Investments, Positive Externalities, and Majority Bargaining

by

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This study analyzes the welfare implications of requiring either unanimity or a simple majority in negotiations to distribute a budget among three agents who could previously invest to generate positive consumption externalities for others. This complements Cardona and Rubí-Barceló (2014), who consider only the unanimity case. We show that reducing the majority requirement reduces the profitability of investments, and consequently alleviates overinvestment, which is predominant under unanimous bargaining. Nevertheless, requiring a simple majority reduces the aggregate surplus attained in the bargaining stage. Therefore, the relative performance of the bargaining rules is uncertain. We show how the size of consumption externalities affects this performance.

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1 Introduction

In many situations, the members of a committee – who must bargain about how to distribute a divisible good among them – have the opportunity to make a previous investment that will increase the positive externalities that their shares will generate for others. These situations range from (i) productivity investments by the branch managers of a firm preceding the negotiations to distribute a given budget among them, to (ii) legislative budgeting environments in which legislators may "shape" the projects they will propose to vote on beforehand. For example, before negotiating how to split a given budget among legislators' projects, each may invest to

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increase the benefits of his or her project for all others. Cardona and Rubí-Barceló (2014) analyze the theoretical framework fitting these and other examples in the case in which unanimous consent is required to approve any proposed distribution of the budget. The present study extends this analysis to a simple-majority setting and examines how this alternative consensus requirement changes the efficiency properties of the equilibria in the three-player case.

We consider the following setting. A committee has a budget to divide among its three members. The share allocated to each individual may generate consumption externalities for all agents, depending on whether he or she carried out a previous investment. After making the investment decisions, agents negotiate the distribution of this budget following a standard legislative bargaining process, Ãă la Baron and Ferejohn (1989), where agreements require the acceptance of at least half of the agents.[1] The existence of positive consumption externalities would lead to underinvestment in the initial stage, as in a classical public-good game. However, investments provide a more favorable bargaining position for investing agents because others are more willing to allocate a larger share to them in the ensuing negotiation. This leads to rent-seeking behavior, which may lead agents' investments to indirectly "hurt" others. This would cause overinvestment in equilibrium, as agents do not consider this negative external effect when deciding. Due to the interaction between these two opposite forces, Cardona and Rubí-Barceló (2014) show that under unanimity, the equilibria can be efficient, but they may also feature either under- or overinvestment. Moreover, since this consensus requirement is particularly profitable for investing agents (because any agreement must always include them), overinvestment is the predominant inefficiency pattern in that setting. The present study illustrates that reducing the consensus requirement to a simple majority has two opposite effects on efficiency. First, some of the benefits from investing are lost because those who invested can now be excluded from the winning coalition. This lower profitability mitigates overinvestment. Second, excluding investing agents from the winning coalition may imply that they receive a lower share of the budget in the bargaining equilibrium. In that case, the aggregate surplus is not maximized. This study intends to illustrate the balance between these two opposite effects. Our results show that requiring a simple majority generates more efficient outcomes than unanimity when consumption externalities are sufficiently large. Otherwise, the rent dissipation caused by the allocation of some budget to noninvesting agents might exceed the positive effects of reducing overinvestment in the initial stage, so outcomes under unanimity are more efficient than under a simple majority.

Our study analyzes how investment decisions and redistribution depend on the specificity of the decision process, particularly the consensus required in the negotiation stage. Thus, this paper contributes to the literature on the performance of alternative majority rules. Additionally, our model analyzes individual investments

[1] Calvert and Dietz (2005) study a three-player simple-majority bargaining game with symmetric positive consumption externalities.

in collective decisions, which is the main issue in the extensive literature addressing the *holdup problem*. This literature analyzes how the nature of investments (basically, their effects on the ensuing bargaining game) alters the possibilities that the rents individuals generate are, at least partially, expropriated (see Che and Hausch, 1999; de Meza and Lockwood, 2010; Grout, 1984; Hart and Moore, 1988; Rogerson, 1992, among others). Harstad's (2005) study of the effects of majority rules on investment decisions in a binary-choice problem is also related to our work. The author models the collective decision process as a protocoalition majority bargaining game that requires unanimity within the coalition. Investments increase the utility of one of the alternatives for the investing agent and therefore the aggregate surplus in the negotiation stage. His results show that when unanimity is required, the usual underinvestment problem arises. Under weaker consensus requirements to form winning protocoalitions, he assumes partial transferability of utility in the sense that utilities are perfectly transferable among agents within the formed coalitions, but not from the excluded agents to the coalition members. This implies that investing agents included in the coalition generate a higher collective surplus. Moreover, as any winning coalition may expropriate outsiders, this generates strong incentives to invest, which can induce overinvestment when the required majority is small enough. The study characterizes the efficient (super)majority rule by finding the consensus requirement that better balances such incentives with the holdup problem of underinvestment. Hence, while Harstad (2005) shows that a simple-majority rule enhances investments, our results suggest the opposite when considering a legislative bargaining game without transferable utilities.

In the next section we present the model. The main object of this study is to analyze the effects of the consensus requirement on the efficiency of equilibria, which we do in section 4. However, we are also interested in disentangling the origin of the inefficiencies in that they can generally arise from two different collective decisions of the game: the investment choices or the budget allocation decision. In section 3, we characterize the inefficiencies arising from the first choice when the negotiation requires either unanimity or a simple majority. Section 5 discusses some assumptions and generalizations, and section 6 concludes. The appendix contains the proofs not included in the main text, and some additional tables.

2 *The Model*

The model we consider builds on Cardona and Rubí-Barceló (2014), henceforth CRB. A committee of three players, $N = \{1,2,3\}$, follows a two-stage game to distribute a fixed divisible budget among them. First, agents individually decide whether to make an investment that costs $c > 0$, and second, they negotiate how to divide the budget, through a standard legislative bargaining process. Each agent benefits from his or her own share and the share allocated to the agents who invested in the first stage. That is, the shares of the investing agents generate positive consumption externalities.

We denote the investment choice of agent i by $m_i \in \{0,1\}$, where $m_i = 1$ when i invests, and $m_i = 0$ otherwise. Let $\mathrm{m} \in M = \{(m_1, m_2, m_3) : m_i \in \{0,1\},\ i = 1,2,3\}$ denote the investment profile, and $m = \sum_{i=1}^{3} m_i$ denote the number of investing agents. Let h and l denote a generic investing and noninvesting member, respectively.

Once m is selected, negotiations proceed over discrete infinite time as follows. In each period $t \geq 0$, one player $j \in N$ is randomly selected as the proposer with equal probability. This player proposes an allocation of the budget

$$\mathrm{x}^j = (x_1^j, x_2^j, x_3^j) \in X = \left\{(x_1, x_2, x_3) \in \mathbb{R}_+^3 : \sum\nolimits_{i=1}^{3} x_i \leq 1\right\},$$

where x_i^j denotes the part of the budget assigned to player i. Then, sequentially, all other players respond by either accepting the proposal or not. A *simple majority* is required for approval, so the budget is divided as prescribed by the proposal when it receives support from at least one responder. In case of approval, the proposed allocation is implemented and the game ends; otherwise, the game moves to period $t+1$ with a new randomly chosen proposer.

We define agents' preferences over both the particular share of the budget they receive and the part of the budget assigned to investing agents. Specifically, upon agreement on $\mathrm{x} \in X$ in period t, player $j \in N$ obtains utility $\delta^t u_j(\mathrm{x}) - c \cdot m_j$, where $\delta < 1$ is the discount factor, and

$$u_j(\mathrm{x}) = x_j + \beta y,$$

with $\beta \in (0,1)$ and $y = \sum_{i \in N} m_i x_i$. Perpetual disagreement yields utility $-c \cdot m_j$. Although all proofs in the appendix are valid for any $\delta < 1$, in the main text we focus on the limiting case $\delta \to 1$.

Let $H_i^P(\mathrm{m})$ [respectively, $H_i^R(\mathrm{m})$] denote the subset of the decision nodes of the extensive-form game where i is the proposer [the responder], when m is played. A *(behavioral) strategy* for a generic player i consists of a triple $s_i = (\mu_i, \pi_i, a_i)$, where (1) $\mu_i \in [0,1]$ is the probability that agent i invests, (2) $\pi_i : H_i^P(\mathrm{m}) \to \mathcal{P}(X)$ is the proposal function of agent i, where $\mathcal{P}(X)$ is the set of Borel probability measures over X, and (3) $a_i : X \times H_i^R(\mathrm{m}) \to [0,1]$ is the response rule, indicating the probability that agent i accepts a proposal at any node. To avoid complicating the analysis with measurability issues, we can assume that π_i has finite support for any $h \in H_i^P(\mathrm{m})$. We use $\pi_i(\mathrm{x}|h)$ to denote the probability that agent i proposes $\mathrm{x} \in X$ at node $h \in H_i^P(\mathrm{m})$.

Our equilibrium concept is a refinement of the subgame-perfect equilibrium, where the equilibrium strategies must also satisfy (i) $\mu_i \in \{0,1\}$, (ii) $a_i(x, h_i^R) \in \{0,1\}$, and (iii) stationarity, meaning that (a) proposals depend only on m – that is, $\pi_i : M \to \mathcal{P}(X)$ – and (b) response rules depend only on m and the proposal itself.[2] This implies that at any bargaining stage, a profile of such stationary equilibrium

[2] Although condition (ii) would arise in any equilibrium where (i) and (iii) are imposed, we assume it directly for simplicity.

strategies induces a constant expected outcome profile $(\overline{u}_1(\mathrm{m}),\overline{u}_2(\mathrm{m}),\overline{u}_3(\mathrm{m}))$. This determines the individual acceptance sets $A_i(\mathrm{m}) = \{\mathrm{x} \in X : u_i(\mathrm{x}) \geq \delta\overline{u}_i(\mathrm{m})\}$, for $i = 1,2,3$, and the simple-majority acceptance set as $A(\mathrm{m}) = A_{12}(\mathrm{m}) \cup A_{13}(\mathrm{m}) \cup A_{23}(\mathrm{m})$, where $A_{ij}(\mathrm{m}) = A_i(\mathrm{m}) \cap A_j(\mathrm{m})$. We refer to this equilibrium as a *refined equilibrium.*

Definition *A refined equilibrium (RE) consists of a profile of stationary behavioral strategies* $\{(\mu_i,\pi_i,a_i)\}_{i=1}^3$ *satisfying the following conditions:*[3]

1. $\mu_i = 1$ *if* $\overline{u}_i(1,m_j,m_k) - c \geq \overline{u}_i(0,m_j,m_k)$*, where* $j,k \neq i$ *for* $i,j,k \in N$*; and* $\mu_i = 0$ *otherwise.*
2. *For all* $\mathrm{m} \in M$*,* $\pi_i(\mathrm{x}^i|\mathrm{m}) > 0$ *only if* $\mathrm{x}^i \in \operatorname{argmax}_{\mathrm{x}\in A(\mathrm{m})} u_i(\mathrm{x})$.
3. *For all* $\mathrm{m} \in M, \mathrm{x} \in X$*, one has* $a_i(\mathrm{x},\mathrm{m}) = 1$ *if* $\mathrm{x} \in A_i(\mathrm{m})$*, and* $a_i(\mathrm{x},\mathrm{m}) = 0$ *otherwise.*

An RE determines both the investment profile m^* and an expected utility of $\overline{u}_j(\mathrm{m}^*)$ for any $j \in N$. Such an equilibrium investment profile is said to be *stable*.

We say that agent i *compensates* agent j when $\mathrm{x}^i \in A_j(\mathrm{m})$. In this context with positive consumption externalities, i might compensate j either *directly* – by offering j a share of the budget – or *indirectly* through the externalities generated by the shares allocated to investing agents. The exact form of compensation determines whether simple-majority bargaining maximizes the aggregate surplus, which occurs when only investing agents share the budget.

Regarding the investment choices, it is also important to see how the presence of externalities affects the efficiency of the individual decision. To allow for this question, we say that m is *r-efficient* (r refers to restricted) if the aggregate surplus at the corresponding equilibrium of the bargaining stage is maximal. That is,

$$W_{\mathrm{m}} = \sum_{j\in N}\overline{u}_j(\mathrm{m}) - c\cdot m \geq \sum_{j\in N}\overline{u}_j(\mathrm{m}') - c\cdot m' = W_{\mathrm{m}'} \quad \text{for any } \mathrm{m}' \in M.$$

The concept of r-efficiency we define above only refers to the investment profile m, as the allocation of the budget is taken from the bargaining equilibrium. A comparison between stable and r-efficient investment profiles, which we carry out in the next section, will illustrate the inefficiencies in the first stage of the game. However, a more general notion of (unrestricted) efficiency would specify both an investment profile and a budget allocation (m,x) that maximizes the aggregate surplus. Note that externalities make utilities (ex post) nontransferable, and as we stated, the budget is efficiently allocated only when noninvesting agents do not receive any positive share. Thus, inefficiencies can generally come from two different sources: the investment profile and the budget allocation. The welfare analysis developed in section 4 compares the aggregate utility attained at equilibrium under unanimity and simple majority, where these two sources of inefficiency will affect the comparison.

[3] Note that we study a game with perfect recall, and therefore the set of equilibria using behavioral strategies is equivalent to that using standard strategies (see Kuhn, 1953).

3 Stability versus Efficiency

In this section, we characterize the RE of the strategic game defined in the previous section and the r-efficient investment profiles. We start by exploring the simple-majority setting and then reproduce CRB's results for the unanimity case.

3.1 Simple-Majority Bargaining

Given an investment profile m $= (m_1, m_2, m_3)$, bargaining proceeds by players alternating offers. Any RE is characterized by three (stationary) expected utilities $(\overline{u}_1, \overline{u}_2, \overline{u}_3)$.[4] Our first result in appendix section A.1 shows that in any RE, agents reach a bargaining agreement in the first round (Lemma A1). In this equilibrium, any player proposes an allocation that maximizes his or her utility among all allocations that at least one responder accepts. Consequently, a minimal winning coalition is formed in equilibrium unless the proposer gets all the budget.[5] In appendix section A.2, we show that any bargaining equilibrium must be symmetric; that is, any investing agent (noninvesting agent) will get the same expected utility, denoted as $\overline{u}_h$ ($\overline{u}_l$). Based upon symmetry and the no-delay property, we can characterize the unique bargaining equilibrium for each m, which depends only on the number of investing agents, m. We next summarize the equilibrium expected utilities for any investment profile.

Proposition 1 *For any* m $\in M$, *the RE expected utilities are* $\overline{u}_h = (3\beta + 1)/3$ *if* $m = 3$*;* $\overline{u}_l = 1/3$ *if* $m = 0$*;*

$$\overline{u}_h = \begin{cases} 1+\beta & \text{if } \beta > \frac{1}{2}, \\ \frac{1+\beta}{3-4\beta} & \text{otherwise} \end{cases} \quad \text{and} \quad \overline{u}_l = \begin{cases} \beta & \text{if } \beta > \frac{1}{2}, \\ \frac{1-\beta}{3-4\beta} & \text{otherwise} \end{cases}$$

if $m = 1$*; and*

$$\overline{u}_h = \frac{(2\beta+1)^2}{4\beta+3}, \quad \overline{u}_l = \frac{2\beta+4\beta^2+1}{4\beta+3}$$

if $m = 2$.

In the proof of this result (see appendix section A.3), we exploit some intermediate findings that deserve to be mentioned. First, the proposer can obtain the approval of a noninvesting agent by allocating a share of the budget to an investing agent (who may possibly reject that offer). In this case, the noninvesting responder is *indirectly compensated*. As Lemma A2 shows, this will occur when $m = 1$ and the externalities are high ($\beta > 1/2$). In this case, it turns out that a noninvesting proposer prefers to take advantage of the externalities by compensating the other noninvesting agent indirectly. A second consideration is that an investing agent

[4] We omit the dependence on m when no confusion arises.

[5] This is not necessarily the case if the committee size is greater than 4, as we explain in section 5.

is able to retain a higher share of the budget when proposing, due to the externalities. This implies that compensating investing agents will be more expensive, and this may induce the proposer to exclude them from the winning coalition in simple-majority bargaining. This will delimit the RE expected utility of the investing agents,[6] and it actually occurs when $m = 1$ and $\beta < 1/2$ (Lemma A6) and when $m = 2$ (Lemma A7), where noninvesting proposers play mixed strategies that exclude investing agents from the winning coalition with a certain probability. Thus, the size of the externalities (β) determines how attractive it is to give a positive share of the budget to investing agents, but also how expensive it is to include them in the winning coalition. Finally, notice from this proposition that when $m = 2$ and when $m = 1$ and $\beta \leq 1/2$, the bargaining outcome does not maximize aggregate utility, because the budget is not entirely allocated among investing agents. Thus, in these cases, the bargaining game generates inefficiencies. The following analysis will allow us to detect whether the investment choice also does so.

Using this result, the next two propositions characterize the stable and r-efficient numbers of investing agents, denoted as m^* and m^0, respectively.

PROPOSITION 2 *The RE number of investing agents is:*

m^*	$\beta \in \left(0, \frac{1}{2}\right]$	$\beta \in \left(\frac{1}{2}, \frac{3}{4}\right]$	$\beta \in \left(\frac{3}{4}, 1\right)$
0	$c \geq \frac{7\beta}{9-12\beta}$	$c \geq \frac{3\beta+2}{3}$	$c \geq \frac{3\beta+2}{3}$
1	$c \in \left[\beta\frac{7-16\beta^2}{9-16\beta^2}, \frac{7\beta}{9-12\beta}\right]$	$c \in \left[\frac{\beta+1}{4\beta+3}, \frac{3\beta+2}{3}\right]$	$c \in \left[\frac{\beta+1}{4\beta+3}, \frac{3\beta+2}{3}\right]$
2	$c \in \left[\frac{7\beta}{12\beta+9}, \beta\frac{7-16\beta^2}{9-16\beta^2}\right]$	$c \in \left[\frac{7\beta}{12\beta+9}, \frac{\beta+1}{4\beta+3}\right]$	$\varnothing$
3	$c \leq \frac{7\beta}{12\beta+9}$	$c \leq \frac{7\beta}{12\beta+9}$	$c \leq \frac{7\beta}{12\beta+9}$

PROOF Immediate from comparing the utilities of the agents who change their status given any m, using the bargaining equilibrium expected utilities given by Proposition 1. *Q.E.D.*

Interestingly, when

$$\beta > \frac{3}{4} \quad \text{and} \quad c \in \left[\frac{\beta+1}{4\beta+3}, \frac{7\beta}{12\beta+9}\right],$$

6 Eraslan (2002) highlights that being stronger in unanimity bargaining games does not necessarily imply stronger positions when consensus requirements are weaker than unanimity. Along the same lines, Montero (2008) shows that altruism (an alternative interpretation of consumption externalities) may increase the agent's material payoff in a three-player majority bargaining game.

either $m^* = 1$ or $m^* = 3$ can be sustained in equilibrium,[7] whereas no parameter range sustains $m^* = 2$ if $\beta > 3/4$. From an intuitive viewpoint, the reason behind these two results is that the aggregate surplus attained in the bargaining stage shrinks when going from $m = 1$ to $m = 2$, but grows when going from $m = 2$ to $m = 3$ (this occurs because noninvesting agents do not obtain any share of the budget when $m = 1$ and $\beta > 1/2$, or, obviously, when $m = 3$, but the noninvesting agent does when $m = 2$, as Lemma A7 shows). This has a negative effect on the marginal utility of the investment of the second investor, but a positive effect on that of the third investor. It turns out that for $\beta > 3/4$, the first marginal utility is higher than the second one, so if a second agent has incentives to invest, then so does the third agent.

PROPOSITION 3 *The r-efficient number of investing agents is:*

m^0	$\beta \in \left(0, \frac{1}{4}\right]$	$\beta \in \left(\frac{1}{4}, \frac{1}{2}\right]$	$\beta \in \left(\frac{1}{2}, 1\right)$
0	$c \geq \frac{3\beta}{3-4\beta}$	$c \geq \frac{3\beta}{3-4\beta}$	$c \geq 3\beta$
1	$c \in \left[3\beta\frac{3-16\beta^2}{9-16\beta^2}, \frac{3\beta}{3-4\beta}\right]$	$c \in \left[3\beta\frac{1-2\beta}{3-4\beta}, \frac{3\beta}{3-4\beta}\right]$	$c \leq 3\beta$
2	$c \in \left[3\frac{\beta}{4\beta+3}, 3\beta\frac{3-16\beta^2}{9-16\beta^2}\right]$	$\varnothing$	$\varnothing$
3	$c \leq 3\frac{\beta}{4\beta+3}$	$c \leq 3\beta\frac{1-2\beta}{3-4\beta}$	$\varnothing$

PROOF Immediate from comparing the sum of the utilities for any possible m, using the bargaining equilibrium expected utilities given by Proposition 1. *Q.E.D.*

Table 1 shows the complete comparison between the stable and r-efficient numbers of investing agents. From this table, we can detect some patterns easily. For example, for any given β, the stable number of investing agents is r-efficient if c is sufficiently large, as $m^* = m^0 = 0$. Similarly, the RE number of investing agents is r-efficient if β and c are sufficiently low, as $m^* = m^0 = 3$. With regard to the other patterns, we can say roughly that overinvestment arises when β is so high that the r-efficient number of investing agents is 1 (as the investing agent obtains the whole budget in these cases, which is the cheapest way to maximize the aggregate surplus), but c is so low that rent-seeking induces a higher number of investing agents in equilibrium. Although underinvestment can arise for any β, in most cases it appears under low values of β, which imply low incentives to invest.

7 For any β, we also obtain multiple equilibria at threshold values of c.

Table 1
Pairs of (m^*, m^0) under Simple-Majority Bargaining for Any Possible Combination of β and c

β \ c	$[0,c_1]$	$[c_1,c_2]$	$[c_2,c_3]$	$[c_3,c_4]$	$[c_4,c_5]$	$[c_5,c_6]$	$[c_6,\infty)$
(0,0.0937]	(3,3)	$\underline{(2,3)}$	$\underline{(1,3)}$	$\underline{(0,3)}$	$\underline{(0,2)}$	$\underline{(0,1)}$	(0,0)
(0.0937,0.1289]	(3,3)	$\underline{(2,3)}$	$\underline{(1,3)}$	$\underline{(1,2)}$	$\underline{(0,2)}$	$\underline{(0,1)}$	(0,0)
(0.1289,1/4]	(3,3)	$\underline{(2,3)}$	$\underline{(1,3)}$	$\underline{(1,2)}$	(1,1)	$\underline{(0,1)}$	(0,0)
(1/4,0.3663]	(3,3)	$\underline{(2,3)}$	$\overline{(2,1)}$	(1,1)	$\underline{(0,1)}$	(0,0)	(0,0)
(0.3663,1/2]	(3,3)	$\overline{(3,1)}$	$\overline{(2,1)}$	(1,1)	$\underline{(0,1)}$	(0,0)	(0,0)
(1/2,3/4]	$\overline{(3,1)}$	$\overline{(2,1)}$	(1,1)	$\underline{(0,1)}$	(0,0)	(0,0)	(0,0)
(3/4,1)	$\overline{(3,1)}$	$(\{1,\overline{3}\},1)$	(1,1)	$\underline{(0,1)}$	(0,0)	(0,0)	(0,0)

Notes: The underlined pairs show underinvestment, and the overlined pairs show overinvestment. Each row corresponds to an interval of β, whereas each column corresponds to a segment of c whose bounds vary for each row. Table A1 in appendix section A.4 specifies these bounds.

3.2 Unanimity Bargaining

When unanimity is required, CRB characterize the symmetric bargaining equilibria for any m. We next summarize their results for the three-player case.[8]

PROPOSITION 4 (CRB) *Under unanimity, the equilibrium expected utilities are given by* $\overline{u}_h(\mathrm{m}) = (3\beta+1)/3$ *if* $m = 3$, $\overline{u}_l(\mathrm{m}) = 1/3$ *if* $m = 0$, *and*

$$\overline{u}_h(\mathrm{m}) = \begin{cases} \frac{\beta m+1}{m} & \text{if } \beta > \frac{1}{3}, \\ \frac{\beta m+1}{3(1-\beta(3-m))} & \text{otherwise} \end{cases} \quad \text{and} \quad \overline{u}_l(\mathrm{m}) = \begin{cases} \beta & \text{if } \beta > \frac{1}{3}, \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

when $m \in \{1,2\}$.

From this proposition, we should note that when $m \in \{1,2\}$, the bargaining outcome maximizes aggregate utility (that is, the budget is entirely allocated among investing agents) only if $\beta \geq 1/3$. Hence, when $m \in \{1,2\}$ and $\beta < 1/3$, inefficiencies arise from the bargaining stage. The following analysis will allow us to detect the inefficiencies in the investment stage.

From CRB, we can also derive the *stable* and *r-efficient* numbers of investing agents (now under unanimous bargaining) as a function of c and β. We summarize these results in Tables 2 and 3. The tables allow us to compare the different combinations of m^* and m^0 for all possible parameter combinations (β, c) for the unanimity case, as we do in Table 4.

[8] Although we might have opted to refer the reader to that paper, we use some of these results in section 4, so we decided to include them explicitly.

Table 2
Parameters that Induce m^*

m^*	$\beta \in \left(0, \frac{1}{3}\right]$	$\beta \in \left(\frac{1}{3}, 1\right)$
0	$c \geq \frac{\beta}{1-2\beta}$	$c \geq \beta + \frac{2}{3}$
1	$c \in \left[\frac{\beta}{1-\beta}, \frac{\beta}{1-2\beta}\right]$	$c \in \left[\frac{1}{2}, \beta + \frac{2}{3}\right]$
2	$c \in \left[\beta, \frac{\beta}{1-\beta}\right]$	$c \in \left[\frac{1}{3}, \frac{1}{2}\right]$
3	$c \leq \beta$	$c \leq \frac{1}{3}$

Table 3
Parameters that Induce m^0

m^0	$\beta \in \left(0, \frac{1}{3}\right]$	$\beta \in \left(\frac{1}{3}, 1\right)$
0	$c \geq \frac{\beta}{1-2\beta}$	$c \geq 3\beta$
1	$c \in \left[\frac{\beta(1-3\beta)}{(1-2\beta)(1-\beta)}, \frac{\beta}{1-2\beta}\right]$	$c \leq 3\beta$
2	$c \in \left[\frac{\beta(1-3\beta)}{1-\beta}, \frac{\beta(1-3\beta)}{(1-2\beta)(1-\beta)}\right]$	$\varnothing$
3	$c \leq \frac{\beta(1-3\beta)}{1-\beta}$	$\varnothing$

Table 4
Pairs of (m^*, m^0) under Unanimity for Any Possible Combination of β and c

β \ c	$[0, c'_1]$	$[c'_1, c'_2]$	$[c'_2, c'_3]$	$[c'_3, c'_4]$	$[c'_4, c'_5]$	$[c'_5, \infty)$
$(0, 1/3]$	$\underline{(3,3)}$	$\overline{(3,2)}$	$\overline{(3,1)}$	$\overline{(2,1)}$	$(1,1)$	$(0,0)$
$(1/3, 1)$	$\overline{(3,1)}$	$\overline{(2,1)}$	$(1,1)$	$\underline{(0,1)}$	$(0,0)$	…

Notes: Each row corresponds to an interval of β, whereas each column corresponds to a segment of c whose bounds vary for each row. Table A2 in appendix section A.4 specifies these bounds.

The pattern in Table 4, which also appears in the n-player case, is the following: when $m^0 \in \{0,n\}$, then $m^0 = m^*$; thus, the RE number of investing agents is r-efficient. When $m^0 \in \{2,\ldots,n-1\}$, then $m^0 < m^*$; thus, any RE entails overinvestment. Finally, when $m^0 = 1$, an equilibrium can be r-efficient or display either over- or underinvestment because any m^* can be sustained under the suitable parameter combination.

4 Welfare Comparisons

Requiring unanimity in the bargaining stage makes investments particularly profitable, because the increase in the expected utility that investing agents obtain when proposing (they can obtain a bigger share of the budget) also has a positive effect on the utility that they obtain when responding (having a larger expected utility implies that they must be compensated accordingly). In contrast, under weaker consensus requirements, proposers can exclude investing agents from the winning coalition if their demands are too high. This has two immediate consequences. First, it erodes the benefits of investing. In particular, the utility of investing is always lower under a simple majority than under unanimity.[9] Then:

COROLLARY *When decisions require a simple majority, the incentives to invest are lower than under unanimity. As a consequence, the equilibrium number of investing agents under unanimity cannot be lower than under simple majority.*

PROOF The first claim follows directly from comparing the RE expected utilities under a simple-majority requirement (Proposition 1) and under unanimity (Proposition 4). See appendix section A.5. From this comparison it follows that m^*_{sm} cannot be equal to $m^*_u + 1$. So, in order to prove the second claim, it remains to show that $m^*_{sm} \geq m^*_u + 2$ is impossible. If $m^*_{sm} = 3$, then the first claim implies that $m^*_u = 3$, but there is no parameter constellation such that both $m^*_u = 3$ and $m^*_u \in \{0,1\}$ coexist. A similar argument shows that $m^*_{sm} = 2$ cannot hold when $m^*_u = 0$.[10] *Q.E.D.*

A second implication of lowering the consensus requirement is the possible reduction of the budget allocated to investing agents. Thus, for any given m, the aggregate surplus under unanimous bargaining is never less than under simple-majority bargaining. We show this in Table 5, which reports the total gross welfare (excluding investment costs) under unanimous and simple-majority bargaining for different values of m, represented by $\overline{W}_u(m)$ and $\overline{W}_{sm}(m)$, respectively. We use the symbol $\overline{W}(m)$ when $\overline{W}_u(m) = \overline{W}_{sm}(m)$.

Direct computations show that $\overline{W}_u(m) \geq \overline{W}_{sm}(m)$, with strict inequality when (i) $m = 2$ and (ii) $m = 1$ and $\beta < 1/2$. Thus, unanimity bargaining generates (weakly) more surplus for any $m \in \{0,1,2,3\}$. Nevertheless, as we showed in the previous

[9] As we noted, this result is in line with Eraslan (2002) and contrasts with that of Harstad (2005).

[10] We thank an anonymous referee for completing the previous version of this proof.

Table 5
Gross Welfare for Any Number of Investing Agents under Unanimity and Simple Majority

β	$\overline{W}(3)$	$\overline{W}_u(2)$	$\overline{W}_{sm}(2)$	$\overline{W}_u(1)$	$\overline{W}_{sm}(1)$	$\overline{W}(0)$
$> \frac{3}{4}$	$3\beta+1$	$3\beta+1$	$\frac{12\beta^2+10\beta+3}{4\beta+3}$	$3\beta+1$	$3\beta+1$	1
$\left(\frac{1}{2},\frac{3}{4}\right)$	$3\beta+1$	$3\beta+1$	$\frac{12\beta^2+10\beta+3}{4\beta+3}$	$3\beta+1$	$3\beta+1$	1
$\left(\frac{1}{3},\frac{1}{2}\right)$	$3\beta+1$	$3\beta+1$	$\frac{12\beta^2+10\beta+3}{4\beta+3}$	$3\beta+1$	$\frac{3-\beta}{3-4\beta}$	1
$< \frac{1}{3}$	$3\beta+1$	$\frac{1+\beta}{1-\beta}$	$\frac{12\beta^2+10\beta+3}{4\beta+3}$	$\frac{1-\beta}{1-2\beta}$	$\frac{3-\beta}{3-4\beta}$	1

section, inefficiencies can also arise in the investment stage: stable investment profiles may involve either over- or underinvestment in terms of r-efficiency. In this respect, due to the Corollary, simple majority contributes to alleviating overinvestment. Hence, to compare the equilibria under different consensus requirements in terms of their (unrestricted) efficiency, we must compare these two opposite effects.

In order to illustrate the forces that interact in this comparison, we next consider two subcases: large externalities ($\beta > 3/4$) and small externalities ($\beta < 1/6$). We provide a complete review of all cases in Table A4 in appendix section A.6. Tables 6 and 7 display the information contained in the first column of Table A4, consisting of the range of investment costs (c) that sustain different pairs of (m^*_{sm}, m^*_u) when $\beta > 3/4$ and $\beta < 1/6$, respectively.[11] Moreover, they also show the net welfare gains $\Delta = \overline{W}_u(m^*_u) - c \cdot m^*_u - [\overline{W}_{sm}(m^*_{sm}) - c \cdot m^*_{sm}]$ for unanimity, where m^*_{sm} and m^*_u denote the stable number of investing agents attained under simple majority and unanimity, respectively.

As shown, for high values of β ($\beta > 3/4$), simple majority cannot be worse than unanimity in terms of aggregate welfare. Instead, Table 7 shows that for low values of β ($\beta < 1/6$), aggregate welfare is generally higher under unanimity, except when

$$\beta \in \left(\frac{1}{9},\frac{1}{6}\right) \quad \text{and} \quad c \in \left(\frac{3\beta(1-2\beta)}{(3-4\beta)},\frac{7\beta}{9-12\beta}\right).$$

As we stated, requiring a simple majority instead of unanimity has two opposite effects on aggregate welfare. On the one hand, as the winning coalition can exclude investing agents, an inefficient allocation of a share of the budget to noninvesting agents in the bargaining stage is more plausible. On the other hand, and due to this effect, the incentives to invest are lower, which reduces the overinvestment inefficiency caused by rent-seeking in the first stage of the game. When $\beta > 3/4$,

[11] In Table 7, we do not include $(m^*_u, m^*_{sm}) \in \{(3,3),(0,0)\}$, to save space. In both cases, the net welfare gains are zero.

Table 6
Welfare Comparison of Unanimity and Simple Majority when $\beta > 3/4$

(m_u^*, m_{sm}^*)	(3,3)	(3,{3,1})	(3,1)	(2,1)	(1,1)	(0,0)
c	$\left[0, \frac{\beta+1}{4\beta+3}\right]$	$\left[\frac{\beta+1}{4\beta+3}, \frac{7\beta}{12\beta+9}\right]$	$\left[\frac{7\beta}{12\beta+9}, \frac{1}{3}\right]$	$\left[\frac{1}{3}, \frac{1}{2}\right]$	$\left[\frac{1}{2}, \frac{3\beta+2}{3}\right]$	$\left[\frac{3\beta+2}{3}, \infty\right)$
Δ	0	$\{0, -2c\}$	$-2c$	$-c$	0	0

Table 7
Welfare Comparisons of Unanimity and Simple Majority when $\beta < 1/6$.

(m_u^*, m_{sm}^*)	(3,2)	(3,1)	(3,0)	(2,0)	(1,0)
c	$\left[\frac{7\beta}{12\beta+9}, \frac{(16\beta^2-7)\beta}{16\beta^2-9}\right]$	$\left[\frac{(16\beta^2-7)\beta}{16\beta^2-9}, \frac{7\beta}{9-12\beta}\right]$	$\left[\frac{7\beta}{9-12\beta}, \beta\right]$	$\left[\beta, \frac{\beta}{1-\beta}\right]$	$\left[\frac{\beta}{1-\beta}, \frac{\beta}{1-2\beta}\right]$
Δ	$-\frac{(3c-3\beta+4c\beta)}{4\beta+3}$	$-\frac{2(3\beta-3c+4c\beta-6\beta^2)}{4\beta-3}$	$3\beta-3c$	$3\beta-2c$	$3\beta-c$
	(+)	(+,−)	(+)	(+)	(+)

the negative effect does not apply, because noninvesting agents do not receive any share of the budget in any stable investment profile; but that will occur only when $m = 2$, which cannot be sustained in equilibrium. Thus, the lower incentives to invest explain why simple majority outperforms unanimity when $\beta > 3/4$. On the contrary, unanimity generally outperforms simple majority when $\beta < 1/6$, because the inefficiencies from allocating a part of the budget to noninvesting agents are offset by the benefits from reducing the overinvestment inefficiency. For intermediate values of β, there is some variability in the dominance relationship between the alternative consensus requirements.

5 *Discussion*

One may wonder whether the results of the three-player case presented here could be extended to the general n-player case. Requiring a simple majority means that w agents should approve any proposal, where $w = \lfloor n/2 \rfloor + 1$ and $\lfloor z \rfloor$ denotes the integer part of z. Any proposer can form $\binom{n-1}{w-1}$ alternative minimal winning coalitions,[12] each with a different distribution of investing and noninvesting agents. Furthermore, winning coalitions may not be minimal. As in the three-player case, this may happen when the proposer keeps the entire budget for him or herself. However, unlike the three-player case, this may also happen if a noninvesting proposer compensates all noninvesting responders indirectly. All this casuistry arising in the n-player case under a simple-majority requirement makes the bargaining-equilibrium

[12] This number will be higher if the symmetry result does not hold.

analysis extremely tedious. However, there are no reasons to suspect that the results in the general case will change qualitatively with respect to the three-player case. Requiring a simple majority instead of unanimity reduces agents' incentives to invest, because the winning coalition might exclude investing agents. Thus, over-investment inefficiency is mitigated. This exclusion has a counterpart in that it will reduce the average budget allocated to investing agents, thereby increasing the inefficiency in the bargaining stage. Thus, as in the three-player case, the effect of reducing the consensus requirement on aggregate welfare is uncertain. For high values of β, this uncertainty disappears because the budget is efficiently allocated to investing agents in the bargaining equilibrium. Therefore, the negative effects of reducing the consensus requirement do not appear, so that simple majority performs better. As we can already derive this intuition from the three-player case, we consider that there is not much to be gained from looking at the general n-player case.

In our setting, the investment decision is binary. In a reasonable alternative, this decision may possibly be continuous. For example, we might consider a framework in which agents have to choose the level of externalities $\beta \in [0,\infty)$ at some cost $c(\beta)$. In such cases, the basic underlying forces that drive the results in the investment stage do not change with respect to the setting we consider here: the positive externalities induce underinvestment, and the private benefits of investment induce the rent-seeking behavior that causes overinvestment. However, this behavior will be exacerbated in that agents will compete Ãă la Bertrand to generate the highest level of externality because this investor will attract a bigger share of the budget. Whether the effects of this investing race, which cause overinvestment, will offset the effect of positive externalities, causing underinvestment, is still an open question that may be explored in future research. In any case, the qualitative effect of reducing the consensus requirement in this setting will be the same as in ours. That is, under simple majority, the benefits of investing will be eroded, so the investing race will not be as aggressive as it would be under unanimity. Consequently, the main result of this study would hold in this respect.

6 Conclusions

We studied a case in which a set of three players must decide whether to invest in generating positive consumption externalities before a budget is distributed through simple-majority bargaining. We showed that in this setting, the level of investment might be either efficient, too low, or too high, depending on both the investment costs and the size of the externalities. Compared to the analogous setting with unanimity bargaining (analyzed in CRB), the overinvestment inefficiency is alleviated because a simple majority generates lower incentives to invest. The investing agents might not need to approve of an allocation for the group to accept the proposal. This has a counterpart: investing agents might receive a smaller share of the budget under simple-majority bargaining, so the aggregate surplus attained in the negotiation stage might also be smaller. Hence, the comparison between the collective surpluses attained under simple-majority and unanimity bargaining is ambiguous, as it depends on the sizes of these two opposite effects. We showed that under unanimity bargaining, the equilibria are generally more efficient when the externalities are sufficiently small. In contrast, the aggregate surplus under unanimity bargaining is less than that under simple-majority bargaining when the externalities are sufficiently large.

Appendix

A.1 No Delay

LEMMA A1 *Any RE is a no-delay equilibrium, and* $\sum_{i\in N} x_i^j = 1$ *for all* $j \in N$.

PROOF Consider an equilibrium yielding expected utilities $(\overline{u}_1,\overline{u}_2,\overline{u}_3)$. Because of stationarity and the linearity of preferences, the expected utility of any agent $i \in N$ is no more than $x_i^e + \beta y^e$, where x_i^e is the expected share obtained by agent i, and y^e is the expected share allocated to investing agents in the equilibrium path starting at any period. This implies that

$$A = \left\{ \mathrm{x} \in \mathbb{R}^3_+, \sum\nolimits_{j\in N} x_j \leq 1 : u_j(\mathrm{x}) \geq \overline{u}_j \text{ for all } j \in N \right\} \neq \varnothing.$$

Hence, for any $\delta < 1$,

$$A(\delta) = \left\{ \mathrm{x} \in \mathbb{R}^3_+, \sum\nolimits_{j\in N} x_j < 1 : u_j(\mathrm{x}) \geq \delta\overline{u}_j \text{ for all } j \in N \right\} \neq \varnothing.$$

Therefore, any proposer j can obtain more than $\delta\overline{u}_j$ when proposing, implying that there is no delay. Moreover, this proposer exhausts all resources when proposing. That is, $\sum_{i\in N} x_i^j = 1$. *Q.E.D.*

A.2 Symmetry

We next show that any bargaining equilibrium is symmetric. Note that this is immediate when $m = 0$ or $m = 3$, as agents play the standard random-proposers bargaining game with surplus of 1 when $m = 0$ and $1+\beta$ when $m = 3$. Thus, we will now focus on cases where $m \in \{1,2\}$.

For any equilibrium, we denote the expected utility of agent i by $\overline{u}_i$ and the expected utility obtained by j when i proposes by u_{ij}; i.e., $u_{ij} = u_j(\mathrm{x}^i)$. Hence, the following holds:

$$3\overline{u}_i = u_{1i} + u_{2i} + u_{3i} \quad \text{for all } i = 1,2,3.$$

Under stationarity, agent i accepts any proposal x^j such that $u_i(\mathrm{x}^j) \geq \delta\overline{u}_i$. When $m = 1$, a noninvesting proposer can get the approval of the noninvesting responder either directly or indirectly, as explained in section 2. The next result shows when each of these two compensations is used.

Lemma A2 *When* $m = 1$*, a noninvesting proposer prefers to compensate the non-investing responder indirectly iff* $\beta > 1/2$ *but directly iff* $\beta < 1/2$*, and she is indifferent between the two when* $\beta = 1/2$*.*

Proof Assume *without loss of generality* (henceforth, w.l.o.g.) that $\mathrm{m} = (1,0,0)$, and consider an equilibrium where agent 2 makes a proposal $\mathrm{x}^2 = (x_1^2, x_2^2, x_3^2)$ to obtain the acceptance of 3, i.e., $u_3(\mathrm{x}^2) = \delta\overline{u}_3$. Using a direct compensation implies $\mathrm{x}^2 = (0, 1-\delta\overline{u}_3, \delta\overline{u}_3)$, so that $u_{22} = 1-\delta\overline{u}_3$. Instead, using an indirect compensation implies $\mathrm{x}^2 = (\delta\overline{u}_3/\beta, 1-\delta\overline{u}_3/\beta, 0)$, so that $u_{22} = 1-\delta\overline{u}_3/\beta+\delta\overline{u}_3$. The result follows from comparing these two utilities. *Q.E.D.*

Now we address the symmetry result.

Lemma A3 *If* $\mathrm{m} = (1,0,0)$ *then* $\overline{u}_2 = \overline{u}_3$.

Proof Assume w.l.o.g. that $\overline{u}_2 > \overline{u}_3$. This implies that agent 1 never offers a positive share to the more demanding agent 2. That is, $\mathrm{x}^1 = (1-x_3^1, 0, x_3^1)$, implying $u_{12} \leq u_{13}$. Regarding the equilibrium proposals of players 2 and 3, we next consider the different possibilities in turn.

Case 1. Suppose first that the optimal proposal of agent 2 is $\mathrm{x}^2 = (\delta\overline{u}_1/(1+\beta), 1-\delta\overline{u}_1/(1+\beta), 0)$. As $\overline{u}_2 > \overline{u}_3$, the optimal proposal of agent 3 is $\mathrm{x}^3 = (\delta\overline{u}_1/(1+\beta), 0, 1-\delta\overline{u}_1/(1+\beta))$. Hence, $u_{22} = u_{33}$ and $u_{23} = u_{32}$. Thus, using $u_{12} \leq u_{13}$, it follows that

$$3\overline{u}_2 = u_{12}+u_{22}+u_{32} \leq u_{13}+u_{23}+u_{33} = 3\overline{u}_3,$$

contradicting $\overline{u}_2 > \overline{u}_3$.

Case 2. Suppose now that the optimal proposal of player 2 is $\mathrm{x}^2 = (\delta\overline{u}_3/\beta, 1-\delta\overline{u}_3/\beta, 0)$, which compensates agent 3 indirectly. By Lemma A2, this implies that $\beta > 1/2$. Then, $u_{23} = \delta\overline{u}_3$ and $u_{22} = \beta(\delta\overline{u}_3/\beta)+1-\delta\overline{u}_3/\beta = 1-[(1-\beta)/\beta]\delta\overline{u}_3$. Moreover, $u_{32} \leq \delta\overline{u}_2$ and $u_{33} \geq 1-[(1-\beta)/\beta]\delta\overline{u}_2$, because agent 3 has also the option to compensate agent 2. Therefore, using $u_{12} \leq u_{13}$, we can write

$$3\overline{u}_2 \leq u_{12}+1-\frac{1-\beta}{\beta}\delta\overline{u}_3+\delta\overline{u}_2 \quad \text{and} \quad 3\overline{u}_3 \geq u_{12}+\delta\overline{u}_3+1-\frac{1-\beta}{\beta}\delta\overline{u}_2.$$

Thus,

$$3(\overline{u}_2-\overline{u}_3) \leq \frac{\delta}{\beta}(\overline{u}_2-\overline{u}_3).$$

As $\beta > 1/2$, then $\delta/\beta < 2$, so that we obtain a contradiction.

Case 3. Suppose now that the optimal proposal of player 2 is $\mathrm{x}^2 = (0, 1-\delta\overline{u}_3, \delta\overline{u}_3)$, which compensates agent 3 directly. Then, $u_{23} = \delta\overline{u}_3$ and $u_{22} = 1-\delta\overline{u}_3$. Moreover,

$u_{32} \le \delta\overline{u}_2$ and $u_{33} \ge 1-\delta\overline{u}_2$, because agent 3 has also the option to compensate agent 2. Therefore, using $u_{12} \le u_{13}$, we can write

$$3\overline{u}_2 \le u_{12} + 1 - \delta\overline{u}_3 + \delta\overline{u}_2 \quad \text{and} \quad 3\overline{u}_3 \ge u_{12} + \delta\overline{u}_3 + 1 - \delta\overline{u}_2.$$

Thus,

$$3(\overline{u}_2 - \overline{u}_3) \le 2\delta(\overline{u}_2 - \overline{u}_3).$$

As $\delta < 1$, we reach a contradiction.

Case 4. If both $(\delta\overline{u}_1/(1+\beta), 1-\delta\overline{u}_1/(1+\beta), 0)$ and $(\delta\overline{u}_3/\beta, 1-\delta\overline{u}_3/\beta, 0)$ are optimal proposals of agent 2, then

$$\overline{u}_1 = \frac{1+\beta}{\beta}\overline{u}_3.$$

Moreover, the optimal proposal of 3 must be $(\delta\overline{u}_1/(1+\beta), 0, 1-\delta\overline{u}_1/(1+\beta))$, as $\overline{u}_2 > \overline{u}_3$. Therefore, $u_{33} = u_{22}$, $u_{32} = \beta[\delta\overline{u}_1/(1+\beta)] = \delta\overline{u}_3$, and $u_{23} = p\beta[\delta\overline{u}_1/(1+\beta)] + (1-p)\delta\overline{u}_3 = \delta\overline{u}_3$ for any $p \in [0,1]$. Then, we can proceed as in case 1 to find a contradiction.

Case 5. If both $(\delta\overline{u}_1/(1+\beta), 1-\delta\overline{u}_1/(1+\beta), 0)$ and $(0, 1-\delta\overline{u}_3, \delta\overline{u}_3)$ are optimal proposals of agent 2, then

$$\overline{u}_1 = \frac{1+\beta}{1-\beta}\overline{u}_3.$$

Moreover, the optimal proposal of 3 must be $(\delta\overline{u}_1/(1+\beta), 0, 1-\delta\overline{u}_1/(1+\beta))$, as $\overline{u}_2 > \overline{u}_3$. That is, $u_{33} = u_{22}$, $u_{32} = \beta[\delta\overline{u}_1/(1+\beta)] = \delta\overline{u}_3$, and $u_{23} = p\beta[\delta\overline{u}_1/(1+\beta)] + (1-p)\delta\overline{u}_3 = \delta\overline{u}_3$, for any $p \in [0,1]$. Then, we can proceed as in case 1 to find a contradiction.

Case 6. If both $(\delta\overline{u}_3/\beta, 1-\delta\overline{u}_3/\beta, 0)$, and $(0, 1-\delta\overline{u}_3, \delta\overline{u}_3)$ are optimal proposals of agent 2, then $\beta = 1/2$ (see Lemma A2), $u_{22} = 1-\delta\overline{u}_3$, and $u_{23} = \delta\overline{u}_3$. As $u_{32} \le \delta\overline{u}_2$ and $u_{33} \ge 1-\delta\overline{u}_2$, we can proceed as in case 3 to reach a contradiction.

Case 7. If $(\delta\overline{u}_1/(1+\beta), 1-\delta\overline{u}_1/(1+\beta), 0)$, $(\delta\overline{u}_3/\beta, 1-\delta\overline{u}_3/\beta, 0)$, and $(0, 1-\delta\overline{u}_3, \delta\overline{u}_3)$ are optimal proposals of agent 2, then reasoning as in case 6 we obtain a contradiction. *Q.E.D.*

LEMMA A4 *If* $\mathrm{m} = (1,1,0)$ *then* $\overline{u}_1 = \overline{u}_2$.

PROOF Let assume w.l.o.g. that $\overline{u}_1 > \overline{u}_2$. Then, agent 3 never compensates agent 1. That is, her optimal proposal x^3 satisfies $u_2(\mathrm{x}^3) = u_{32} \ge u_{31} = u_1(\mathrm{x}^3)$. Regarding the equilibrium proposals of 1 (and 2), we next consider the different possibilities in turn.

Case 1. If the optimal proposal of agent 1 is to compensate agent 3, then the optimal proposal of agent 2 will also compensate agent 3, because $\overline{u}_1 > \overline{u}_2$. This implies that $u_{22} = u_{11}$ and $u_{21} = u_{12}$. Thus, using $u_{32} \geq u_{31}$, it follows that

$$3\overline{u}_1 = u_{11} + u_{21} + u_{31} \leq u_{12} + u_{22} + u_{32} = 3\overline{u}_2,$$

contradicting $\overline{u}_1 > \overline{u}_2$.

Case 2. If the optimal proposal of agent 1 compensates agent 2, we need to distinguish three different cases:

(a) $\beta \geq \delta\overline{u}_1 > \delta\overline{u}_2$. It is immediate that $u_{11} = u_{22} = 1 + \beta$ and $u_{12} = u_{21} = \beta$. As $u_{32} \geq u_{31}$, we can proceed as in case 1 to reach a contradiction.

(b) $\delta\overline{u}_1 > \beta \geq \delta\overline{u}_2$. In this case, the optimal proposal of 1 is $x^1 = (1,0,0)$, so that $u_{11} = 1 + \beta$ and $u_{12} = \beta$. If agent 2's proposal $x^2 = (0,1,0)$ is accepted by agent 3, then $u_{22} = 1 + \beta$ and $u_{21} = \beta$. As $u_{32} \geq u_{31}$, we can proceed as in case 1 to reach a contradiction. If $x^2 = (0,1,0)$ is not accepted by agent 3, then $u_{22} \geq 1 + 2\beta - \delta\overline{u}_1$, as agent 2 can compensate agent 1. Moreover, given that $\delta\overline{u}_1 > \beta$, we have $u_{21} \leq \delta\overline{u}_1$. Therefore, as $u_{32} \geq u_{31}$,

$$3\overline{u}_1 \leq 1 + \beta + \delta\overline{u}_1 + u_{32} \quad \text{and} \quad 3\overline{u}_2 \geq \beta + (1 + 2\beta - \delta\overline{u}_1) + u_{32}.$$

Given that $\beta \geq \delta\overline{u}_2$, from these two inequalities we obtain that

$$3(\overline{u}_1 - \overline{u}_2) \leq 2\delta(\overline{u}_1 - \overline{u}_2),$$

contradicting $\overline{u}_1 > \overline{u}_2$.

(c) $\delta\overline{u}_1 > \delta\overline{u}_2 > \beta$. In this case, $x^1 = (1 - x_2^1, x_2^1, 0)$. Then, $u_2(x^1) = u_{12} = \beta(1 - x_2^1) + (1+\beta)x_2^1 = \beta + x_2^1 = \delta\overline{u}_2$ and $u_{11} = \beta x_2^1 + (1+\beta)(1 - x_2^1) = 1 + 2\beta - \delta\overline{u}_2$. Regarding agent 2's proposal x^2, as the most favorable x^2 for player 1 will compensate her, we have $u_{21} \leq (1+\beta)x_1^2 + \beta(1 - x_1^2) = \delta\overline{u}_1$. Moreover, 2 cannot obtain less than what she would obtain in this situation, so $u_2(x^2) = u_{22} \geq \beta x_1^2 + (1+\beta)(1 - x_1^2) = 1 + 2\beta - \delta\overline{u}_1$. As $u_{32} \geq u_{31}$,

$$3\overline{u}_1 \leq (1 + 2\beta - \delta\overline{u}_2) + \delta\overline{u}_1 + u_{32} \quad \text{and} \quad 3\overline{u}_2 \geq \delta\overline{u}_2 + (1 + 2\beta - \delta\overline{u}_1) + u_{32}.$$

Thus,

$$3(\overline{u}_1 - \overline{u}_2) \leq 2\delta(\overline{u}_1 - \overline{u}_2),$$

which is a contradiction.

Case 3. A mix of the two proposals can occur when agent 1 is indifferent between compensating agent 2, so that

$$(1 + \beta)x_2^1 + \beta(1 - x_2^1) = \delta\overline{u}_2 \;\Rightarrow\; u_1(x^1) = 1 + 2\beta - \delta\overline{u}_2,$$

and compensating agent 3, so that

$$x_3^1 + \beta(1 - x_3^1) = \delta\overline{u}_3 \;\Rightarrow\; u_1(\mathrm{x}^1) = \frac{1+\beta}{1-\beta}(1 - \delta\overline{u}_3).$$

As $\overline{u}_2 < \overline{u}_1$, this means that agent 2 strictly prefers compensating agent 3. Hence, $u_{22} = u_{11}$ and $u_{12} \geq u_{21}$, which added to $u_{32} \geq u_{31}$ contradicts $\overline{u}_1 > \overline{u}_2$. *Q.E.D.*

A.3 Proof of Proposition 1

In the case that all agents are identical, they obtain the same expected utilities. That is, $u_i(\mathrm{m}) = 1/3$ if $m = 0$ and $u_i(\mathrm{m}) = (1 + 3\beta)/3$ if $m = 3$. Next, we consider the remaining investment profiles, m = (1,0,0) and m = (1,1,0).

Using symmetry, the no-delay property of equilibria, and the linearity of utilities, we can next specify the relationship between the agents' equilibrium expected utilities when $m \in \{1,2\}$.

LEMMA A5 *In any bargaining equilibrium, as* $\delta \to 1$,

$$\overline{u}_h = \begin{cases} \frac{(1+2\beta)}{2(1-\beta)}(1 - \overline{u}_l) & \text{when } m = 2, \\ \frac{(\beta+1)(2\overline{u}_l - 1)}{2\beta - 1} & \text{when } m = 1 \text{ and } \beta \neq \frac{1}{2}. \end{cases}$$

If $m = 1$ *and* $\beta = 1/2$, *then* $\overline{u}_l = 1/2$ *and* $\overline{u}_h = 3/2$.

PROOF Symmetry implies that expected shares are $(1-2y, y, y)$ when m = (1,0,0), for some $y \in [0,1/2]$. This implies that $\overline{u}_h = (1+\beta)(1-2y)$ and $\overline{u}_l = y + \beta(1-2y)$ when $m = 1$. Rearranging terms, the result follows directly when $\beta \neq 1/2$. When $\beta = 1/2$ and $m = 1$, we have $\overline{u}_l = 1/2$ for all $y \in [0,1/2]$. In these cases, it is easy to see that responders will accept agent 1's proposal $\mathrm{x}^1 = (1,0,0)$, since $\beta > \delta\overline{u}_l$. Thus, $u_{11} = 1 + \beta = 3/2$. It can also be seen that noninvesting proposers always compensate agent 1 with some positive probability, because $\overline{u}_h \leq 3/2$. When $\overline{u}_h = 3/2$, the investing agent might be excluded from the winning coalition with some positive probability, but this contradicts $\overline{u}_h = 3/2$. Hence, noninvesting proposers compensate agent 1 with probability 1. Therefore, $\mathrm{x}^2 = (\delta\overline{u}_h/(1+\beta), 1 - \delta\overline{u}_h/(1+\beta), 0)$, and $\mathrm{x}^3 = (\delta\overline{u}_h/(1+\beta), 0, 1 - \delta\overline{u}_h/(1+\beta))$. In that case,

$$3\overline{u}_h = 3/2 + \delta\overline{u}_h + \delta\overline{u}_h$$

and consequently

$$\overline{u}_h = \frac{3/2}{3 - 2\delta},$$

which converges to $\overline{u}_h = 3/2$ as $\delta \to 1$.

The utilities in the case m = (1,1,0) can be obtained similarly. In that case, expected shares are $(y, y, 1-2y)$ for some $y \in [0,1/2]$. This implies that $\overline{u}_h = y(1+\beta) + y\beta$ and $\overline{u}_l = 1 - 2y + 2y\beta$ when $m = 2$. Rearranging terms, the result follows directly. *Q.E.D.*

Next, we derive the bargaining expected utilities through two lemmata.

LEMMA A6 *When $m = 1$, the limiting expected utilities when $\delta \to 1$ are*

1. $\overline{u}_h = 1+\beta$ *and* $\overline{u}_l = \beta$ *when* $\beta > 1/2$, *and*
2. $\overline{u}_h = (1+\beta)/(3-4\beta)$ *and* $\overline{u}_l = (1-\beta)/(3-4\beta)$ *otherwise.*

PROOF Let assume w.l.o.g. m = (1,0,0).

Case 1. Consider that $\beta = 1/2$. The result follows immediately from Lemma A5, as $(1-\beta)/(3-4\beta) = \beta = 1/2$ and $(1+\beta)/(3-4\beta) = 1+\beta = 3/2$. That lemma proves the existence of this equilibrium.

Case 2. Suppose now that $\beta > 1/2$. Symmetry implies that $\overline{u}_l = y+\beta(1-2y)$ for some $y \in [0,1/2]$. So, $\overline{u}_l \in [1/2,\beta]$. Consequently, agent 1's proposal $x^1 = (1,0,0)$ will be accepted because $u_l(x^1) = \beta > \delta\overline{u}_l$. As $\beta > 1/2$, we know that noninvesting agents will prefer to compensate other noninvesting agents indirectly rather than directly (Lemma A2). Therefore, noninvesting agents will either compensate the investing agent directly or the noninvesting agent indirectly. That is, their strategy proposals will be either $x^2 = (\delta\overline{u}_h/(1+\beta), 1-\delta\overline{u}_h/(1+\beta), 0)$ and $x^3 = (\delta\overline{u}_h/(1+\beta), 0, 1-\delta\overline{u}_h/(1+\beta))$, or $x^2 = (\delta\overline{u}_l/\beta, 1-\delta\overline{u}_l/\beta, 0)$ and $x^3 = (\delta\overline{u}_l/\beta, 0, 1-\delta\overline{u}_l/\beta)$, or a convex combination of those two. In the first case, the expected utility of the investing agent satisfies

$$3\overline{u}_h = 1+\beta+\delta\overline{u}_h+\delta\overline{u}_h$$

and therefore

$$\overline{u}_h = \frac{1+\beta}{3-2\delta}.$$

In the second case, the expected utilities satisfy

$$3\overline{u}_h = (1+\beta)\left(1+2\frac{\delta\overline{u}_l}{\beta}\right) \quad \text{and} \quad 3\overline{u}_l = \beta\left(1+2\frac{\delta\overline{u}_l}{\beta}\right)+\left(1-\frac{\delta\overline{u}_l}{\beta}\right),$$

yielding

$$\overline{u}_h = (\beta+\delta)\frac{1+\beta}{3\beta+\delta-2\beta\delta}.$$

In those two cases, $\overline{u}_h$ converges to $1+\beta$ as $\delta \to 1$. Then $\overline{u}_l$ can be obtained following Lemma A5.

In cases where the noninvesting agent uses a mixed-strategy proposal, then $\overline{u}_h/(1+\beta) = \overline{u}_l/\beta$. Using Lemma A5, this implies that $\overline{u}_h = 1+\beta$ and $\overline{u}_l = \beta$.

Case 3. Finally, assume that $\beta < 1/2$. From Lemma A2, we know that in equilibrium noninvesting agents prefer to compensate the other noninvesting agent directly rather than indirectly. A noninvesting proposer prefers to compensate the noninvesting responder rather than compensating agent 1 if

$$\beta\frac{\delta\overline{u}_h}{1+\beta}+1-\frac{\delta\overline{u}_h}{1+\beta}<1-\delta\overline{u}_l \;\Leftrightarrow\; \overline{u}_h>\frac{1+\beta}{1-\beta}\overline{u}_l.$$

(a) $\overline{u}_h > [(1+\beta)/(1-\beta)]\overline{u}_l$. In that case, the investing agent does not receive anything from others' proposals. Consequently, $\overline{u}_l \geq (\beta+1)/3$ and $\overline{u}_h \leq (\beta+1)/3$, contradicting $\overline{u}_h > [(1+\beta)/(1-\beta)]\overline{u}_l$.

(b) $\overline{u}_h < [(1+\beta)/(1-\beta)]\overline{u}_l$. Here, we consider two alternative cases. First, if $\delta\overline{u}_l \leq \beta$ then $x^1 = (1,0,0)$, $x^2 = (\delta\overline{u}_h/(1+\beta), 1-\delta\overline{u}_h/(1+\beta), 0)$, and $x^3 = (\delta\overline{u}_h/(1+\beta), 0, 1-\delta\overline{u}_h/(1+\beta))$, as in case 2. Thus, the limiting utilities when $\delta \to 1$ are $\overline{u}_h = 1+\beta$ and $\overline{u}_l = \beta$. Therefore,

$$\overline{u}_h-\frac{1+\beta}{1-\beta}\overline{u}_l=(\beta+1)\left(1-\frac{\beta}{1-\beta}\right)>0,$$

contradicting $\overline{u}_h < [(1+\beta)/(1-\beta)]\overline{u}_l$. Second, if $\delta\overline{u}_l > \beta$, the equilibrium proposals must be

$$x^1\in\left\{\left(1-\frac{\delta\overline{u}_l-\beta}{1-\beta},\frac{\delta\overline{u}_l-\beta}{1-\beta},0\right),\left(1-\frac{\delta\overline{u}_l-\beta}{1-\beta},0,\frac{\delta\overline{u}_l-\beta}{1-\beta}\right)\right\},$$
$$x^2=\left(\frac{\delta\overline{u}_h}{1+\beta},1-\frac{\delta\overline{u}_h}{1+\beta},0\right),\quad\text{and}\quad x^3=\left(\frac{\delta\overline{u}_h}{1+\beta},0,1-\frac{\delta\overline{u}_h}{1+\beta}\right),$$

yielding

$$3\overline{u}_h=(1+\beta)\left(1-\frac{\delta\overline{u}_l-\beta}{1-\beta}\right)+2\delta\overline{u}_h,$$
$$3\overline{u}_l=\beta\left(1-\frac{\delta\overline{u}_l-\beta}{1-\beta}\right)+\frac{1}{2}\frac{\delta\overline{u}_l-\beta}{1-\beta}+\beta\frac{\delta\overline{u}_h}{1+\beta}+1-\frac{\delta\overline{u}_h}{1+\beta}+\beta\frac{\delta\overline{u}_h}{1+\beta},$$

and therefore,

$$\overline{u}_l=\frac{2-\beta-2\delta+2\beta\delta}{6-6\beta-5\delta+6\beta\delta}\quad\text{and}\quad\overline{u}_h=\frac{2\beta-\delta-\beta\delta+2}{6-6\beta-5\delta+6\beta\delta}.$$

Considering again the limiting utilities when $\delta \to 1$, it follows that

$$\overline{u}_h-\frac{1+\beta}{1-\beta}\overline{u}_l=\frac{\beta+1}{1-\beta}(2\beta+1)>0,$$

contradicting again $\overline{u}_h < [(1+\beta)/(1-\beta)]\overline{u}_l$ for sufficiently high values of δ.

(c) $\overline{u}_h = [(1+\beta)/(1-\beta)]\overline{u}_l$. Any noninvesting proposer is indifferent between compensating either of the two other agents. For instance, agent 2 is indifferent

between proposals $(\delta\overline{u}_h/(1+\beta), 1-\delta\overline{u}_h/(1+\beta), 0)$ and $(0, 1-\delta\overline{u}_l, \delta\overline{u}_l)$. This implies that

$$1-\frac{\delta\overline{u}_h}{1+\beta}+\beta\frac{\delta\overline{u}_h}{1+\beta}=1-\delta\overline{u}_l,$$

yielding

$$\overline{u}_h=\frac{1+\beta}{1-\beta}\overline{u}_l.$$

Using Lemma A5, the limiting expected utilities when $\delta\to 1$ are $\overline{u}_h=(1+\beta)/(3-4\beta)$ and $\overline{u}_l=(1-\beta)/(3-4\beta)$. *Q.E.D.*

Lemma A7 *When $m=2$, the limiting expected utilities when $\delta\to 1$ are $\overline{u}_h=[(2\beta+1)^2]/[4\beta+3]$ and $\overline{u}_l=[2\beta+4\beta^2+1]/[4\beta+3]$.*

Proof Assume w.l.o.g. that m = (1,1,0). Any proposal of the noninvesting agent $\mathrm{x}^3=(x_1^3, x_2^3, x_3^3)$ must satisfy

$$(1-x_3^3)(1+\beta)=\delta\overline{u}_h \;\Leftrightarrow\; 1-x_3^3=\frac{1}{1+\beta}\delta\overline{u}_h,$$

implying

$$\begin{aligned}
u_{31}&=p\delta\overline{u}_h+(1-p)\frac{\beta}{1+\beta}\delta\overline{u}_h=\frac{1}{1+\beta}(p+\beta)\delta\overline{u}_h,\\
u_{32}&=(1-p)\delta\overline{u}_h+p\frac{\beta}{1+\beta}\delta\overline{u}_h=\frac{1}{1+\beta}(1-p+\beta)\delta\overline{u}_h
\end{aligned}$$

for some $p\in[0,1]$.

Regarding the equilibrium proposal of any investing agent, say 1, suppose first that $\mathrm{x}^1=(1,0,0)$, which is possible if either $\delta\overline{u}_2\leq\beta$ or $\delta\overline{u}_3\leq\beta$. In these cases, as $\overline{u}_1=\overline{u}_2=\overline{u}_h$, we obtain that $u_{11}=u_{22}=1+\beta$ and $u_{21}=u_{12}=\beta$. Moreover, symmetry implies that $p=1/2$. So, applying Lemma A5,

$$\begin{aligned}
&3\overline{u}_1=(1+\beta)+\beta+\frac{1}{1+\beta}\left(\frac{1}{2}+\beta\right)\delta\overline{u}_h\\
\Rightarrow\quad &\overline{u}_1=\frac{2(\beta+1)(2\beta+1)}{6-\delta+6\beta-2\beta\delta}\quad\text{and}\quad\overline{u}_3=\frac{(2\beta-\delta+2)(2\beta+1)}{6-\delta+6\beta-2\beta\delta}.
\end{aligned}$$

Note that $\overline{u}_1>\overline{u}_3$ and

$$\delta\overline{u}_3-\beta=\frac{-(2\beta+1)\delta^2+(6\beta^2+7\beta+2)\delta-6\beta(\beta+1)}{6-\delta+6\beta-2\beta\delta}>0\;\Leftrightarrow\;\delta>\hat{\delta},$$

where

$$\widehat{\delta}=\frac{1}{4\beta+2}\left(7\beta+6\beta^2-\sqrt{4\beta+\beta^2+36\beta^3+36\beta^4+4}+2\right)\in(0,1).$$

Hence, assuming $\delta > \widehat{\delta}$, we have that $\delta\overline{u}_h > \delta\overline{u}_3 > \beta$, reaching a contradiction.

Therefore, when agents are sufficiently patient, investing proposers cannot get the whole budget for them. Thus, these agents must choose compensating agent 3 or compensating the other investing agent. An investing agent prefers to compensate the investing responder rather than agent 3 when

$$\text{(A1)} \qquad \frac{1+\beta}{1-\beta}(1-\delta\overline{u}_l) < 2\beta+1-\delta\overline{u}_h,$$

whereas an investing proposer will prefer to compensate the noninvesting responder when

$$\text{(A2)} \qquad \frac{1+\beta}{1-\beta}(1-\delta\overline{u}_l) > 2\beta+1-\delta\overline{u}_h.$$

Let us consider the possible cases in turn:

Case 1. Consider that the inequality (A1) holds. In this case, agent 1 prefers to compensate agent 2 (similarly 2 compensates 1). This yields $u_{11} = u_{22} = 2\beta + 1-\delta\overline{u}_h$, $u_{13} = u_{23} = \beta$, and $u_{21} = u_{12} = \delta\overline{u}_h$, and symmetry implies $p = 1/2$. So, applying Lemma A5,

$$3\overline{u}_1 = (2\beta+1-\delta\overline{u}_h)+\delta\overline{u}_h+\frac{1}{1+\beta}\left(\frac{1}{2}+\beta\right)\delta\overline{u}_h$$
$$\Rightarrow \quad \overline{u}_1 = \frac{2(\beta+1)(2\beta+1)}{6-\delta+6\beta-2\beta\delta} \quad \text{and} \quad \overline{u}_3 = \frac{(2\beta-\delta+2)(2\beta+1)}{6-\delta+6\beta-2\beta\delta}.$$

The inequality (A1) holds only if

$$K(\beta,\delta) = (\beta+1)(2\beta+1)\delta^2-2\beta(3\beta+2)(2\beta+1)\delta+12\beta^2(\beta+1) < 0,$$

which never holds, because $K(\beta,\delta)$ has roots $\delta_+ > \delta_- > 1$ for all $\beta \in (0,1)$. Hence, this case is not possible.

Case 2. Suppose now that the inequality (A2) holds. In this case, it is optimal for agent 1 to compensate agent 3 (similarly, 2 compensates 3). This yields $u_{11} = u_{22} = [(1+\beta)/(1-\beta)](1-\delta\overline{u}_l)$, $u_{13} = u_{23} = \delta\overline{u}_l$, and $u_{21} = u_{12} = [\beta/(1-\beta)](1-\delta\overline{u}_l)$. Moreover, symmetry implies $p = 1/2$. So, applying Lemma A5,

$$3\overline{u}_1 = \frac{1+\beta}{1-\beta}(1-\delta\overline{u}_l)+\frac{\beta}{1-\beta}(1-\delta\overline{u}_l)+\frac{1}{1+\beta}\left(\frac{1}{2}+\beta\right)\delta\overline{u}_h$$
$$\Rightarrow \quad \overline{u}_h = \frac{-6\beta+2\delta-4\beta^2+6\beta\delta+4\beta^2\delta-2}{5\delta+6\beta^2+\beta\delta-6\beta^2\delta-6} \quad \text{and} \quad \overline{u}_l = \frac{2\beta-\delta-2\beta\delta+2}{6\beta-5\delta-6\beta\delta+6}.$$

The inequality (A2) holds if

$$H(\beta,\delta) = (\beta+1)(2\beta+1)\delta^2+2\beta(4\beta+6\beta^2-1)\delta-12\beta^2(\beta+1) < 0.$$

As $H(\beta,1) = (2\beta+1)(1-\beta) > 0$ and $H(\beta,0) = -12\beta^2(\beta+1) < 0$, it is immediate that $H(\beta,\delta) > 0$ for all $\delta > \delta_+$, where $\delta_+ \in (0,1)$ is the positive root of $H(\beta,\delta) = 0$. Hence, such strategies cannot be sustained in equilibrium when $\delta > \delta_+$.

Case 3. Finally, consider that $[(1+\beta)/(1-\beta)](1-\delta\overline{u}_l) = 2\beta+1-\delta\overline{u}_h$. Using Lemma A5, we obtain that

$$\overline{u}_l = \frac{\delta+4\beta^2+2\beta\delta}{\delta(4\beta+3)} \quad \text{and} \quad \overline{u}_h = \frac{(2\beta+1)(\delta-2\beta^2+\beta\delta)}{-\delta(4\beta+3)(\beta-1)}.$$

It is not difficult to show that such an equilibrium is sustained when investing agents compensate the noninvesting agent with probability

$$q = \frac{(\beta+1)(2\beta+1)\delta^2-2\beta(3\beta+2)(2\beta+1)\delta+12\beta^2(\beta+1)}{2(\beta+1)(2\beta+1)\delta^2-6\beta(\beta+1)\delta}.$$

The limiting values as $\delta \to 1$ are

$$q = \frac{1}{2(1+\beta)}, \quad \overline{u}_h = \frac{(2\beta+1)^2}{4\beta+3}, \quad \text{and} \quad \overline{u}_l = \frac{2\beta+4\beta^2+1}{4\beta+3}.$$

Q.E.D.

A.4 Specifications for Tables 1 and 4

Table A1
Specifications for Table 1

β \ c	c_1	c_2	c_3	c_4	c_5	c_6
(0,0.0937]	$\frac{7\beta}{12\beta+9}$	$\beta\frac{7-16\beta^2}{9-16\beta^2}$	$\frac{7\beta}{9-12\beta}$	$\frac{3\beta}{4\beta+3}$	$3\beta\frac{3-16\beta^2}{9-16\beta^2}$	$\frac{3\beta}{3-4\beta}$
(0.0937,0.1289]	$\frac{7\beta}{12\beta+9}$	$\beta\frac{7-16\beta^2}{9-16\beta^2}$	$\frac{3\beta}{4\beta+3}$	$\frac{7\beta}{9-12\beta}$	$3\beta\frac{3-16\beta^2}{9-16\beta^2}$	$\frac{3\beta}{3-4\beta}$
$\left(0.1289,\frac{1}{4}\right]$	$\frac{7\beta}{12\beta+9}$	$\beta\frac{7-16\beta^2}{9-16\beta^2}$	$\frac{3\beta}{4\beta+3}$	$3\beta\frac{3-16\beta^2}{9-16\beta^2}$	$\frac{7\beta}{9-12\beta}$	$\frac{3\beta}{3-4\beta}$
$\left(\frac{1}{4},0.3663\right]$	$\frac{7\beta}{12\beta+9}$	$3\beta\frac{1-2\beta}{3-4\beta}$	$\beta\frac{7-16\beta^2}{9-16\beta^2}$	$\frac{7\beta}{9-12\beta}$	$\frac{3\beta}{3-4\beta}$	∞
$\left(0.3663,\frac{1}{2}\right]$	$3\beta\frac{1-2\beta}{3-4\beta}$	$\frac{7\beta}{12\beta+9}$	$\beta\frac{7-16\beta^2}{9-16\beta^2}$	$\frac{7\beta}{9-12\beta}$	$\frac{3\beta}{3-4\beta}$	∞
$\left(\frac{1}{2},\frac{3}{4}\right]$	$\frac{7\beta}{12\beta+9}$	$\frac{\beta+1}{4\beta+3}$	$\frac{3\beta+2}{3}$	3β	∞	∞
$\left(\frac{3}{4},1\right)$	$\frac{\beta+1}{4\beta+3}$	$\frac{7\beta}{12\beta+9}$	$\frac{3\beta+2}{3}$	3β	∞	∞

Table A2
Specifications for Table 4

β \ c	c'_1	c'_2	c'_3	c'_4	c'_5
$\left(0,\frac{1}{3}\right]$	$\beta\frac{1-3\beta}{1-\beta}$	$\beta\frac{1-3\beta}{(1-2\beta)(1-\beta)}$	β	$\frac{\beta}{1-\beta}$	$\frac{\beta}{1-2\beta}$
$\left(\frac{1}{3},1\right)$	$\frac{1}{3}$	$\frac{1}{2}$	$\beta+\frac{2}{3}$	3β	∞

Table A3
Profitability of Investment

$m \to m+1$	Simple majority		Unanimity	
$0 \to 1$	$\frac{7\beta}{9-12\beta}$	if $\beta \leq \frac{1}{2}$	$\frac{\beta}{1-2\beta}$	if $\beta \leq \frac{1}{3}$
	$\frac{2}{3}+\beta$	if $\beta > \frac{1}{2}$	$\frac{2}{3}+\beta$	if $\beta > \frac{1}{3}$
$1 \to 2$	$\beta\frac{16\beta^2-7}{16\beta^2-9}$	if $\beta \leq \frac{1}{2}$	$\frac{\beta}{1-\beta}$	if $\beta \leq \frac{1}{3}$
	$\frac{\beta+1}{4\beta+3}$	if $\beta > \frac{1}{2}$	$\frac{1}{2}$	if $\beta > \frac{1}{3}$
$2 \to 3$	$\frac{\beta}{12\beta+9}$		β	if $\beta \leq \frac{1}{3}$
			$\frac{1}{3}$	if $\beta > \frac{1}{3}$

A.5 Proof of the First Claim of the Corollary

Using Propositions 1 and 4, we can compare the expected utilities of the agents for any investment profile, and thus the profitability of investment. The results are summarized next. The expressions $m \to m+1$ refer to the individual profitability of investing when there already are m investing agents. Such a profitability might depend on the exact value of β.

The value in the unanimity column of Table A3 is always weakly greater than that in the simple-majority column, in any case and for any value of β.

A.6 Stable Numbers of Investing Agents

Using Table 2 and Proposition 2, and after tedious comparisons, we display in the next table the stable number of investing agents under unanimous bargaining (m_u^*) and simple-majority bargaining (m_{sm}^*) that are obtained for different values of (β, c). Intervals in the table refer to values of c for which (m_u^*, m_{sm}^*) are attained, and the symbol $\varnothing$ refers to situations where (m_u^*, m_{sm}^*) can never be attained.

Table A4

Ranges of c that Sustain Any Possible Pair of Stable Numbers of Investing Agents (m_u^*, m_{sm}^*) for Any Possible Value of β

(m_u^*, m_{sm}^*)	$\beta > \frac{3}{4}$	$\beta \in \left(\frac{1}{2}, \frac{3}{4}\right]$	$\beta \in \left(0.3461, \frac{1}{2}\right]$	$\beta \in \left(\frac{1}{3}, 0.3461\right]$	$\beta \in \left(\frac{1}{6}, \frac{1}{3}\right]$	$\beta \le \frac{1}{6}$
(3,3)	$\left[0, \frac{7\beta}{12\beta+9}\right]$	$\left[0, \frac{7\beta}{12\beta+9}\right]$	$\left[0, \frac{7\beta}{12\beta+9}\right]$	$\left[0, \frac{7\beta}{12\beta+9}\right]$	$\left[0, \frac{7\beta}{12\beta+9}\right]$	$\left[0, \frac{7\beta}{12\beta+9}\right]$
(3,2)	$\varnothing$	$\left[\frac{7\beta}{12\beta+9}, \frac{\beta+1}{4\beta+3}\right]$	$\left[\frac{7\beta}{12\beta+9}, \frac{[16\beta^2-7]\beta}{16\beta^2-9}\right]$	$\left[\frac{7\beta}{12\beta+9}, \frac{[16\beta^2-7]\beta}{16\beta^2-9}\right]$	$\left[\frac{7\beta}{12\beta+9}, \frac{[16\beta^2-7]\beta}{16\beta^2-9}\right]$	$\left[\frac{7\beta}{12\beta+9}, \frac{[16\beta^2-7]\beta}{16\beta^2-9}\right]$
(3,1)	$\left[\frac{\beta+1}{4\beta+3}, \frac{1}{3}\right]$	$\left[\frac{\beta+1}{4\beta+3}, \frac{1}{3}\right]$	$\left[\frac{[16\beta^2-7]\beta}{16\beta^2-9}, \frac{1}{3}\right]$	$\left[\frac{[16\beta^2-7]\beta}{16\beta^2-9}, \frac{1}{3}\right]$	$\left[\frac{[16\beta^2-7]\beta}{16\beta^2-9}, \beta\right]$	$\left[\frac{[16\beta^2-7]\beta}{16\beta^2-9}, \frac{7\beta}{9-12\beta}\right]$
(3,0)	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\left[\frac{7\beta}{9-12\beta}, \beta\right]$
(2,2)	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
(2,1)	$\left[\frac{1}{3}, \frac{1}{2}\right]$	$\left[\frac{1}{3}, \frac{1}{2}\right]$	$\left[\frac{1}{3}, \frac{1}{2}\right]$	$\left[\frac{1}{3}, \frac{7\beta}{9-12\beta}\right]$	$\left[\beta, \frac{7\beta}{9-12\beta}\right]$	$\varnothing$
(2,0)	$\varnothing$	$\varnothing$	$\varnothing$	$\left[\frac{7\beta}{9-12\beta}, \frac{1}{2}\right]$	$\left[\frac{7\beta}{9-12\beta}, \frac{\beta}{1-\beta}\right]$	$\left[\beta, \frac{\beta}{1-\beta}\right]$
(1,1)	$\left[\frac{1}{2}, \frac{3\beta+2}{3}\right]$	$\left[\frac{1}{2}, \frac{3\beta+2}{3}\right]$	$\left[\frac{1}{2}, \frac{7\beta}{9-12\beta}\right]$	$\varnothing$	$\varnothing$	$\varnothing$
(1,0)	$\varnothing$	$\varnothing$	$\left[\frac{7\beta}{9-12\beta}, \frac{3\beta+2}{3}\right]$	$\left[\frac{1}{2}, \frac{3\beta+2}{3}\right]$	$\left[\frac{\beta}{1-\beta}, \frac{\beta}{1-2\beta}\right]$	$\left[\frac{\beta}{1-\beta}, \frac{\beta}{1-2\beta}\right]$
(0,0)	$\left[\frac{3\beta+2}{3}, \infty\right)$	$\left[\frac{3\beta+2}{3}, \infty\right)$	$\left[\frac{3\beta+2}{3}, \infty\right)$	$\left[\frac{3\beta+2}{3}, \infty\right)$	$\left[\frac{\beta}{1-2\beta}, \infty\right)$	$\left[\frac{\beta}{1-2\beta}, \infty\right)$

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