How we move is universal: scaling in the average shape of human activity

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Human motor activity is constrained by the rhythmicity of the 24 hours circadian cycle, including the usual 12-15 hours wake-sleep cycle. However, activity fluctuations also appear over a wide range of temporal scales, from days to few seconds, resulting from the concatenation of myriad of individual smaller motor events. Furthermore, individuals present different propensity to wakefulness and thus to motor activity throughout the circadian cycle. Are activity fluctuations across temporal scales intrinsically different, or is there an universal description encompassing them? Is this description also universal across individuals, considering the aforementioned variability? Here we establish the presence of universality in motor activity fluctuations based on the empirical study of a month of continuous wristwatch accelerometer recordings. We study the scaling of average fluctuations across temporal scales and determine an universal law characterized by critical exponents α, τ and $1/\mu$. Results are highly reminiscent of the universality described for the average shape of avalanches in systems exhibiting cracking noise. Beyond its theoretical relevance, the present results can be important for developing objective markers of healthy as well as pathological human motor behavior.

INTRODUCTION

Circadian modulation results in the most obvious periodicity of human motor activity. However, distinct fluctuations are evident in a wider range of temporal scales, from days to the 24 hs of the aforementioned rhythm and to a few seconds. Indeed, human activity evolves in bursts of all sizes and durations and is known to be scale-free [1–7] (regardless of the origins and intended consequences of said activity) both in terms of the timing between consecutive human actions and in its spectral density.

Despite these results, the mechanisms underlying the aforementioned scale-free behavior of motor activity remain to be elucidated. In most cases, demonstration of scale invariance have been based on the computation of critical exponents from different observables obeying power-law distributions [1–7]. Considering the intermittent nature of human motor activity - comprising brief activity excursions separated by periods of quiescence - a natural approach is the characterization of the average shape of an event. Previous work [9–12] demonstrated that the average shape of a fluctuation obeys, for a large class of processes, scaling laws admitting a scaling function encoding information about temporal correlations in the process.

In the present work we study the application of this approach to time series encoding human motor activity recorded via wristwatch accelerometer recordings over approximately one month. We establish the presence of scale-free behavior in the distribution of the durations of the events as well as in the power spectral density. Afterwards we study, for the first time, scaling in of the shape of an average burst of activity and derive the scaling function in order to determine the associated exponents.

MATERIALS AND METHODS

Recordings analyzed were part of a larger study and included six healthy, non-smokers, drug-free volunteers (mean age 50.1 years, S.D. = 6.8). The study was approved by the Bioethics Commission of the University of Isles Baleares (Spain). Participants were informed about the procedures and goals of the study, and provided their written consent. After determining their handedness each subject was provided with a wristwatch-sized activity recorder (Actiwatch from Mini- Mitter Co., OR, USA) measuring acceleration changes in the forearm in any plane. Each data point of activity corresponded to the number of zero crossings in acceleration larger than 0.01 G (sampled at 32 Hz and integrated over a 30-second window length).

Records of several thousands of data points were kept in the devices internal memory until downloaded to a personal computer every week. Subjects wore the device in their non-dominant arm continuously for up to several weeks (mean 28.1 days, S.D.= 4.). After careful visual inspection of the data to exclude sets with gaps (due to subject non-compliance) a combined total of 280 days of data were available for further analysis.

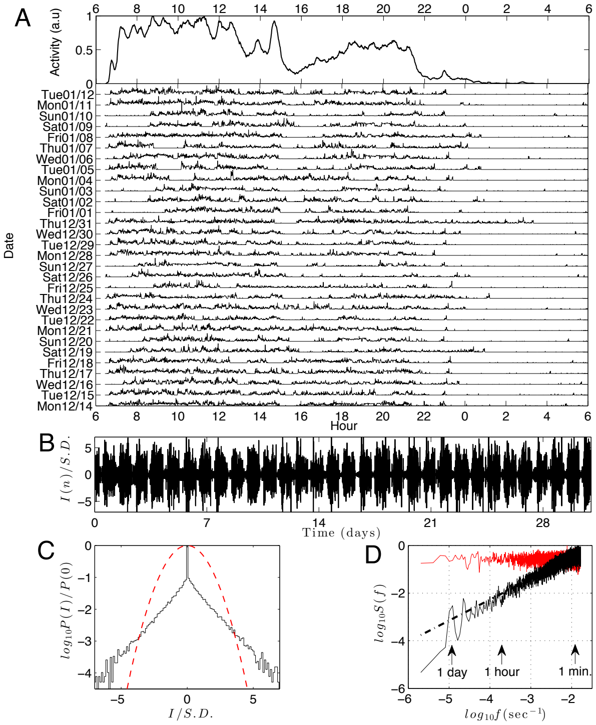

FIG. 1: Example data set, distribution of successive increments and their spectral power. Panel A: Time series of activity $x(n)$ recorded continuously from a subject during a month. Individual traces correspond to consecutive days. The top subpanel depicts daily activity averaged over the entire month. Panel B: Time series of successive increments $I(n) = x(n+1) - x(n)$ (normalized by its SD) for the same data. Panel C: Density function of the increments (continuous line), exhibiting exponential tails (compare with the dotted line, a Gaussian of the same variance). Panel D: Power spectral density (black line) for the signal in panel B. This density is scale invariant $S_d \sim f^{\gamma}$ with $\gamma = 0.9$ (dashed line), corresponding to a decay $S_d \sim f^{-\beta}$ with $\beta = 2 - \gamma = 1.1$ for the raw signal of panel A. In contrast, for the randomly shuffled increments, the serial correlations vanish and a flat spectral density is obtained (red).

RESULTS

For ease of presentation, we will use recordings from a single subject to describe the main results. Nevertheless, results are robust as well as similar for the entire group of subjects in the study. A typical recording is presented in Figure 1. Panel A shows a full month of continuously recorded activity from this subject, who is particularly regular in her daily routines. The subject wakes up with the alarm clock at 6:45 AM on week days and has lunch followed by a short nap each day (at between 14:00 and 16:00). Panel B displays the time series of the successive increments of the signal $x(n)$, defined as $i(n) = x(n+1) - x(n)$.

The large-scale statistical features of the time series presented in Figure 1 are well known. The density distribution of the successive increments $i(n)$ is non-gaussian, as can be appreciated by a joint plot with a gaussian distribution of the same variance (Figure 1, Panel C). The power spectrum of the activity decays as $S_d \sim f^{-\beta}$. This power-law decay was observed over four orders of magnitude for the time series of successive increments. The exponent obtained for this time series was $\gamma = 0.9$ which corresponds to an exponent of $\beta = 2 - \gamma = 1.1$ for the original time series. For comparison, the spectral densities of the actual signal and a surrogate obtained after randomly shuffling the increments are jointly displayed in Panel D of Figure 1.

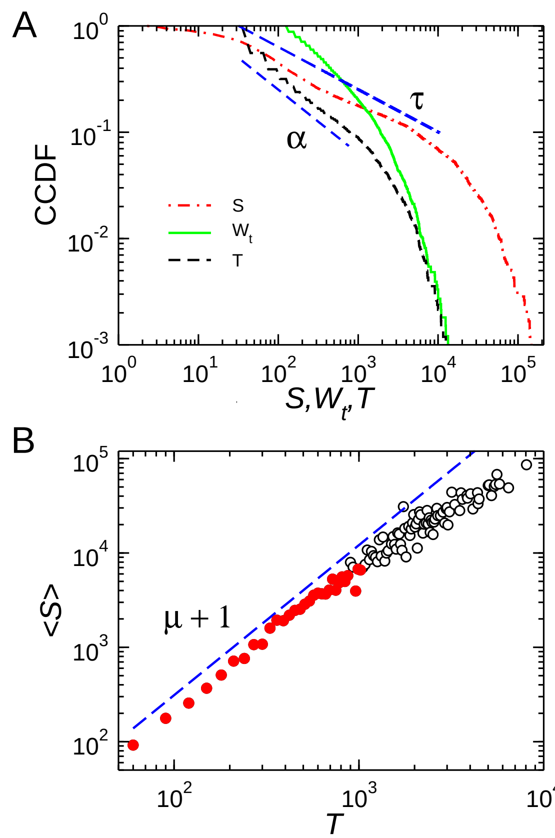

FIG. 2: Scaling of activity events in a single subject (same dataset as in Fig. 1). Panel A: The complementary cumulative distribution function (CCDF) for event durations and sizes can be well described by power-laws with exponents $\alpha = 1.70$ and $\tau = 1.44$ respectively (dashed lines). The waiting time between events falls of exponentially. Panel B: The average size of a given duration is well described (for small T) by $\langle S\rangle(T) \sim T^{\mu+1}$ with $\mu + 1 = 1.59$ (blue dashed line) comparable with results obtained from fitting within the scaling region (red filled symbols) giving $\mu + 1 = 1.61$.

To further study the time series from the perspective of individual bursts of activity we introduce the definition of an event. We consider the time series of activity $x(n)$ and select a threshold value U, to be vanishingly small. An event is defined by the consecutive points starting when $x(n) > U$ and ending when $x(n) < U$. This is equivalent to the definition of avalanches in other contexts [9, 13]. Here, we are concerned primarily with the statistics of event lifetimes T, as well as of their average size S and shape.

In all subjects we found that the distributions of event durations and sizes (defined by the area, i.e. the integral of the signal corresponding to the individual events) can be well described, for relatively small values, by a power-law (Figure 2, Panel A). In contrast, the distribution of waiting times between events demonstrated an exponential decay. In addition to the scale invariance, we found that the longer an event lasted, the stronger the motor activity executed by the subject, as shown in Panel C of Figure 2. The plot of average event size $\langle S\rangle$ as a function of duration T follows a power-law (for small values of T) described by $\langle S\rangle(T) = T^{\mu+1}$ with $\mu + 1 = 1.59$. The exponents in this power-law are robust across subjects and to changes of threshold over a reasonable range of values.

This type of scaling is well-known in the statistical mechanics of critical phenomena [13]. Examples range from earthquakes [14] to active transport processes in cells [15], crackling noise [12], the statistics of Barkhausen noise in permalloy thin films [10] and plastic deformation of metals [16]. In all these cases, the distributions obey universal functional forms:

$$f(S) \sim S^{\tau} \tag{1}$$

$$f(T) \sim T^{\alpha} \tag{2}$$

$$\langle S\rangle(T) \sim T^{1/\sigma\nu z} \tag{3}$$

where f denotes the probability density functions of the size of the event S and its duration T, and $\langle S\rangle(T)$ is the expected size for a given duration. The parameters τ, α and $1/\sigma\nu z$ are the critical exponents of the system and are expected to be independent of the details, being related to each other by the scaling relation:

$$\frac{\alpha - 1}{\tau - 1} = \frac{1}{\sigma\nu z} \tag{4}$$

We found that the empirical exponents very closely fulfill the expression above. Using the fitting approach introduced by Clauset [17] in the scaling regions depicted in Panel A of Figure 2, we found $\tau = -1.44$ and $\alpha = -1.70$. Thus, from Eq. 4 a value of $1/\sigma\nu z = \mu + 1 = 1.59$ is expected. The experimental data points are very close to this theoretical expectation (dashed line), especially for the relatively small T values within the scaling region of Panel A (where a linear fit estimates $\mu + 1 = 1.61$), while those for relatively larger T values (corresponding to the cutoff of the distributions) are a bit apart, probably due to undersampling. After repeating this analysis for all subjects in our sample, the average exponents were all within 5% of the reported values.

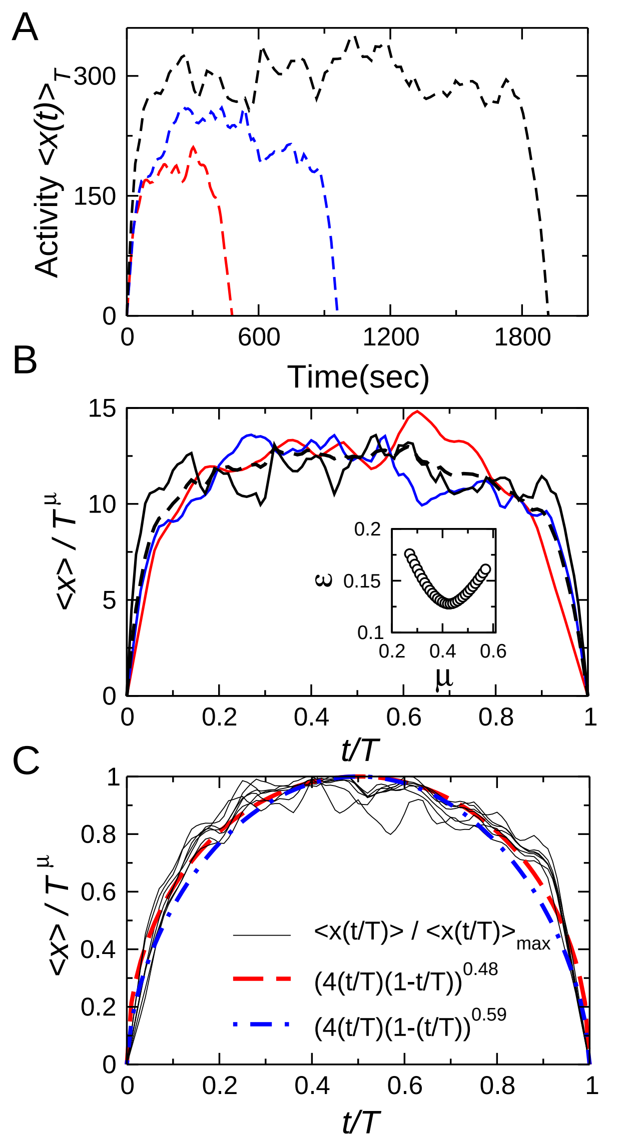

FIG. 3: Collapse of events of different duration into a single functional form. Panel A: Three examples of typical events of duration T=480, 960 and 1920 sec.. Panel B: The heterogeneous events shown in Panel A can be collapsed onto the average shape (dashed black line) by normalizing t to t/T and $\langle x(t)\rangle$ to $\langle x(t)\rangle/T^{\mu}$. The inset shows the cumulative variance for a range of μ. Panel C: The average event shape, i.e., $f_{shape}(t/T)$, recovered from six data sets (thin lines). The best fit using an inverted parabola is shown as a red dashed line ($\mu = 0.49$) as well as the one expected from the critical exponent $\mu = 0.59$ as a dot-dashed blue line.

From scaling arguments, it is expected that the average shape of an event of duration T $\langle x(T,t)\rangle$ scales as :

$$\langle x(T,t)\rangle = T^{\mu} f_{shape}(t/T), \tag{5}$$

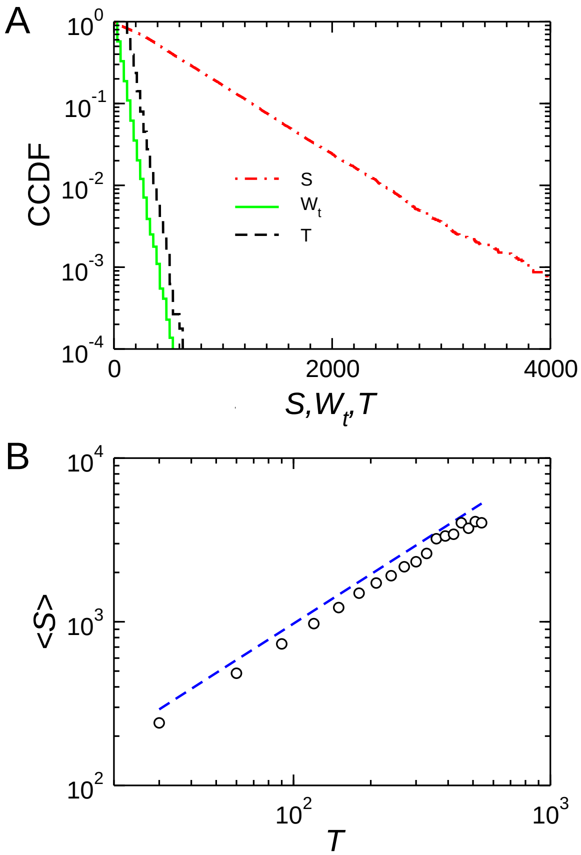

FIG. 4: Scaling is absent in a null model resulting from defining events after randomly reordering the time series $x(n)$. Panel A: Density distributions for event duration, size, and waiting time. All the distributions are exponential (note the logarithmic-linear scale). Panel B: The expected average size for a given duration in the null model is a linear function of T (the dashed line represents the fit with slope 1), therefore, $\mu = 0$ and there is no collapse.

Thus, the shapes of events of different durations T rescaled by μ should collapse on a single scaling function given by $f_{shape}(t/T)$. Note that μ corresponds in this context to the wandering exponent (i.e., the mean squared displacement) of the activity.[18, 19].

Examples of this collapse are presented in Panels A and B of Figure 3. Considering the number of events here averaged (in the order of $N \sim 10^2$) the data collapse is quite satisfactory, while the value of the exponent ($\mu = 0.48$) does not exactly match the one predicted in Eq. 4 ($\mu = 0.59$), likely a consequence of insufficient sampling. To determine the generality of our results we extended this analysis to six other data sets. For each data set the value of μ was first determined. Subsequently, the $x(T, t)$ obtained from the events were rescaled with T^{μ} and their average computed. To account for individual differences in mean activity, shape functions were normalized by their mean value. The results for the six datasets are presented in Panel C of Figure 3. They can be accurately described by an inverted parabola, as in other systems previously studied using this method. The best fit disagrees with the empirical functions near their peak, being the latter flatter, likely an effect related to saturation observed in long events.

Finally we turn to discuss potential null models. We consider two extreme cases, in both of them the raw time series are randomly shuffled to remove serial correlations. In the first case we remove all temporal correlations by randomly reordering $x(n)$, thus attaining a flat power spectral density. After repeating the above analysis in this surrogate data set, become clear, as shown in Figure 4, that the scale invariance is absent in all the statistics under study: size S, waiting time W_t and duration T of events (note that the distributions are here plotted using a logarithmic-linear scale). Results in Panel B show that $\mu + 1 = 1$, thus $\mu = 0$, implying that there is not collapse, because with $T^{\mu} = 1$ in Eq. 5, the amplitude of the individual events remains invariable. To consider the second case, we need first to reorder randomly the time series of increments $I(n)$ and then proceed to integrate the increments. Since each increment is a random variable, the power spectral density for this surrogate process is known to obey $1/f^{\beta}$ with $\beta = 2$, and as shown analytically by Baldassari et al. [18] for this case $\mu = 1/2$ and the scaling function is demonstrated to be proportional to a semicircle. We remark that the fluctuations of human activity described here differ from a simple autoregressive process: indeed successive increments $I(n)$ are anti-correlated and the power spectral density corresponds to non-trivial power law correlations (i.e., $\beta \neq 2$).

DISCUSSION

The present findings can be summarized by six stylized facts describing bursts of human activity : I)The spectral density of the time series of activity $x(t)$ obeys a power law, with exponent $1 < \beta > 0$; II) successive increments $i(t)$ are anti-correlated with a spectral density obeying a power law with exponent $\gamma = \beta + 2$; III) the PDF of the increments $i(t)$ is definitely non-gaussian; IV) the PDF of duration and sizes of events obeys truncated power laws with exponents $2 < \tau > 1$ and $2 < \alpha > 1$; V) the average size of the events scales with its lifetime T as $\langle S \rangle (T) \sim T^{\mu}$, where $\mu + 1 = (\alpha - 1)/(\tau - 1)$; VI) the time series of individual events can be appropriately rescaled via a transformation of its duration T and amplitude $x(t)$ onto a unique functional shape: $\langle x(T, t) \rangle = T^{\mu} f_{shape}(t/T)$.

We are aware that these observations are novel only for human activity, because similar statistical regularities of avalanching activity are well known for a large variety of inanimate systems [9–12]. The rescaling of the average shape is not surprising either because, placed in the appropriate context, can be traced back to Mandelbrot' study of the fractal properties of self-affine functions [20]. A curve or a time series is said to be self-affine if a transformation can be found such that rescaling their x, y coordinates by k and k^μ respectively the variance in y is preserved (with $\mu = 1$ corresponding to self-similarity). In that sense the successful collapse of the events shape is a trivial consequence of the overall self-affinity of the $x(t)$ time series.

Thus, it is clear that the existence of the scaling uncovered here is not informative per se of the type of mechanism behind: scale-invariance can be constructed via different processes, ranging from critical phenomena [13] to simple stochastic auto-regresive dynamics [18, 19]. What is then the mechanism by which the above six facts are generated?

It seems that this question can not be easily answered by the type of experiments reported here. Fluctuations of this type could have either an intrinsic (i.e., brain-born) origin but also being the reflection of a collective phenomena. In either case the correlations observed seem to reject the case of independent random events starting and stoping human action, because neither the distribution of the increments $I(n)$, nor the exponents match the case of a random walk. In terms of brain-born process, is hard to accept some of the implications of the scaling function in the shape. The average parabolic shape means that the very beginning of the motion' activity contains information about how long the activity will last, in the same sense that the initial trajectory of a projectile predicts when and where it will land. This proposal is hardly realistic, because there is not physiological argument to support motor planing for the length of time we are observing ($\sim 10^3$ secs). In terms of collective processes, the results here suggest that the interaction with other humans could determines when and where on the average we start and stop moving. Further experiments and analysis should shed light on these possibilities, the present results providing a guide and six important constraints for the models that should best capture the physics (and biology) of the process.

Acknowledgments

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