07.2014	334
ES0000012916 O EST	104,621	4,40	3,74	31.01.2015	153
ES00000120G4 O EST	95,228	3,15	3,76	31.01.2016	153

Taking the first example as a base, we will use its data to continue giving examples of how to proceed to obtain the data we need. Knowing the value for the dependent variable (Y_T), we can start calculating the independent variable. By this, for the first bond we have that

$$-9,5233 = 2,90 \cdot v(t) + 2,90 \cdot v(t) + 102,90 \cdot v(t)$$

From this, we will extract the independent variable $v(t)$, where the discount function (or value function) corresponds to $v(t) = \beta_1 t + \beta_2 t^2 + \beta_3 t^3$:

$$-9,5233 = 2,90\left(\beta_1 t + \beta_2 t^2 + \beta_3 t^3\right) + 2,90\left(\beta_1 t + \beta_2 t^2 + \beta_3 t^3\right) + 102,90\left(\beta_1 t + \beta_2 t^2 + \beta_3 t^3\right)$$

We prepare the components that will form the matrix, where we will estimate the value of the unknown parameters (β_1, β_2 and β_3). By this way, the first row of the matrix:

$$\begin{pmatrix} 2,90 \cdot t_1 + 2,90 \cdot t_2 + 102,90 \cdot t_3 & 2,90 \cdot t_1^{\;2} + 2,90 \cdot t_2^{\;2} + 102,90 \cdot t_3^{\;2} & 2,90 \cdot t_1^{\;3} + 2,90 \cdot t_2^{\;3} + 102,90 \cdot t_3^{\;3} \\ \ldots & \ldots & \ldots \\ a_{81} & a_{82} & a_{83} \end{pmatrix}$$

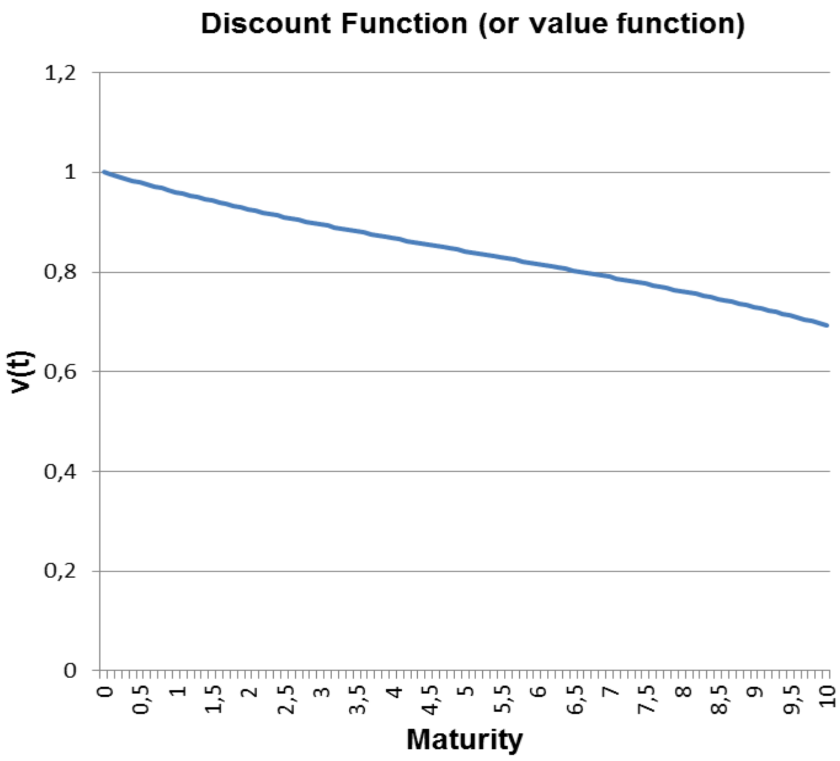
The seven remaining rows of the matrix correspond to the rest of selected bonds. Knowing the values for t, as we do, we will solve the equations where the unknown parameters are the betas, through the following matrix:

$$Y_j = \begin{pmatrix} 2,90\cdot t_1 + 2,90\cdot t_2 + 102,90\cdot t_3 & 2,90\cdot t_1^{\ 2} + 2,90\cdot t_2^{\ 2} + 102,90\cdot t_3^{\ 2} & 2,90\cdot t_1^{\ 3} + 2,90\cdot t_2^{\ 3} + 102,90\cdot t_3^{\ 3} \\ \ldots & \ldots & \ldots \\ a_{81} & a_{82} & a_{83} \end{pmatrix} \cdot \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

Once the matrix is solved through ordinary least squares method (OLS), we obtain the following results for the betas:

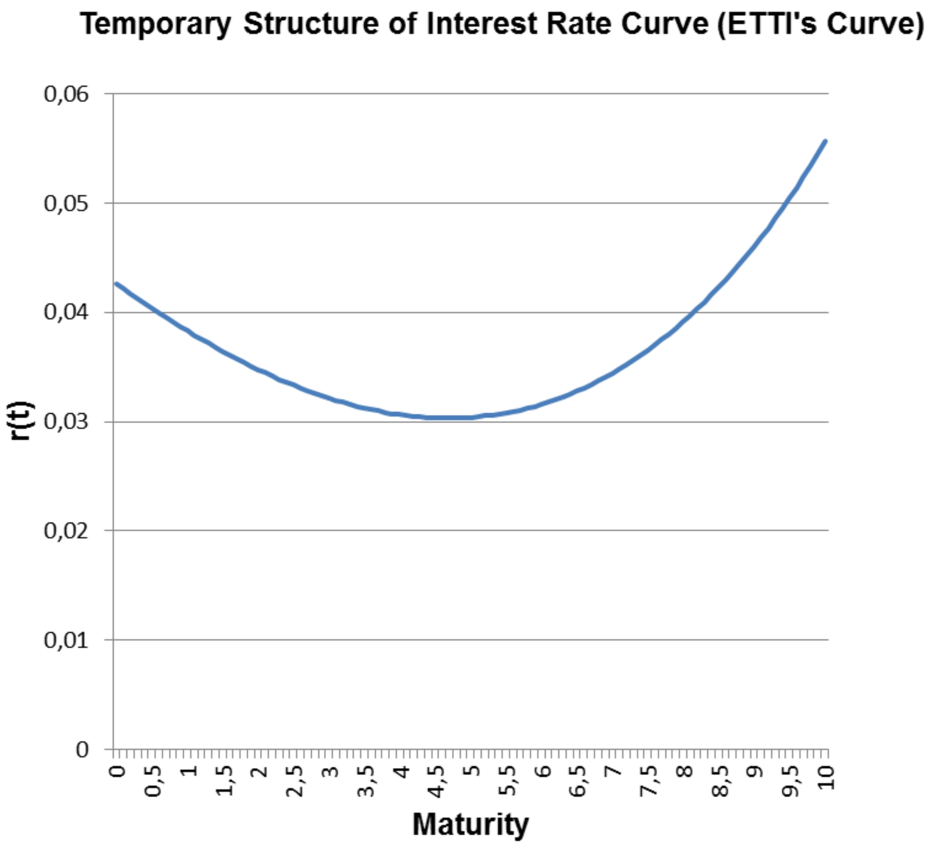
$$\beta_0 = 1$$
$$\beta_1 = -0,0426$$
$$\beta_2 = 0,0032$$
$$\beta_3 = -0,0002$$

According to the initial assumptions, $\beta_0 = 1$. Parameters have economic sense since $\beta_0 = 1$. So the chart we obtain for the discount function is:



As we can observe in the chart, the discount function is a decreasing function, less than a unit and greater than zero. Then, there is nothing indicating that the obtained result is wrong. Now, we can proceed calculating the results for the second period of time (01.09.2011) in order to be able to compare both charts and data. We take the first bond in the second period as an example to show how to proceed with the data.

Then, we can represent the ETTI's curve as follows:



In the chapter called "comparing results", we will analyse and compare the charts obtained for both periods.

6.4.2 Second period of time: in-crisis (2011)

For the second period of time, bonds and liabilities available as of 01.09.2011 are the following:

E M I S I O N	NUMERO OPERACS	IMPORTE CONTRATADO	PRECIO (EX-CUPON) MEDIO	MAXIMO	MINIMO	RENDTO. INTERNO MEDIO	ANTERIOR PRECIO MEDIO (FECHA)
ES00000120L4 B EST 3.90 31.10.12	1	1,00	100,850	100,850	100,850	3,13	100,830 (19/08/2011)
ES0000011660 O EST 6.15 31.01.13	7	80,00	103,780	103,830	103,720	3,33	103,974 (30/08/2011)
ES00000121T5 B EST 2.30 30.04.13	6	49,00	98,468	98,500	98,385	3,26	98,472 (31/08/2011)
ES0000012866 O EST 4.20 30.07.13	2	15,00	101,412	101,435	101,365	3,42	101,550 (31/08/2011)
ES00000121H0 B EST 4.25 31.01.14	5	68,60	100,384	101,581	98,838	4,07	101,601 (31/08/2011)
ES00000123D5 B EST 3.40 30.04.14	4	19,75	99,151	99,160	99,150	3,74	99,272 (31/08/2011)
ES0000012098 O EST 4.75 30.07.14	7	45,60	102,741	102,790	102,700	3,73	102,920 (31/08/2011)
ES00000121P3 B EST 3.30 31.10.14	3	40,00	98,749	98,850	98,720	3,73	98,860 (31/08/2011)
ES0000012916 O EST 4.40 31.01.15	1	5,00	101,660	101,660	101,660	3,86	101,913 (31/08/2011)
ES00000122F2 B EST 3.00 30.04.15	8	40,00	96,961	97,000	96,895	3,91	97,180 (31/08/2011)
ES00000120G4 O EST 3.15 31.01.16	6	40,00	96,071	96,200	96,000	4,14	96,380 (31/08/2011)
ES00000122X5 B EST 3.25 30.04.16	40	420,00	95,539	95,765	95,385	4,32	95,917 (31/08/2011)
ES00000123J2 B EST 4.25 31.10.16	112	983,30	98,951	99,135	98,750	4,49	99,399 (31/08/2011)
ES00000120J8 O EST 3.80 31.01.17	13	319,81	96,803	97,115	96,726	4,48	97,280 (31/08/2011)
ES0000012783 O EST 5.50 30.07.17	4	45,04	104,954	105,102	104,100	4,52	105,650 (25/08/2011)
ES00000121A5 O EST 4.10 30.07.18	1	0,06	97,000	97,000	97,000	4,62	97,100 (30/08/2011)
ES00000122D7 O EST 4.00 30.04.20	10	65,00	94,509	94,760	94,395	4,79	94,787 (30/08/2011)
ES00000122T3 O EST 4.85 31.10.20	11	80,50	99,107	99,555	98,900	4,97	99,534 (31/08/2011)
ES00000123B9 O EST 5.50 30.04.21	29	243,50	103,096	103,400	102,845	5,08	103,509 (31/08/2011)

The selected bonds and liabilities are those with due date greater than 01.09.2012 or less than 01.09.2021. Within these have been selected those eight that have at least 10 trades and among them, those with more hiring. Therefore, the chosen ones are the following:

Bond	**Excoupon Price**	**Coupon**	**IRR**	**Due date**	**Days until first coupon**
ES0000011660 O EST	103,830	6,15	3,33	31.01.2013	153
ES00000122F2 B EST	96,961	3,00	3,91	30.04.2015	242
ES00000122X5 B EST	95,539	3,25	4,32	30.04.2016	242
ES00000123J2 B EST	98,951	4,25	4,49	31.10.2016	61
ES00000120J8 O EST	96,803	3,80	4,48	31.01.2017	153
ES00000122D7 O EST	94,509	4,00	4,79	30.04.2020	242
ES00000122T3 O EST	99,107	4,85	4,97	31.10.2020	61
ES00000123B9 O EST	103,096	5,50	5,08	30.03.2021	211

As in the first example, we have the following information related with the liability ES0000011660 O EST:

Due date	Coupon	Average price	Date of last coupon received	Days after	Days left
31.01.2013	6,15	103,830	31.01.2011	212	153

From this information, in order to calculate the dependent variable Y_T, we have to obtain P, CC and $\sum_{\tau=1}^{T} c$. To calculate CC:

$$CC = \frac{c}{365} \cdot t_{daysleft} \text{, so we have that } CC = \frac{6{,}15}{365} \cdot 153$$

The obtained result for the accrued interest (CC) is 2,57794521. Now, we have to obtain the value for the summation of coupons (plus the nominal invested initially):

	t0	t1	t2	Summation
ES0000011660 O EST	6,15	6,15	106,15	118,45

Once we know the values for P, CC and $\sum_{\tau=1}^{T} c$, we can proceed calculating the value of the dependent variable. From $Y_T = P + CC - \sum_{\tau=1}^{T} c$, and replacing the values we have, we obtain:

$$Y_T = 103{,}83 + 2{,}57794521 - 118{,}45$$

The result for the dependent variable is -12,042055. The same steps would be repeated for the next seven bonds and liabilities.

From this point, knowing the value of Y_T, we can continue calculating the independent variable. For this, we have that

$$-12{,}042 = 6{,}15 \cdot v(t) + 6{,}15 \cdot v(t) + 106{,}15 \cdot v(t)$$

The next step is to extract the independent variable $v(t)$ and replace it with the functional form of the discount function (or value function) that is $v(t) = \beta_1 t + \beta_2 t^2 + \beta_3 t^3$:

$$-12,042055 = 6,15(\beta_1 t + \beta_2 t^2 + \beta_3 t^3) + 6,15(\beta_1 t + \beta_2 t^2 + \beta_3 t^3) + 106,15(\beta_1 t + \beta_2 t^2 + \beta_3 t^3)$$

Once we have this, we have to prepare the components that will form the matrix, where we will estimate the value of the unknown parameters (β_1 , β_2 and β_3). The first row of the matrix:

$$\begin{pmatrix} 6,15 \cdot t_1 + 6,15 \cdot t_2 + 106,15 \cdot t_3 & 6,15 \cdot t_1^2 + 6,15 \cdot t_2^2 + 106,15 \cdot t_3^2 & 6,15 \cdot t_1^3 + 6,15 \cdot t_2^3 + 106,15 \cdot t_3^3 \\ \ldots & \ldots & \ldots \\ a_{81} & a_{82} & a_{83} \end{pmatrix}$$

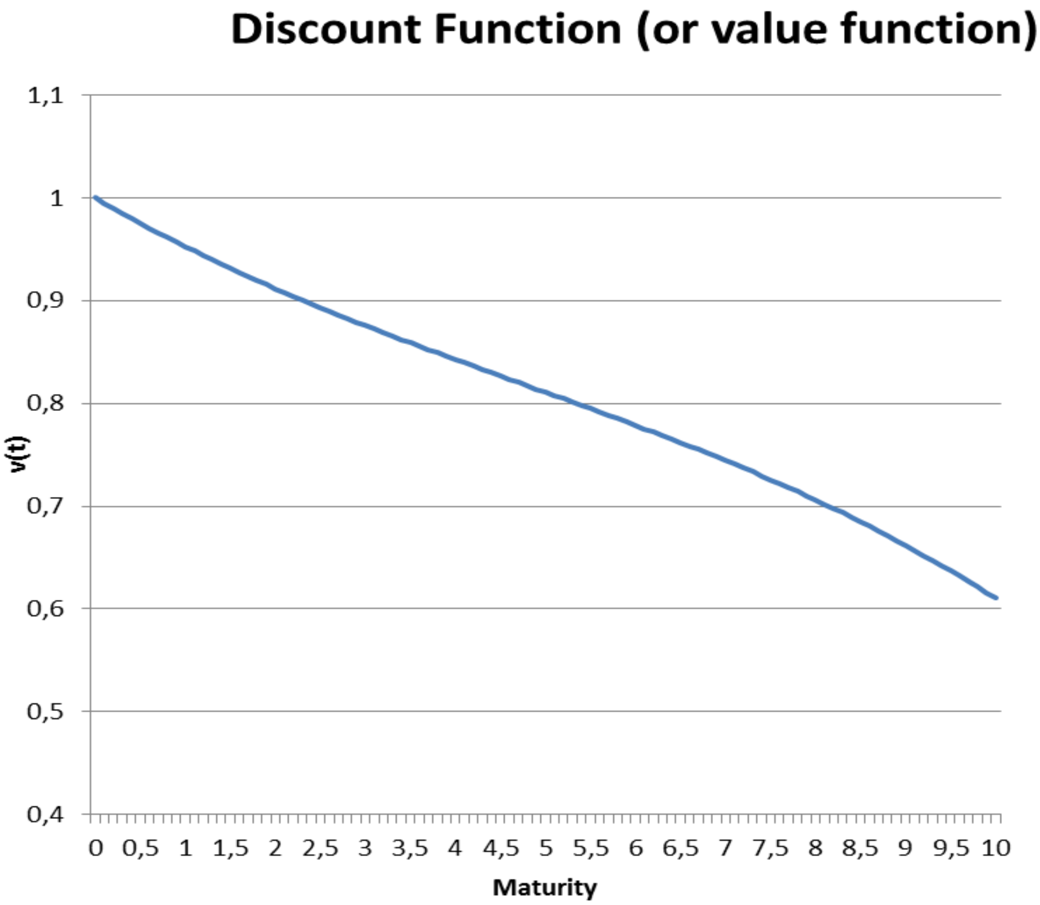
The seven remaining rows of the matrix correspond to the rest of selected bonds. Knowing the values for t, as we do, we will solve the equations where the unknown parameters are the betas, through the following matrix:

$$Y_T = \begin{pmatrix} 6,15 \cdot t_1 + 6,15 \cdot t_2 + 106,15 \cdot t_3 & 6,15 \cdot t_1^2 + 6,15 \cdot t_2^2 + 106,15 \cdot t_3^2 & 6,15 \cdot t_1^3 + 6,15 \cdot t_2^3 + 106,15 \cdot t_3^3 \\ \ldots & \ldots & \ldots \\ a_{81} & a_{82} & a_{83} \end{pmatrix} \cdot \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

Once we solve the matrix applying the ordinary least squares method (OLS), we obtain the following results for the betas:

$$\beta_0 = 1$$
$$\beta_1 = -0,0512437$$
$$\beta_2 = 0,0041406$$
$$\beta_3 = -0,0002912$$

If we remember the initial assumptions, we have that $\beta_0 = 1$. These parameters have economic sense since $\beta_0 = 1$. So the chart we obtain for the discount function is:

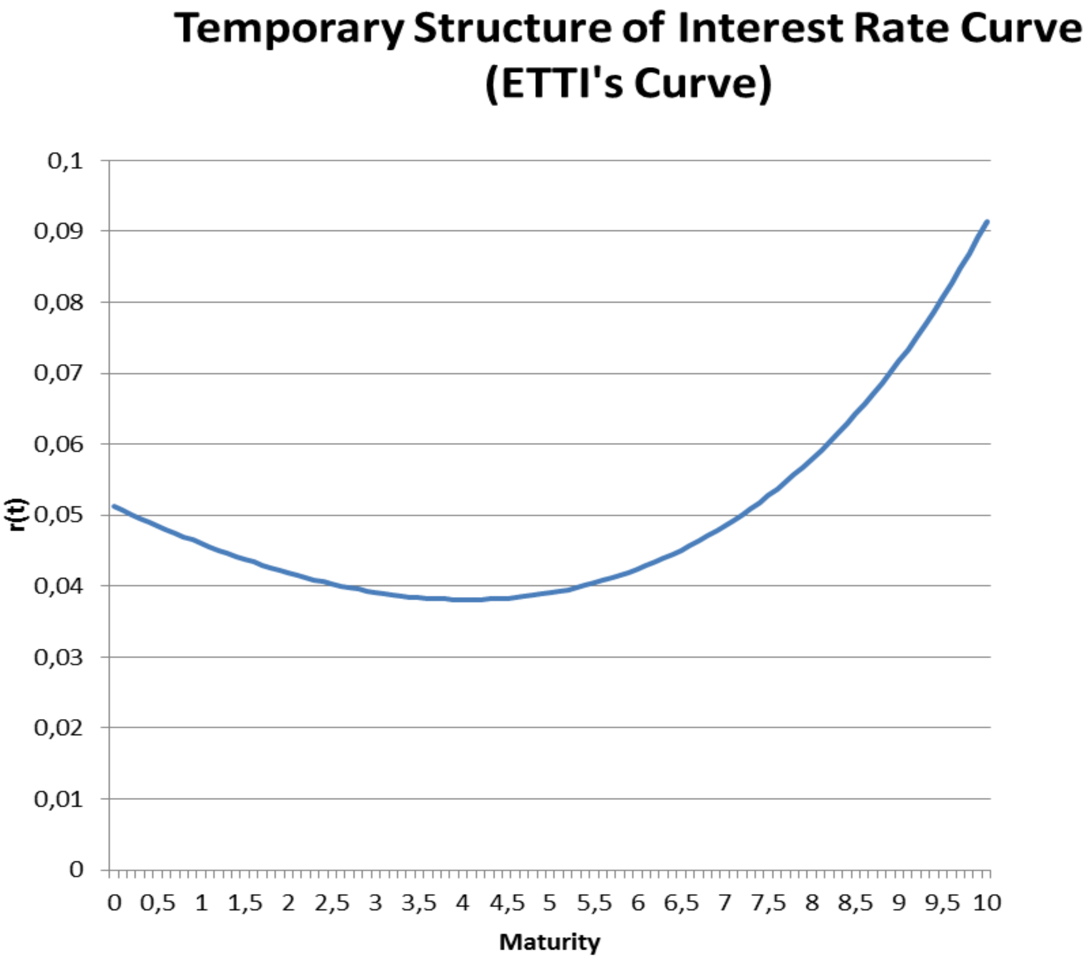


As we can observe in the chart, the discount function is a decreasing function, less than one unit and greater than zero. Then, there is nothing indicating that the result is wrong.

Arrived to this point, we can calculate $r(t)$ since we have all required data. Remember that we obtained the $r(t)$ functional form some chapters before (1.1). Knowing $v(t)$, as we do, we can solve the $r(t)$ function. By this way, the function of the ETTI corresponds to a $v(t)$ function, given by:

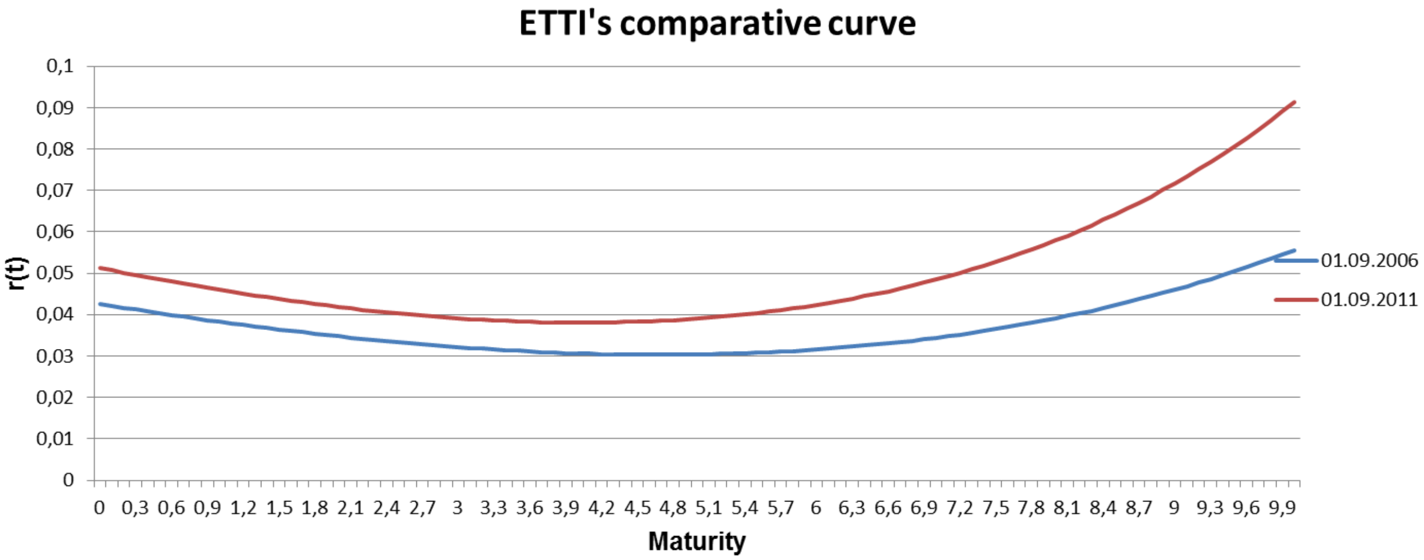
$$r(t) = -\frac{d(\ln(v(t))}{dt} = -\frac{\beta_1 + 2\beta_2 t + 3\beta_3 t^2}{\beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3}$$

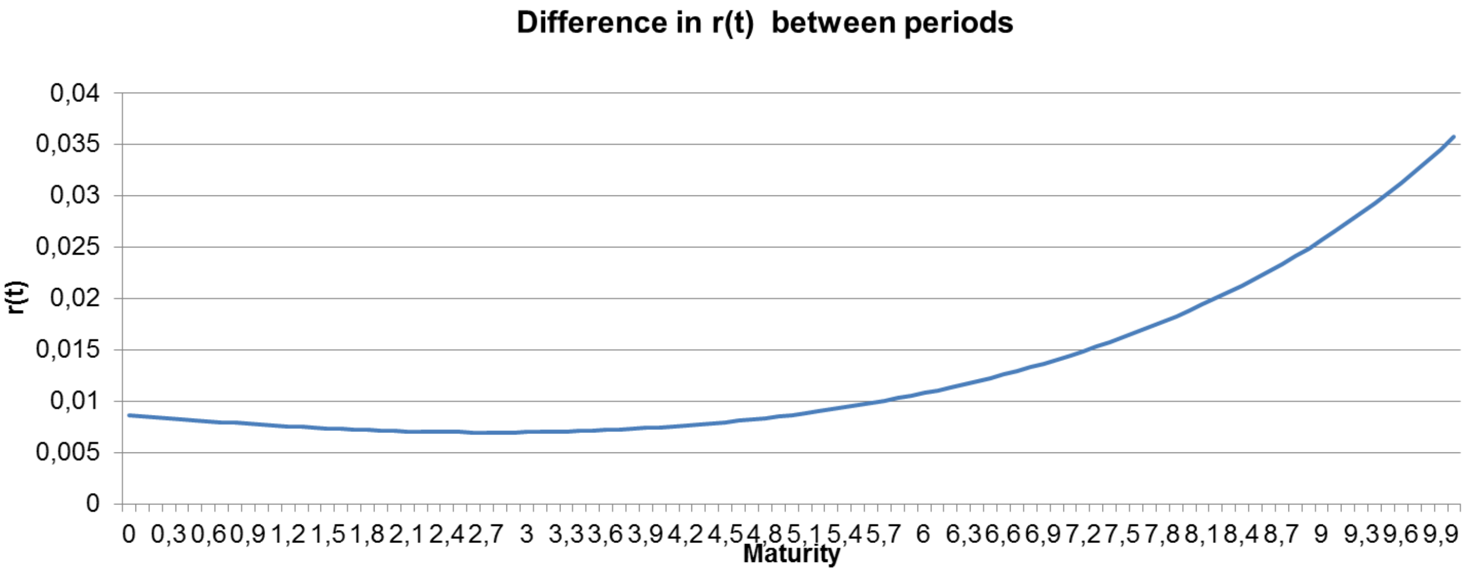
Now, we just have to replace the value of the betas obtained before and we wil get the results for the ETTI. Then, we can represent the chart of the ETTI for the second period as follows:



As we explained with the previous example, we will analyse and compare the charts for both periods in the following chapter.

6.4.3 Comparing results





The difference between the longer term, r(t = 10), and the shorter-term rate, r(t=0), is higher in 2011 than in 2006. This is as an approximation to the slope of the temporary structure. A steeper slope means that for a longer period, in 2011 required a further increase in bond yields. This could be because it has increased the degree of risk aversion of individuals.

It seems that from 2006 to 2011 there is a parallel shift up of the ETTI for short maturities, around 1%.

For long maturities (7, 8, 9 or 10 years), the difference between the interest rates of 2006 and 2011 is much higher (up to 3.5%).

7 Conclusions

The yield curve is a tool used to describe the rates of returns of a set of papers, with the same credit structure but with different maturity, as a means to represent in an approximate way, the temporary structure of interest rates. Its main use is given in monetary policy decisions on the projection of the cycles of expansion or contraction of the economy. Also, its knowledge is of great interest to those who plan their investments, making decisions based on valuation, trading or hedging financial instruments.

The phenomenon of financial globalization creates the need for the proliferation of new techniques and instruments of a financial nature for decision-making by the agents; it should also be considered in this area the development of computer and communication systems that facilitate the interconnection of markets to be targeted more to the relationship risk-performance, and in this sense the temporary structure is suggested as a useful technique for making investment decisions, since it has been adopted by the financial systems of several countries.

It has been shown in various studies the great predictive power of the model of Nelson and Siegel (1987), as it has found a high correlation between the prices of securities estimated using the yield curve and the observed ones, which provides the reliability to use the ETTI as a source of information to make investment decisions by the agents.

It is noted that although the model of Nelson and Siegel is the most applied for the estimation of the yield curve, this can also be estimated by other non-parametric models such as model of Svenson and Splines' theories, which show good behaviour in this regard.

Moreover, it has been shown that not always the movements of short term interest rates affect the long term ones in the same proportion or direction, since, although they are certainly related, each part of the curve can have independent behaviour.

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