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Understanding the effects of COVID-19 on Financial Market Structures: A study of the USA & Brazil

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Beggar that I am, I am even poor in thanks

Hamlet

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Abstract

This study investigates the effects of the Covid-19 pandemic on the topological properties of both American and Brazilian stock markets. We build a minimal spanning tree before and during the pandemic using the correlation matrix of stocks available at the S&P500 and IBOV stock indices. We use network measures to assess the most central sectors and quantify the changes they undergo during the pandemic. Our results show that normalized tree length decreases as the correlation coefficient increases during crises; however, the mean occupational layer increases during a pandemic contrary to regular financial crises. We find the Financial, Industrial, and Information Technology sectors maintain dominance over the American market during and after the pandemic, while in Brazil, the Consumer Discretionary sector shows dominance before and after the pandemic, while the Energy and Industrial sectors become more central during the pandemic.

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CHAPTER 1

Introduction

In December 2019, a group of patients in Wuhan, China, reported the first side effects of Covid-19[5], this airborne virus with a high transmission rate forced the USA and Brazil, among many other countries, to declare a national emergency by March 2020, resulting in around 470 million reported cases worldwide[15]. A number comparable to the Great Influenza Pandemic of 1918 - more commonly known as the Spanish flu[1]- that infected around 500 million individuals[6]. During the almost two years of the Great Influenza Pandemic, about 2.1% of the population died, yet the economic ramifications of such a colossal event were not well studied[2]; however, recent technical reports attempt to shed more light[9, 4]. The lack of thorough analysis made predicting how the global economy would react to the Covid pandemic increasingly difficult. Additionally, other 18th and 19th century epidemics were localized and had little effect on the global economy[1], leaving us little resources to draw direct comparisons.

On the other hand, recent financial market crises have been well documented, with the 2008 crash taking the spotlight, [7, 24, 23] closely examine the 2008 crash and the 1987 Black Monday crash from a network perspective, drawing a set of features at times of financial crises, this set includes: stronger positive correlation between stock prices, higher betweenness centrality, higher strength, higher closeness centrality and the lower average distance between of the constructed stock network, in addition, J.P.Onnela adds that during Black Monday both the Mean Occupational Layer and Normalized Length decrease[23].

This set of features and lack of previous direct comparison motivate our analysis; we investigate the Covid-19 effect on the comovement of stock prices. We adopt Mantenga's analysis[18] and construct a Minimal Spanning Tree of the American stocks available in the S&P500 index and The Brazilian stocks available in the IBOV index. Our choice of countries is motivated by a desire to test the validity of our findings in two economies at different levels of diversity and sophistication. On the other hand, our choice of indices is motivated by market capitalization and public access to data of each. It is worth mentioning

[1]Spain has been unfairly linked to the 1918 influenza pandemic; at the time, Spain was one of very few European countries neutral during WWI, which allowed the media to report the pandemic in gory details without censorship. The lack of media coverage elsewhere allowed the pandemic to be linked to Spain. It is worth noting that there is still controversy about the country of origin.

[2]The lack of studies is attributed to the overlap of the first World War 1914-1919, as it was hard to separate the impact of both events on the economy.

that a similar analysis in methodology was performed on the South African stock market [20]. We find that during the pandemic, naturally, the Health Care sector assumes a more central node; however, not all of its industries follow the same pattern in the case of the USA. Additionally, the Financial, Industrial, and Information Technology sectors maintained dominance over the market during the pandemic and resumed their central role. In Brazil, the Energy and Industrial sector emerged as central sectors, while the Consumer Discretionary sector maintained its dominance before and during the pandemic.

The remaining of this analysis is structured as follows. Chapter 2 describes the data collection process, the mathematical treatments used, and the network measures applied to it. In Chapter 3, we examine the companies in the S&P500 index for two periods to highlight the market structure before and after Covid and analyze in-depth each industry sector; in Chapter 4, we discuss the findings for the IBOV index and compare it with S&P500, and in Chapter 5 we conclude and summarize our findings.

CHAPTER 2

Methodology

2.1 Data

In the analysis for the United States of America USA, we used The Standard and Poor's 500, known as the S&P 500, a stock market index tracking the performance of 500 large companies listed on stock exchanges in the United States; the motivation was to get a broader cross-section of the American market while still having a manageable network size. We used the python package **Alpha Vantage Stock API** to download our data [26].The data used here is the daily adjusted close prices for all the S&P500 companies traded over two periods. period one is the Pre-Covid period, which covers October 2018 to October 2019, while period two is the Covid period covering March 2020 to March 2021. In our selection of companies, we choose to work with the list of companies that have data covering both periods, amounting to only 490 as provided in Appendix A.In the analysis of Brazil, we used the Bovespa index also known as Ibovespa and IBOV. In the analysis of Brazil, we used the Bovespa index, also known as Ibovespa and IBOV. The data used is the daily adjusted close prices available on yahoo finance[27]; we used the same two time periods as our USA analysis. In cleaning the data, some companies were traded under two classes of shares: The common shares (**ON**) with voting rights and the preferred shares (**PN**) with non-voting rights; both shares would have very similar price history, in our selection if both shares exist in IBOV the common shares are selected. The list of companies with data covering both periods amounts to only 77 and is provided in Appendix A.We classified the business sectors into 11 sectors for both countries according to The Global Industry Classification Standard GICS [14].

2.2 Methods

Correlation Matrix

Stock prices are a collection of time series. A standard way of analyzing the similarities between two time series is to compute the correlation between an observable measure, our choice of an observable measure depends on the nature of our analysis [19]. A common observable would be the price change where P(t) is stock price at time t.

$$Y(t) = P(t + \Delta t) - P(t) \tag{2.1}$$

However, this observable would be problematic later on, as it is sensitive to the price scale chosen. Another observable would be the natural logarithm of the price.

$$S(t) = \ln P(t + \Delta t) - \ln P(t) \tag{2.2}$$

This observable has two major benefits, the first being that it is not sensitive to the price scale, while the second comes from the price distribution itself, since the price is approximated by a log-normal distribution[8], this transformation brings it back to a normal distribution. Our time scale would be one trading day following the analysis of Mantenga[18], Kantor[16], and Coletti [7], by incorporating our time scale we can rewrite our observable $S_i(t)$ for stock i from equation 2.2 to be

$$S_i(t) = \ln P_i(t) - \ln P_i(t-1) \tag{2.3}$$

where P_i is the daily adjusted close price. We then begin our investigation by measuring the correlation between every pair of stocks.

$$\rho_{ij} = \frac{\langle S_i S_j \rangle - \langle S_i \rangle \langle S_j \rangle}{\sqrt{\langle S_i^2 - \langle S_i^2 \rangle^2 \rangle - \langle S_j^2 - \langle S_j^2 \rangle^2 \rangle}}, \rho_{ij} \in [-1, 1] \tag{2.4}$$

With 1 indicating complete correlation, -1 complete anti-correlation, and 0 uncorrelation. This results in a $n \times n$ symmetrical matrix with $\rho_{ij} = 1$ as a diagonal, where n is the number of stocks in our sample. This leaves us with $\frac{n \times (n-1)}{2}$ unique matrix elements. So for USA we would have 119805 matrix elements and for Brazil we would have 2926 matrix elements. This is a large amount of information to process, so filtering it would be our priority.

Distance and Minimal Spanning Tree

Now that we have the correlation coefficient between each pair of stocks, we are interested in finding the distance between them, we follow [18] to construct a distance metric d_{ij} where

$$d_{ij} = \sqrt{2(1 - \rho_{ij})} \tag{2.5}$$

our distance fulfills the three metric axioms such that

i $d_{ij} = 0 \iff i = j$

ii $d_{ij} = d_{ji}$

iii $d_{ij} \leq d_{ik} + d_{kj} \ \forall k$

The first and second property are easily verified from the symmetry of the correlation matrix. We set the stocks as nodes, and create an undirected weighted edge between each two stocks, where the weight is the distance calculated by equation 2.5. This would result in a fully dense graph where crucial information is hidden, to avoid that we construct a minimal spanning tree MST. An MST is a subset of a connected undirected weighted graph with n nodes and n-1 edges that has no closed loops and minim total edges weight. To construct the MST we follow Kruskal's algorithm [17] explained below:

i Sort all edges in ascending order of their weight.

ii Connect the 2 nodes with the lowest weighted edge. If and only if it does not create a closed loop.

iii If two or more edges have equal weight, choose one of them arbitrarily.

iv Repeat step (ii) until all nodes are connected.

We find that the distance between two nodes $\hat{d}_{ij}$ following the edges of the MST satisfies the first two properties of a metric distance where $\hat{d}_{ij} = 0 \iff i = j$, and $\hat{d}_{ij} = \hat{d}_{ji}$, additionally the triangular inequality is now replaced by the the ultra-metric inequality:

$$\hat{d}_{ij} \leq max\{\hat{d}_{ik}, \hat{d}_{kj}\} \tag{2.6}$$

Ultra-metric spaces allow us to describe systems in a hierarchical way, commonly used in phylogenetics and later adopted by physicist [25] and ecno-physicists in portfolio taxonomy [19].With our graph now in place, we move to the analysis phase by adopting certain measures from graph theory which will be applied to our MST.

Network measures

In our analysis we are interested in understanding the topology of our network [22], and in doing so find the most influential companies and understand the role each sector played during the pandemic.

Degree

The first measure we use is the node's degree in the MST, the number of edges connected to the node. The higher the degree the more central a node is, since we have a tree, the minimum degree for a node is one, while the maximum is $(n - 1)$.

Closeness Centrality

The second measure we use is the closeness centrality. The closeness centrality measures the mean geodesic distance from one node to another in the MST [22]. The geodesic distance, more commonly known as the shortest path length between two nodes - not to be confused with the distance defined earlier by equation 2.5- is calculated by measuring the minimum number of edges between one node to another without regards to its weight. Unlike the degree, the lowest the closeness centrality the more central a node is and the more access it has to other nodes. We calculate the closeness centrality C_i for node i as

$$C_i = \frac{n}{\sum_j l_{ij}} \tag{2.7}$$

Where l_{ij} is the shortest path length between node i and j, disregarding the edges weight and n is the total number of nodes.

Betweeness Centrality

The third measure we use is the betweeness centrality[12], which measures how much a node lies in the shortest path between other nodes in the MST. The higher the betweeness the more influential a node is. We calculate the betweeness centrality for a node i as

$$B_i = \sum_{jk} n^i_{jk} \tag{2.8}$$

Where n^i is 1 if node i lies in the shortest path between node j and k and zero if not.

Average Distance

The fourth measure we use is the average distance, which measures the average weight defined by equation 2.5 in the shortest path from the node of interest to any other node.

Strength

The fifth measure is the strength, we define it on our MST for node i as the sum of correlation coefficients calculated from its edges.

$$S_i = \sum_{i \neq j} \rho_{ij} \tag{2.9}$$

Mean Occupation Layer and Normalized Length

In this section we aim to investigate the changes to the tree topology and whether it shrinks in times of crisis as reported by J.P.Onnela during Black Monday [24]. The first step in our analysis is to determine the central node, a node with the highest influence on the tree, thus any changes to its stock price are strongly felt by the rest of the nodes. To determine the central node we have several centrality measures. We follow [24] logic and explain the three methods of selection. The first two are straight forward from our definition of a central node. If a node has the highest number of connections and these connections are of the highest weight it is usually the most influential node thus a central is defined as:

i the node with the highest degree in the MST.

ii the node with the highest correlation coefficient weighted degree in the MST.

Usually both definitions lead to the same central node but in the occasion that it does not, we use the third definition explained later in this section. After finding our central node we are now interested in the topology of the tree with respect to it. We start by allocating a level to each node such that:

1. The central node belongs to level Zero.
2. All the nodes directly connected to the central node belong to level One.

3. All the nodes directly connected to nodes of level One belong to level Two.

4. Proceed until all nodes have been allocated to a level.

Now that we have the total number of layers and number of nodes occupying each layer we proceed by finding the average mean occupational layer defined as:

$$L_m = \frac{1}{N} \sum_{i=1}^{N} level(v_i) \tag{2.10}$$

where N is the total number of nodes and $level(v_i)$ measures how far node i is form the central node v. We can find the mean occupational layer with respect to any node not just the central node, however the central node yields the least mean occupational layer, which makes the third definition of the central node. Next we move to the normalized tree length, defined by the weighted edge sum in equation 2.11 where $\hat{d}_{ij}$ is the ultrametric distance between node i and j.

$$L_n = \frac{1}{N-1} \sum_{\hat{d}_{ij}} \hat{d}_{ij} \tag{2.11}$$

Linkage Reliability

As the MST sacrifices important information from the correlation matrix for the sake of readability, it is usually accompanied by the linkage reliability. The linkage reliability is calculated using the **Row Bootstrap** method proposed by [11], and further investigated and validated in MSTs by [21], and applied in similar analysis by [7] and [16]. The Row Bootstrap method uses the collected data in matrix X with n columns representing the companies and T rows representing the length of the period in question. We construct 100 replicas of the original X matrix, each replica is constructed by randomly selecting T rows from X while allowing repetition, a schematic description is provided in figure 2.1. The MST of each replica is then constructed and the linkage reliability is calculated as the percentage of the appearance of each original edge in the replicas.

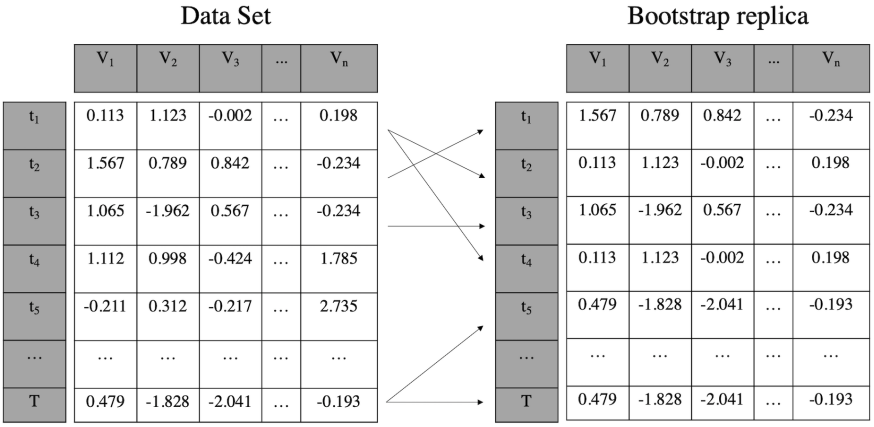

Figure 2.1: Schematic description of the **Row Bootstrap** method copied from [21]. Rows of different time records are sampled with replacement and the bootstrap replica is obtained. The correlation matrix is then computed from the bootstrap replica of vector of data.

CHAPTER 3

Analysis of S&P 500 (USA)

In 2020 USA had a gross domestic product (GDP) of 20.953 Trillion USD, accounting for 24.7% of the world's GDP, making it the largest economy in the world [3]. To represent it, we select the S&P500 stock index, a capitalization-weighted index of the top 500 companies in America. The companies are primarily chosen according to their market size, but other factors such as liquidity and industry group representation also factor in the selection process. As of March 2022, the S&P500 index represents approximately 80% of American market capitalization [13]. In this chapter, we construct a minimal spanning tree using the S&P500 companies selected according to section 2.1, analyze it using our network measures and investigate each sector and its industries in detail.

3.1 Minimal Spanning Tree Construction

Correlation Distribution

Following the methodology section, we construct the correlation matrix for the S&P500. Figure 3.1 shows the distribution of the correlation coefficients in the two periods considered. We notice a broader correlation distribution during the Covid period with a shift towards higher positive correlation coefficient in addition to longer and more defined negative tale quantified by a higher excess kurtosis of around 1.8, even though the negative correlation area is bigger in the Pre-Covid periods, the correlation distribution extends to more negative values in the Covid period with a negative skewness of around -1.1 as shown in table 3.1.

	Min	Max	Av	Var	SK	Kur
Oct 2018 - 2019	-0.223	0.932	0.324	0.028	-0.174	-0.104
Mar 2020 - 2021	-0.368	0.963	0.499	0.033	-1.128	1.838

Table 3.1: Minimum, Maximum, Average, Variance, adjusted Fisher-Pearson coefficient of skewness and excess kurtosis of the correlation matrix for S&P500, Pre-Covid period (top) and Covid period (down).

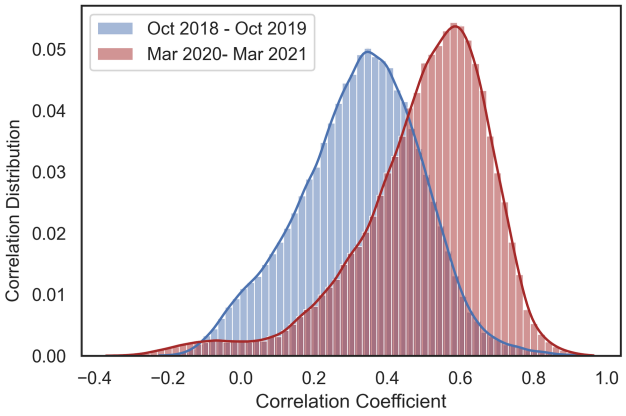

Figure 3.1: Density distribution of the correlation coefficients of the Pre-Covid period (Blue) and the Covid period (Red) the bin width used is 0.02

Construction of the Minimum Spanning Tree

Next, we construct the distance matrix according to equation 2.5. Using Kruskal's algorithm, we first construct the MST for the Pre-Covid period as shown in figure 3.2. Each node represents a company- with colors representing each sector- edges represent the distance between the companies, and their thickness reflects the reliability of the link. Looking closely at figure 3.2, we could find that, for the most part, sectors are disjoint communities and represent different branches in the MST. The Industrial, Information Technology and Financial sectors dominate the center of the MST. Moving from the center, we find the entire utilities and health care sectors lying at the outer layer of the MST. During the Covid period in figure 3.3, we notice the separation between sectors has been reduced, and a redistribution has taken place with the Industrial sector dominating the center of the MST. In addition, the Healthcare sector moves from a single branch occupying an outer layer to a well-mixed sector occupying the majority of the levels, while the Utilities move to a more central position. More detailed sector analysis will be provided later in section 3.4.

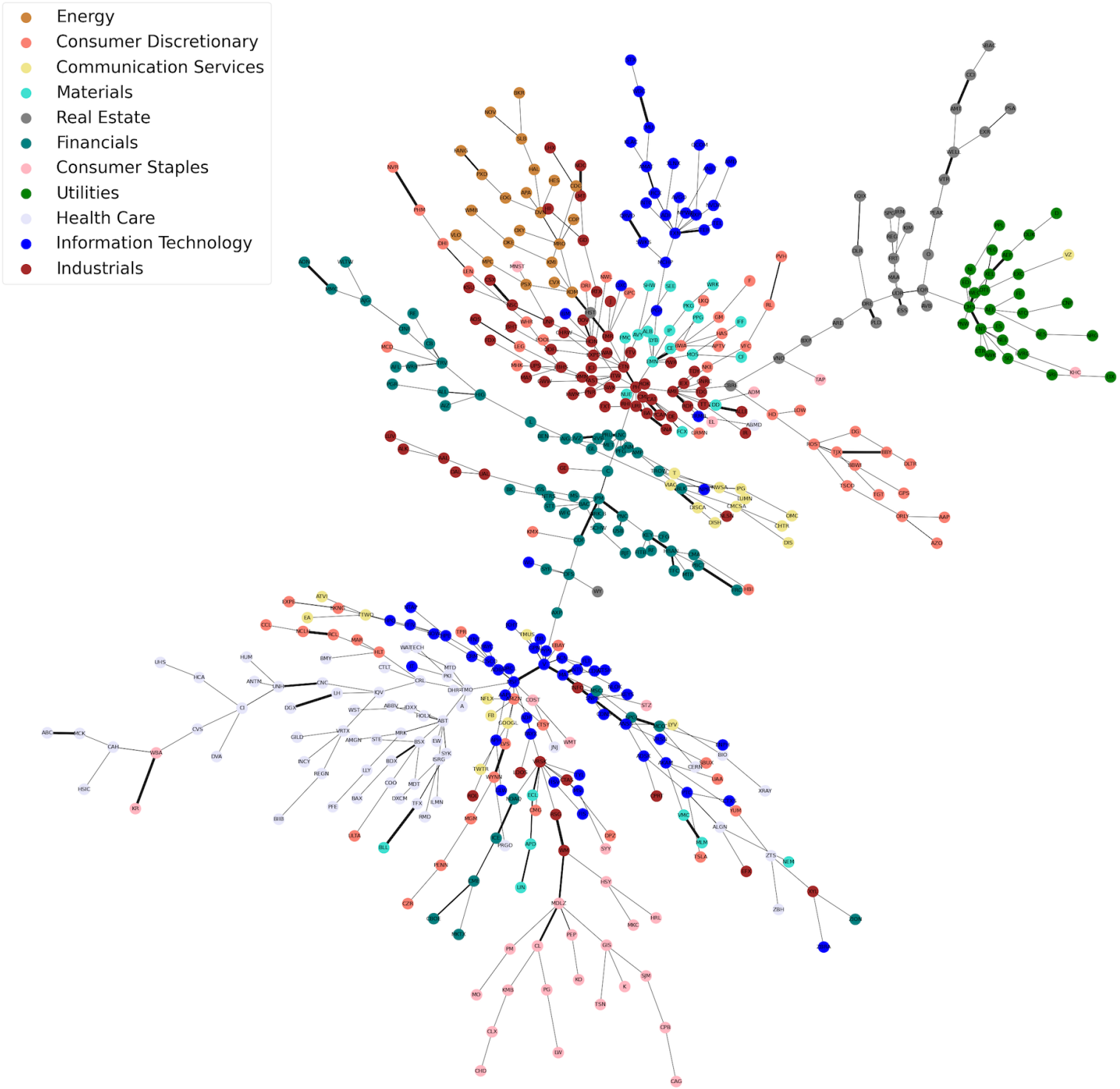

Figure 3.2: MST during the Pre-Covid period October 2018 - October 2019, the colors of the nodes indicate their sector, the labels represent their trading symbo and the thickness of the link indicate the reliability of the link

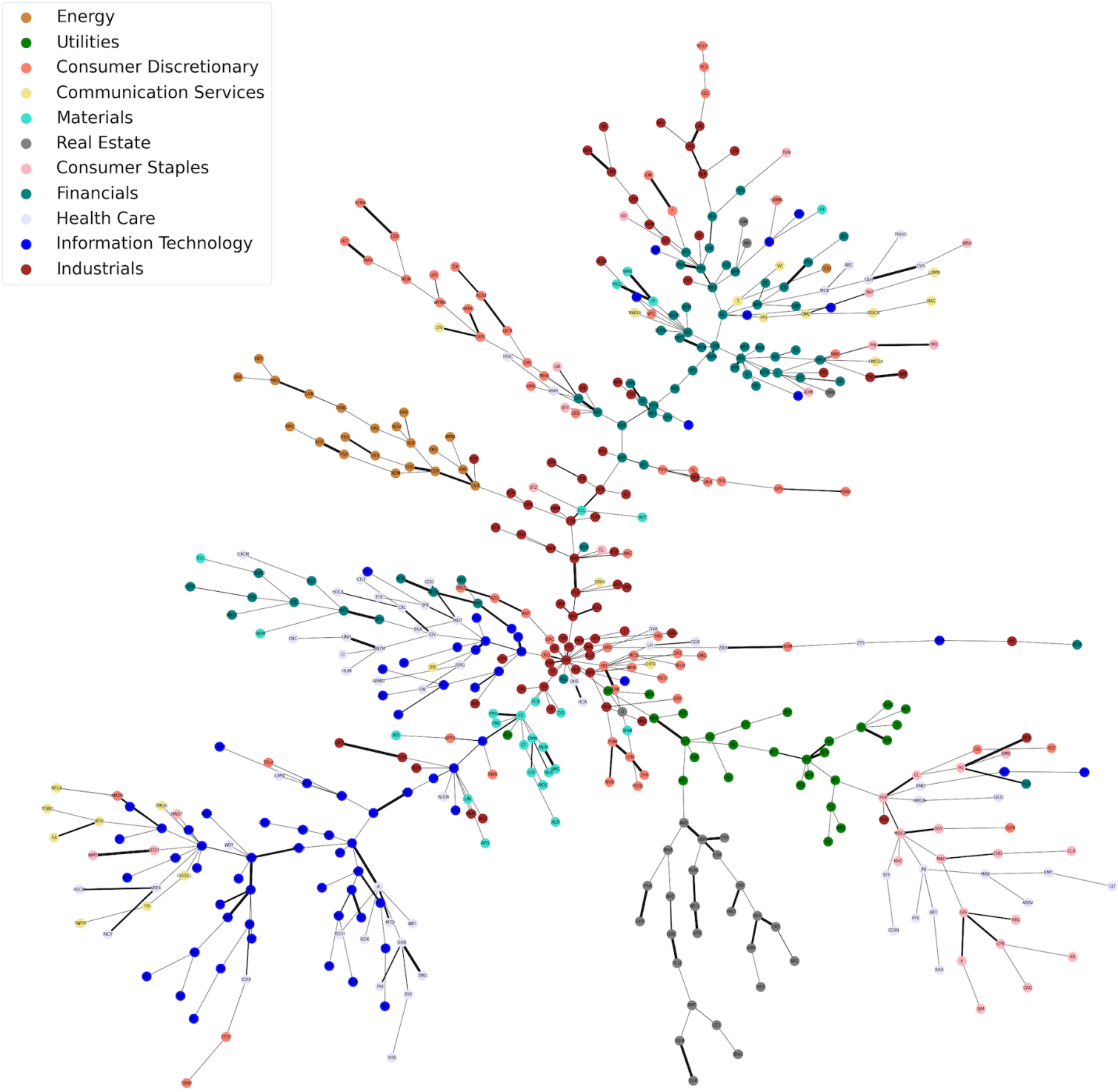

Figure 3.3: MST during the Covid period March 2020 - March 2021, the colors of the nodes indicate their sector, the labels represent their trading symbol and the thickness of the link indicate the reliability of the link

3.2 Measures

In our analysis of the minimum spanning trees, we use two sets of measures, the first a set of topological measures which does not include the distance: Nodes' degree distribution and betweenness centrality. The second is a set of measures that deal with the distance: the nodes' average distance, strength, and closeness centrality.

Node's Degree Distribution

In figure 3.4 we find a similar degree distribution for both the Pre-Covid and Covid period except for the Covid period we have a node **DOV:Dover Corporation** with a degree of 21 dictating the disappearance of any other node with a degree above 10 - to preserve the total number of edges in both MST. Which makes it a strong candidate for the central node. In table 3.2 we list the nodes with highest degree, notice that the Industrials and Information Technology sectors remain central in both periods.

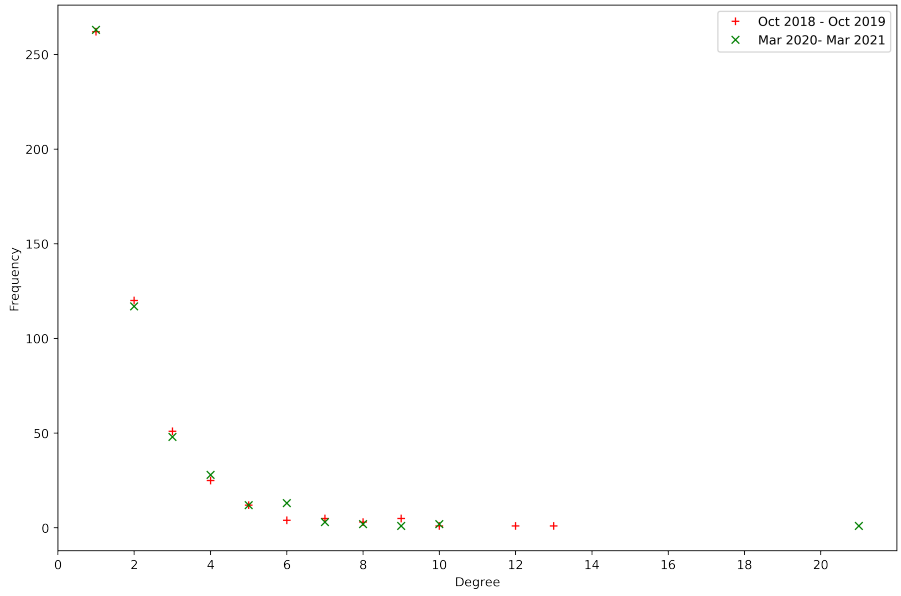

Figure 3.4: Degree Distribution of the nodes of the MSTs during the Pre-Covid and Covid period

	Node	Degree	Sector
Oct 2018 - 2019	**PH**	13	Industrials
	AME	12	Industrials
	TXN	10	Information Technology
	ABT	9	Health Care
	EMN	9	Materials
	MSFT	9	Information Technology

	V	9	Information Technology
	VRSK	9	Industrials
Mar 2020 - 2021	**DOV**	21	Industrials
	MSFT	10	Information Technology
	APH	10	Information Technology
	BAC	9	Financials
	CE	8	Materials
	RJF	8	Financials

Table 3.2: Nodes with the largest degree and their corresponding industry sector. We have listed nodes with degrees above 8 during the Pre-Covid period (top) and degrees above 7 during the Covid period (down)

Betweenness Centrality

Using the Betweenness centrality as shown in figure 3.5 we find that the centrality of the nodes during the Covid period has slightly increased up to node number 50 - with **DOV** taking the lion's share - after which both periods display similar behavior with a betweenness centrality approaching zero. In table 3.3 we can find that during Covid the Industrials sector has the top betweeness centrality while in Pre-Covid period we have both Industrials, Financials and Information Technology.

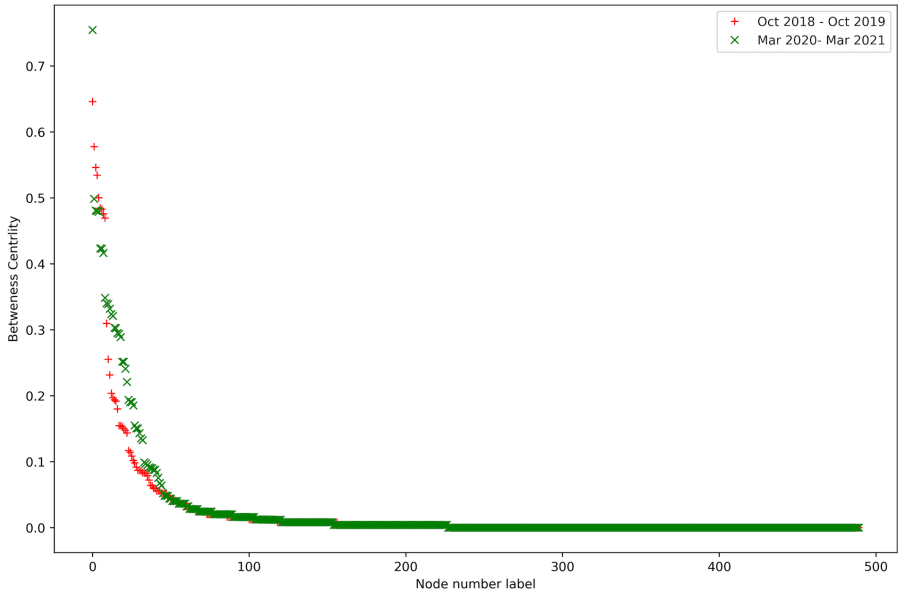

Figure 3.5: Betweenness centrality for the MSTs during the Pre-Covid and Covid period, the nodes are labeled from 1 to 490

	Node	Betweenness Centrality	Sector
Oct 2018 - 2019	**PH**	0.65	Industrials
	LNC	0.58	Financials
	JPM	0.55	Financials
	V	0.53	Information Technology
	C	0.50	Financials
Mar 2020 - 2021	**DOV**	0.75	Industrials
	ETN	0.50	Industrials
	AME	0.48	Industrials
	NSC	0.48	Industrials
	CSX	0.47	Industrials

Table 3.3: Nodes with the largest Betweenness Centrality and their corresponding industry sector during the Pre-Covid period (top) and the Covid period (down)

Strength Distribution

Following figure 3.1 and table 3.1 we quantified a higher correlation distribution in the Covid period and following the strength definition we expect to have a higher strength distribution. In figure 3.6 we notice again the central node **DOV** having the highest strength with the remaining nodes having a slightly larger ones than their Pre-Covid counter part. We also notice in table 3.4 that the central sectors are: Industrials, Information Technology and Financials are the same before and during Covid.

	Node	Strength	Sector
Oct 2018 - 2019	**PH**	9.97	Industrials
	AME	8.3	Industrials
	TXN	7.77	Information Technology
	CMS	6.73	Utilities
	LNC	6.71	Financials
Mar 2020 - 2021	**DOV**	17.09	Industrials
	BAC	8.04	Financials
	MSFT	7.87	Information Technology
	APH	7.38	Information Technology
	CE	6.65	Materials

Table 3.4: Nodes with largest Strength and their corresponding industry sector during the Pre-Covid period (top) and the Covid period (down)

Average Distance

In figure 3.7 we have a general decrease of the average distance during the Covid period showing that the nodes are more closely connected. In table 3.5 we show the sectors with the smallest Average distance.

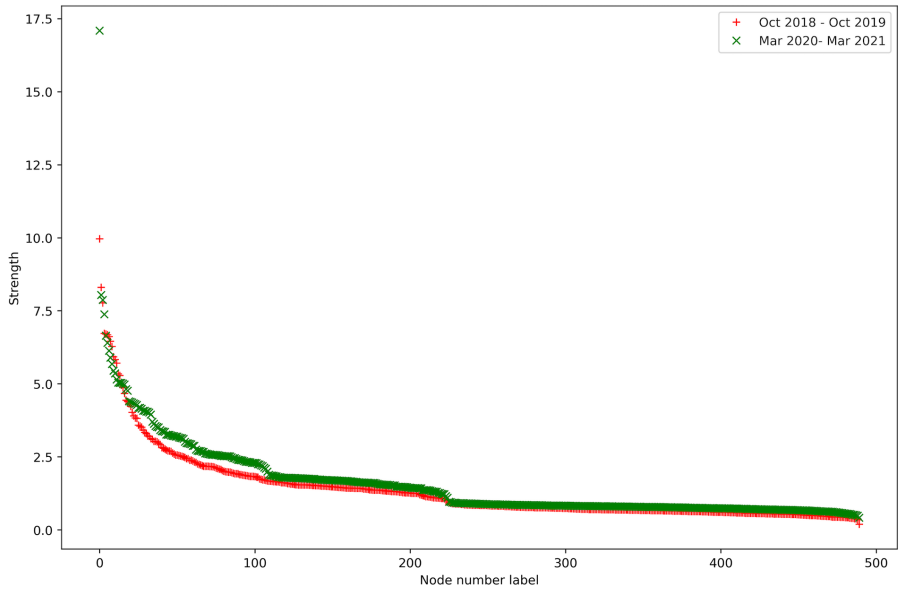

Figure 3.6: Strength distribution for the MSTs during the Pre-Covid and Covid period, the nodes are labeled from 1 to 490 according to their strength.

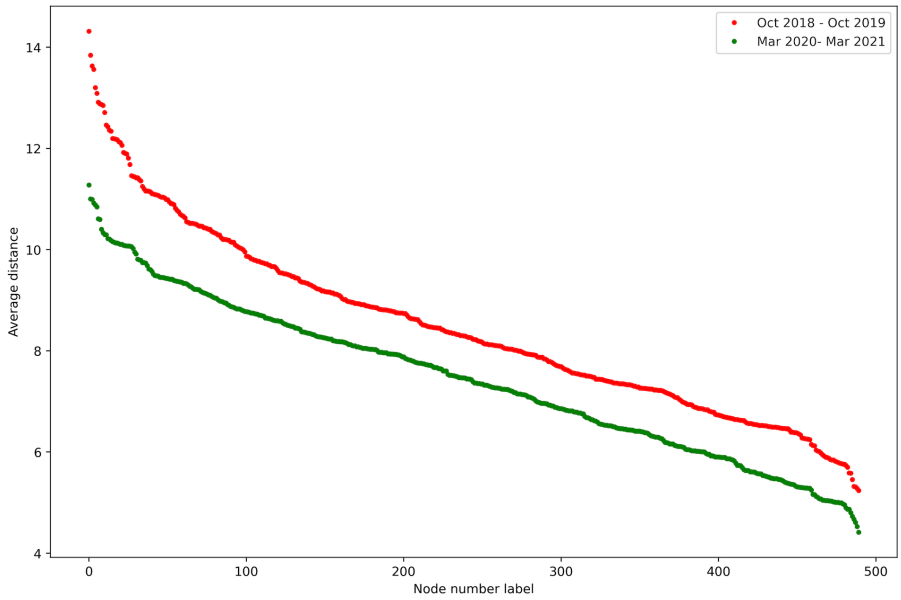

Figure 3.7: Average distance for the MSTs during the Pre-Covid and Covid period

	Node	Av. Distance	Sector
	AME	0.10	Industrials
	PH	0.11	Industrials
Oct 2018 - 2019	**VRSK**	0.13	Industrials
	DOV	0.07	Industrials
	APH	0.13	Information Technology
Mar 2020 - 2021	**MSFT**	0.15	Information Technology

Table 3.5: Nodes with smallest Average Distance and their corresponding industry sector during the Pre-Covid period (top) and the Covid period (down)

Closeness Centrality

A node's closeness centrality is the inverse of its average distance, which is why when comparing figure 3.5 with figure 3.6 we find the lines are now shifted where the Covid period on top of the Pre-Covid period in agreement with [7]. In table 3.6 we list the nodes with lowest closeness centrality.

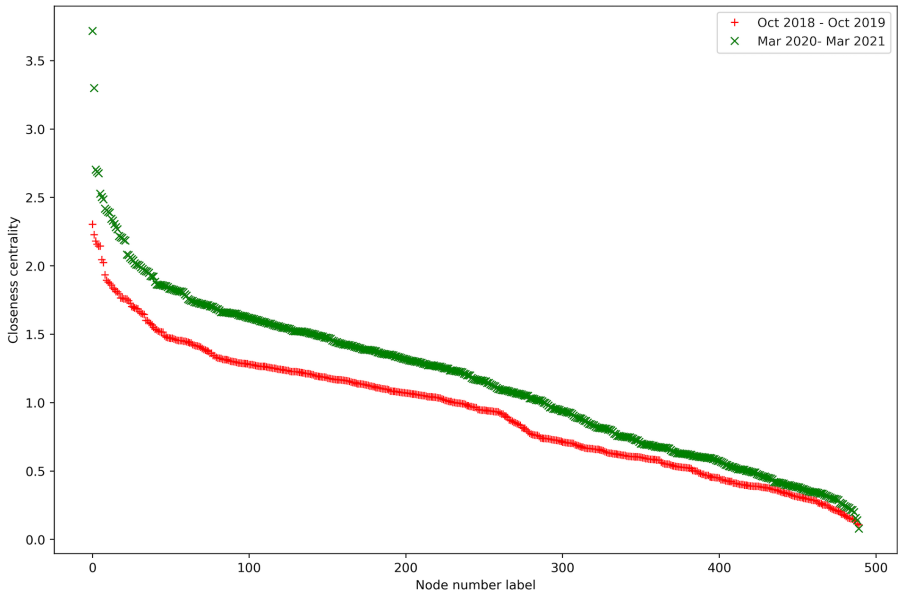

Figure 3.8: Closeness Centrality for the MSTs during the Pre-Covid and Covid period

	Node	Closeness Centrality	Sector
	AME	0.107	Industrials
	PH	0.114	Industrials
Oct 2018 - 2019	**VRSK**	0.136	Industrials

	ABT	0.145	Health Care
	EMN	0.148	Materials
	DOV	0.078	Industrials
	APH	0.14	Information Technology
Mar 2020 - 2021	**MSFT**	0.156	Information Technology
	RJF	0.2	Financials
	A	0.215	Health Care

Table 3.6: Nodes with lowest Closeness Centrality and their corresponding industry sector during the Pre-Covid period (top) and the Covid period (down)

3.3 Finding the Central node

To measure the structural differences between our two MSTs, we follow our methods section and start by finding the central node. During the Covid period, the central node was **DOV: Dover Corporation** with a degree of 21 which satisfied all three definitions mentioned in section 2.2, making it the center mass of the tree; during the Pre-Covid period, we had two choices for the central node, **PH: Parker Hannifin** and **AME: AMETEK** with **PH** having a higher degree and **AME** having a higher correlation coefficient weighted degree. Testing both options, we calculated that **PH** would result in a lower mean occupation layer of $L_m = 7.8$ as opposed to $L_m = 8.5$ calculated with **AME**, and consequently, we choose **PH**. Consequently, any other choice of a central node for both periods would result in higher mean occupational layers than those calculated in table 3.7. To emphasize this statement, we calculated the mean occupational layer for the Pre-covid period with **DOV** as a central node instead of **PH**, which resulted in $L_m = 10.7$; on the other hand, if we calculated it for the Covid period with **PH** instead of **DOV**, we get $L_m = 9.2$. After selecting the central node, we set its level to zero and take the position of every other node relative to it. We find that during the Pre-Covid period, we have a total of 19 layers, while we have 22 layers during the Covid period, resulting in a higher mean occupational layer during the Covid period, as seen in table3.7. A higher mean occupational layer does not fully agree with J.P.Onnela's analysis of Black Monday [23] where both the normalized length and mean occupational layer, while the normalized tree length did shrink during the pandemic as an indication of strong correlation often noticed during crises. One reason to explain this could be that the pandemic caused significant variations on an intra-sector level, later investigated in section 3.4.

	Mean Occupation layer	Normalized Tree Length
Oct 2018 - 2019	$L_m = 7.870$	$L_n = 0.366$
Mar 2020 - 2021	$L_m = 8.614$	$L_n = 0.269$

Table 3.7: Mean Occupation Layer and Normalized Tree Length for S&P500 with **PH** as the central node for Pre-Covid period (top) and **DOV** as the Covid period (down)

3.4 Analysis per sector

In this section, we aim to investigate each sector closely. As we will be averaging per sector size going forward, we plot the relevant size of each sector to put our results in perspective, as can be seen in 3.9. We begin with averaging three

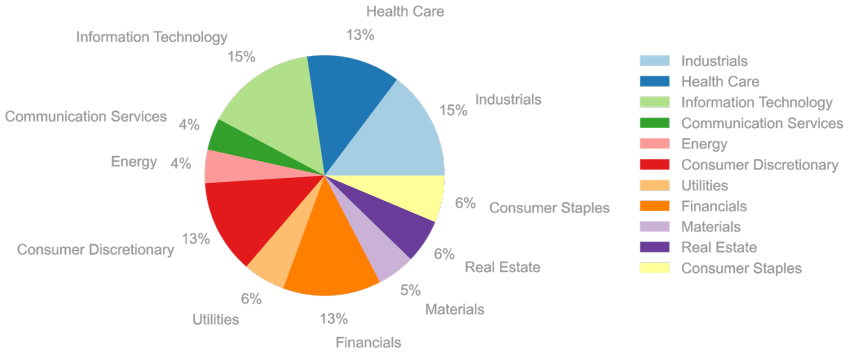

Figure 3.9: Relative size of each sector in the S&P500 index. The right color palette indicates the color of each sector. The size was measured by calculating the number of companies per sector/total number of companies and it is constant throughout the two periods.

measures per sector: distance, degree, and betweenness. We list the values of the Covid period in table 3.8 while the Pre-Covid period is listed in table 3.9. To put the above measures in perspective, we evaluate the percentage of change in the averaged measures of the two periods in table **??**. In the Average distance, we have from figure 3.7 that the Covid period has more closely connected nodes and thus a generally lower average distance; however, the Health-Care sector shows apparent defiance with an average distance of 7.685 exhibiting a 43.8% increase and owning to its larger size in the market we visibly detect it in figure 3.3. Moving on to the average degree, figure 3.4 shows that we have almost the same behavior during the two periods except for the central node of the Covid period **DOV**, which belongs to the industrial sector. We have a general agreement with three sectors having a 0% increase while the financial sector shows a slight increase of 18.4%. Later betweenness centrality shows a slight increase in figure 3.5; however, the utility sector shows a change that is not slight, with a 595.2% increase.

	Count	Av. Distance	Av. Degree	Av. Betweeness
Industrial	72	5.090	2.042	0.052
Health Care	62	7.685	1.774	0.006
Information Technology	73	6.039	2.247	0.032
Communication Services	21	8.284	1.476	0.003
Consumer Staples	31	8.459	1.903	0.012
Consumer Discretionary	62	6.863	1.677	0.006
Utilities	28	2.494	2.036	0.064
Financial	65	4.775	2.569	0.051
Materials	25	4.752	1.720	0.015

Real Estate	29	4.566	1.828	0.0129
Energy	22	2.658	1.955	0.0128

Table 3.8: Summary of all measures per sector during the Covid period March 2020-March 2021, Count is the number of companies representing each sector.

	Count	Av. Distance	Av. Degree	Av. Betweeness
Industrial	72	5.077	2.319	0.027
Health Care	62	5.344	1.968	0.014
Information Technology	73	6.423	2.123	0.027
Communication Services	21	7.126	1.619	0.003
Consumer Staples	31	7.884	1.710	0.006
Consumer Discretionary	62	8.205	1.677	0.006
Utilities	28	2.118	2.036	0.009
Financial	65	5.042	2.169	0.059
Materials	25	6.135	1.760	0.008
Real Estate	29	3.811	2.000	0.052
Energy	22	2.712	1.955	0.010

Table 3.9: Summary of all measures per sector during the Pre-Covid period October 2018 - October 2019, Count is the number of companies representing each sector.

	% Av. Distance	% Av. Degree	% Av. Betweeness
Industrial	0.268	-11.976	95.092
Health Care	43.802	-9.836	-58.189
Information Technology	-5.985	5.806	18.988
Communication Services	16.258	-8.824	0.048
Consumer Staples	7.286	11.321	104.494
Consumer Discretionary	-16.355	0.000	-6.431
Utilities	17.763	0.000	595.233
Financial	-5.286	18.440	-13.745
Materials	-22.541	-2.273	83.330
Real Estate	19.823	-8.621	-75.333
Energy	-1.991	0.000	22.689

Table 3.10: Percentage of the difference in measures per sector for the Covid compared to the Pre-Covid period

Closer Look: Sector Inhomogeneities

In this subsection, we wish to zoom into each sector, quantify the differences, and explain why our measures do not agree with a standard market crash. First, we introduce a new measure not directly related to our MSTs but directly related to the investors and companies themselves: price growth. We define it in each period of investigation as:

$$G(i) = \frac{P(i)_{Av.} - P(i)_{Int.}}{P(i)_{Int.}} \tag{3.1}$$

Where $G(i)$ is the price growth of company i, $P(i)_{Av.}$ is the closing price averaged over the length of the selected period, and $P(i)_{Int.}$ is the initial closing price chosen with respect to the selected period. Following equation 3.1, we average the price growth per sector; as shown in table 3.11, we list the sectors according to their average price growth in descending order. As expected, we have the Health Care, Information Technology, and Communication services - the leading sectors during the pandemic having the greatest growth. However, the Material sector - the second smallest sector in size - leads the price growth list. We also have expected stagnation in Real Estate, Utilities, and Financial sectors due to the worldwide lock-down.

	Covid %	Pre-Covid%	Difference%
Materials	10.793	-2.713	13.506
Health Care	10.543	-1.876	12.419
Information Technology	16.140	4.807	11.333
Communication Services	8.461	-2.526	10.987
Consumer Discretionary	10.492	1.963	8.529
Industrials	4.499	-3.560	8.059
Energy	-19.987	-20.341	0.354
Consumer Staples	1.854	3.917	-2.063
Financial	-5.169	-1.971	-3.198
Utilities	-11.678	13.335	-25.013
Real Estate	-14.263	12.945	-27.208

Table 3.11: The average growth of the price of each sector during the Covid period from March 2020-March 2021 and the Pre-Covid period from October 2018 - October 2019, and the difference between them in growth

Health Care

We start by addressing the biggest elephant in the room, the Health Care sector, comprising 13% of the market, a previously well-balanced sector in terms of price growth with a contracting average of -1.8%, as figure 3.11 shows. It contains two main industry groups: Health Care Equipment & Services and Pharmaceuticals, Biotechnology & Life Sciences; both were vital in mitigating the pandemic. Yet, when investigating the average price growth in each company, we can see that not all industries played an equal role. In figure 3.10, we see the expected average growth, but the vaccine-producing companies are not leading it; we have **ABMD: Abiomed, CTLT: Catalent, WST: West Pharmaceutical Services** instead.

Interestingly **ABMD** was not even involved in the production or packaging of the vaccine but rather specialized in heart pumps, which were used to help patients through serious heart and lung complications from Covid[1]. While **WST** and **CTLT** were involved in the packaging and manufacturing of the vaccine, as **WST** a maker of rubber stoppers was involved in vaccine packaging [2] and **CTLT** had a collaboration with **Moderna** with regards to high-speed

[1] More on ABMD trajectory here
[2] More on their vaccine related business here

filling lines[3]. The absence of vaccine producing companies like **PFE: Pfizer** or **JNJ: Jonhson and Johnson**[4], could also be attributed to our period choice. In terms of inequality in price growth, we have **BIIB: Biogen Inc.**, **PRGO: Perrigo Company**, and **CI: Cigna Corp** with negative growth. With **BIIB** specializing in Neuroscience medical solutions, **PRGO** specializes is Self-Care Products - a sub-sector taking a revenue hit during Covid [5]- and **CI** a healthcare insurance provider - another sub-sector negatively affected by Covid[6]. This non-consistent behavior of the Health Care industries could be the cause of the increase in the average distance in table 3.10.

[3]More on CTLT collaboration here

[4]MRNA: Moderna was not included in the analysis as it joined the S&P500 in July 2021 see Bloomberg coverage here

[5]PRGO reporting revenue decline here

[6]Bloomberg coverage here

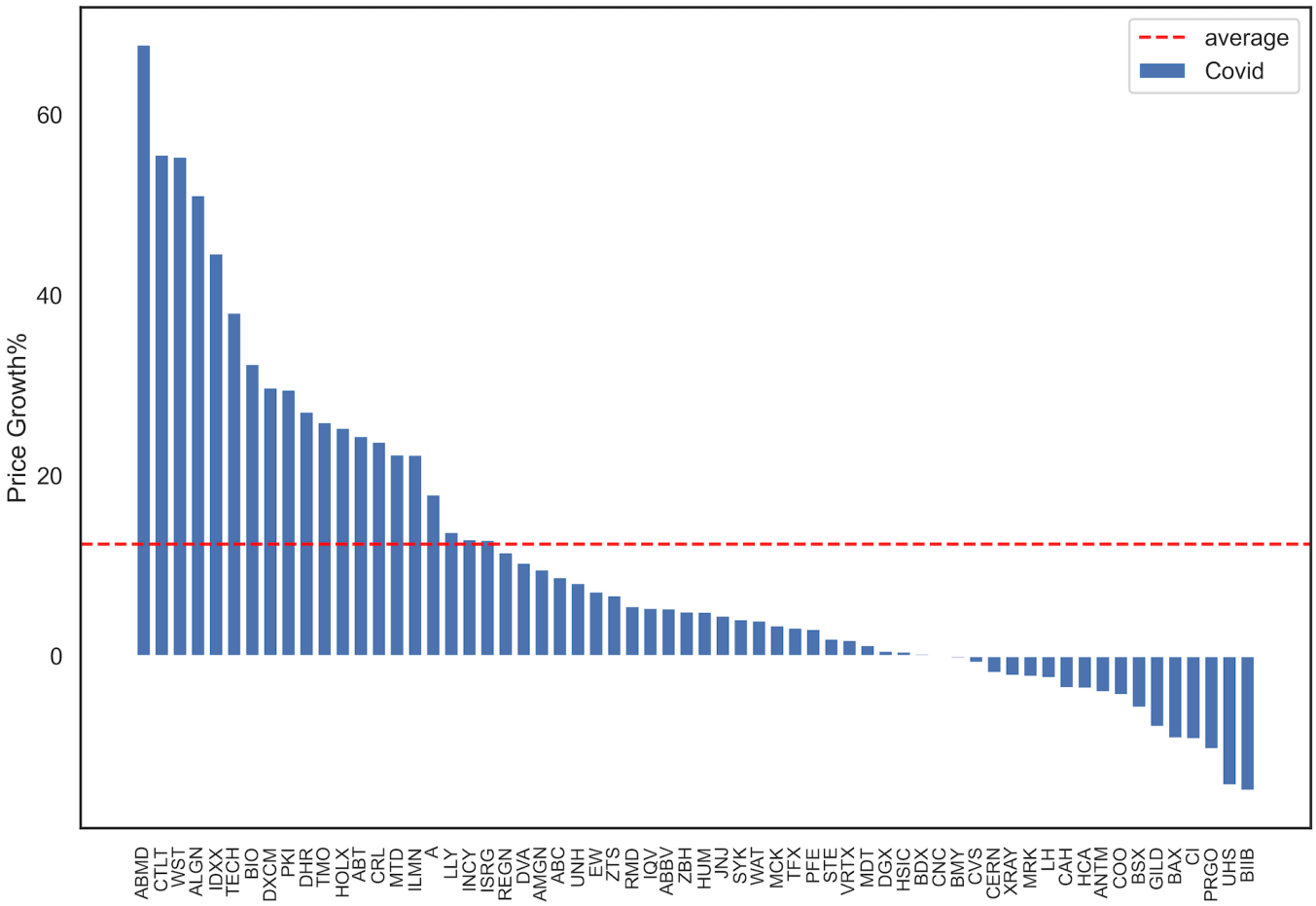

Figure 3.10: Heath Care sector price growth per company during the Covid period March 2020 - March 2021. The x-axis represents the company symbols and the y-axis represents the price growth percentage. The majority of the companies experience a positive growth with an average of 10.543%. Leading the growth is **ABMD: Abiomed** with 67.8% while vaccine producing companies **PFE: Pfizer** or **JNJ :Jonhson and Johnson** have 3% and 4.5% respectively

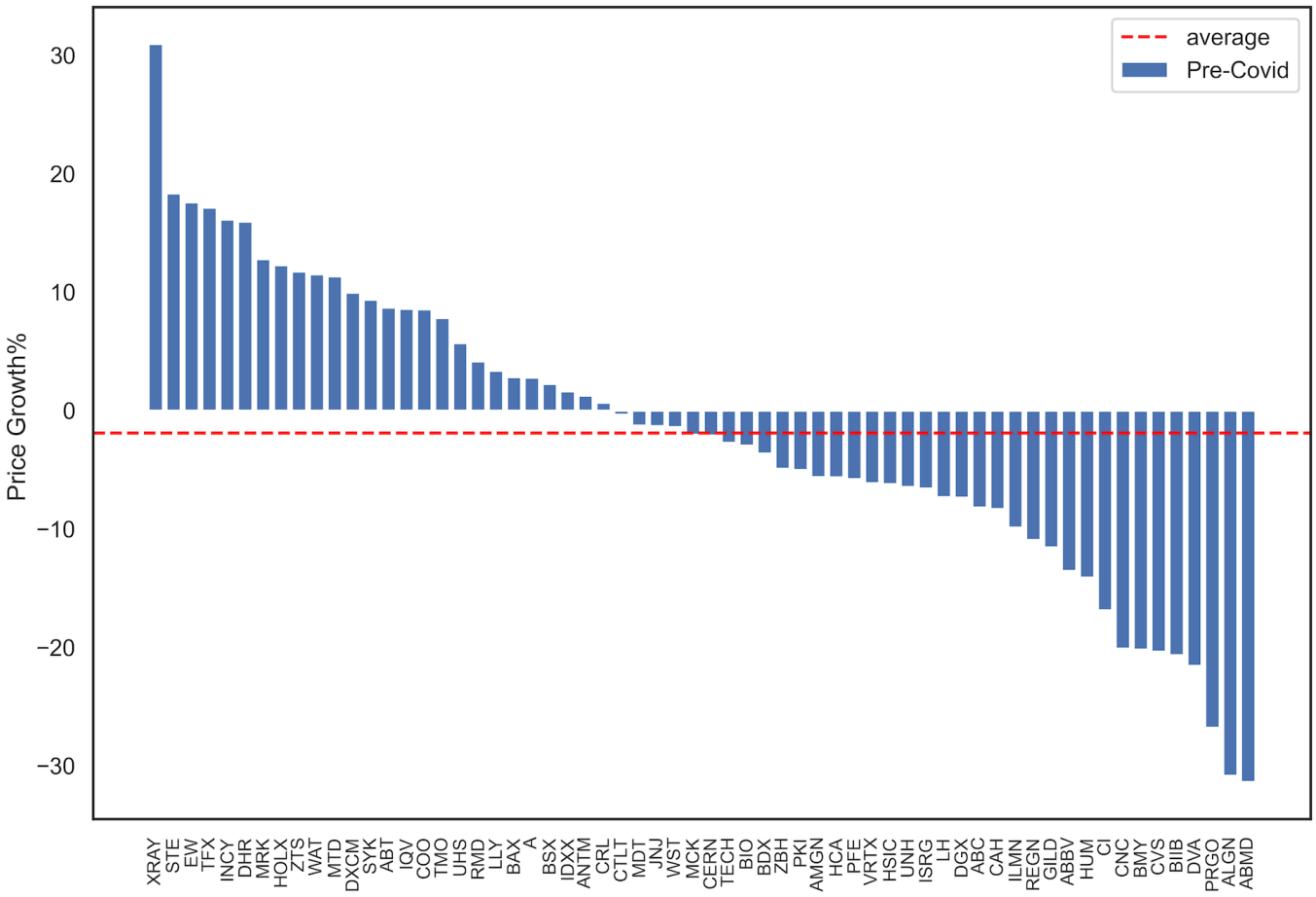

Figure 3.11: Health Care sector price growth per company during the Pre-Covid period October 2018 - October 2019.The x-axis represents the company symbols and the y-axis represents the price growth percentage. The growth is more balanced with a slight tendency towards negative growth with an average of -1.876%. The highest growth belongs to **XRAY: Dentsply Sirona** and **STE: Steris** a dental equipment manufacturer and dental consumables producer and a sterilization and surgical service provider. The highest negative growth belongs to **ABMD: Abiomed** with -31.4%

Information Technology

The biggest sector in terms of size and third in terms of average growth the Tech sector was the second anchor during the pandemic, as many companies found it necessary to digitize, and regular consumer behavior shifting towards e-commerce. In contrast with the healthcare sector the Information Technology was already prospering in the Pre-Covid period as can be seen in figure 3.13 with 4.807% growth, the pandemic seemed to only stimulate it, pushing it to a 16.140% growth putting semi-conductor producing companies at the forefront

with **NVDA: Nvidia Corporation** reversing its trajectory from a -39.3% to a 56.1% average price growth and **MPWR: Monolithic Power Systems** rising up to third place.

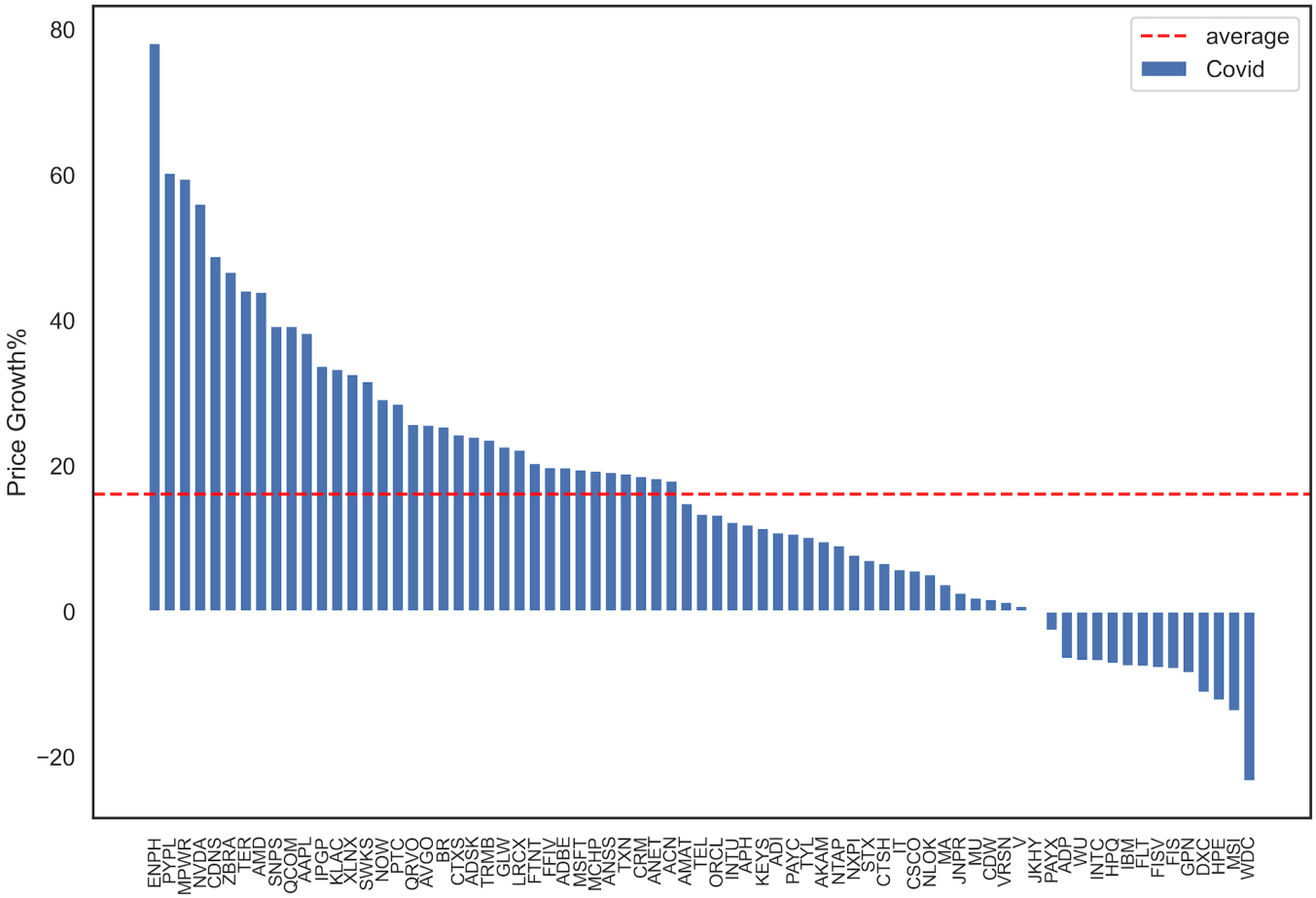

Figure 3.12: Information sector price growth during the Covid period March 2020 - March 2021; we have a shift towards positive growth with **ENPH: Enphase Energy, PYPL:PayPal, MPWR:Monolithic Power Systems Inc, NVDA: Nvidia Corporation** leading the growth with 78.1%,60.3%,59.5% and 56.5% average growth respectively

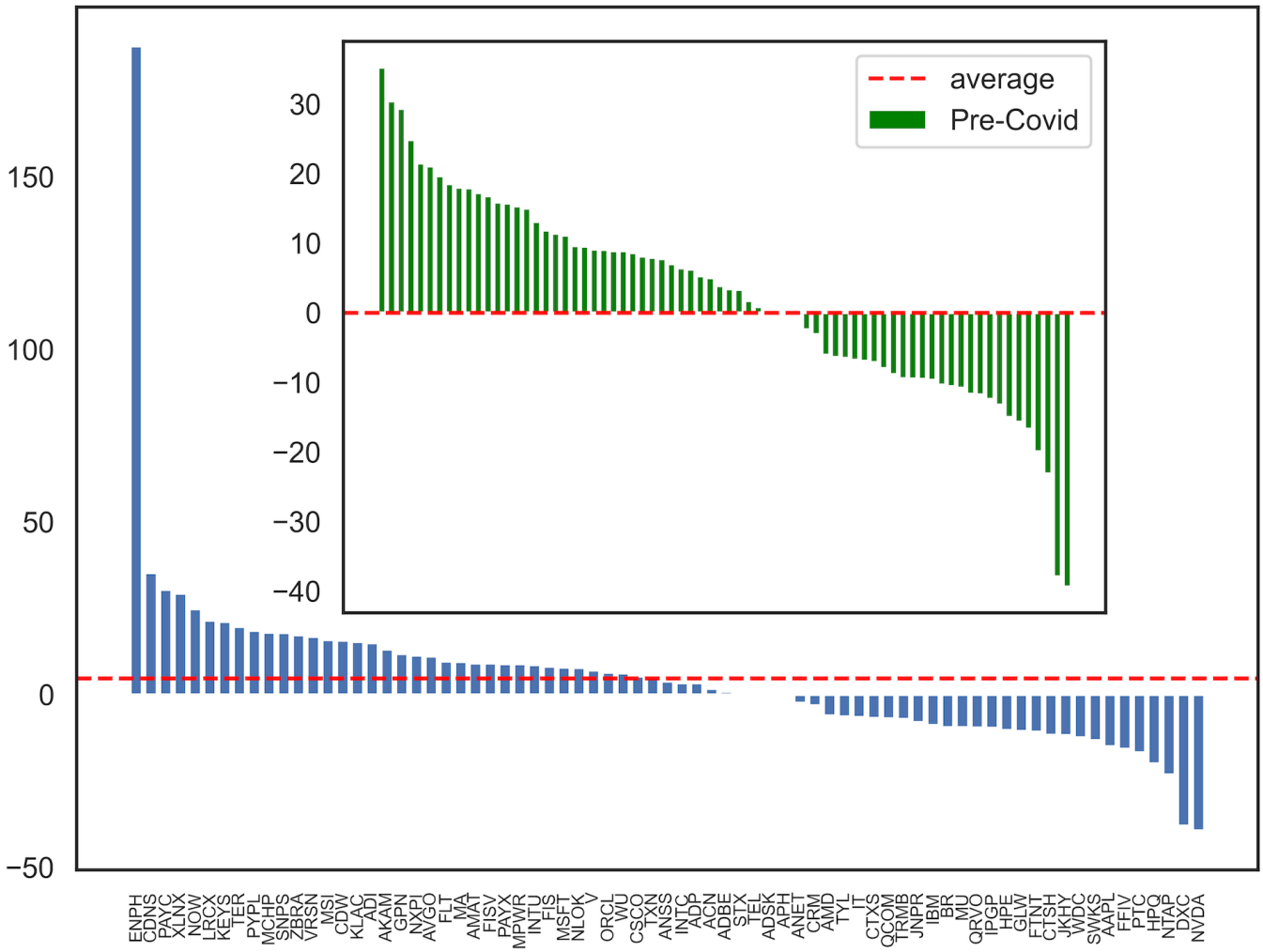

Figure 3.13: Information Technology sector price growth during the Pre-Covid period October 2018 - October 2021; we have a semi-balanced growth for all of the companies with an average of 4.807% except for **ENPH: Enphase Energy**'s shooting growth of 187.8%, in the green sub figure we excluded **ENPH** and plotted the rest of the companies with the intention of estimating the behavior of the rest of the sector, the recalculated average was 0.02%

Industrials

With approximately the same size as The Information Technology sector the industrial sector shows less average price growth ranking in the 6th place. The Industrial sector serves different sectors and contains a variety of sub-industries making it hard to predict. With an initial negative growth in the Pre-Covid period as can be seen in 3.15, the industrial sector moves to a slightly positive growth even with shipping companies **FDX: FedEX and UPS: United**

Parcel Service leading the growth whilst the Airline and Aerospace related companies took a hit during Covid as we can see them representing the bottom ten companies in figure 3.14.

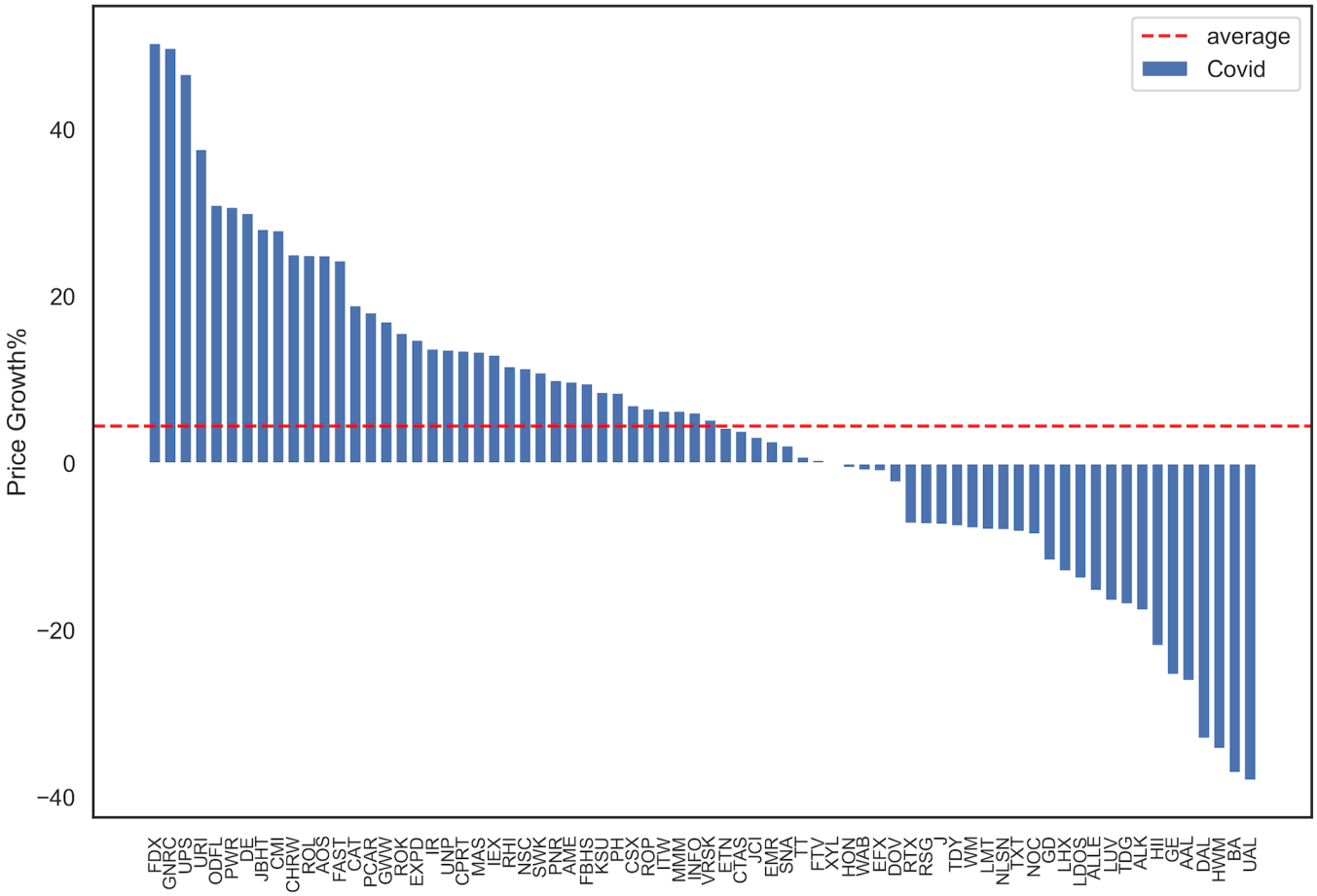

Figure 3.14: Industrials sector price growth during the Covid period March 2020 - March 2021; we have a shift towards positive growth with **FDX: FedEx, GNRC: Generac Holdings, Inc., UPS: United Parcel Service** leading the growth with 50.3%,49.7% and 46.6% respectively while the bottom 10 are airline and aerospace companies.

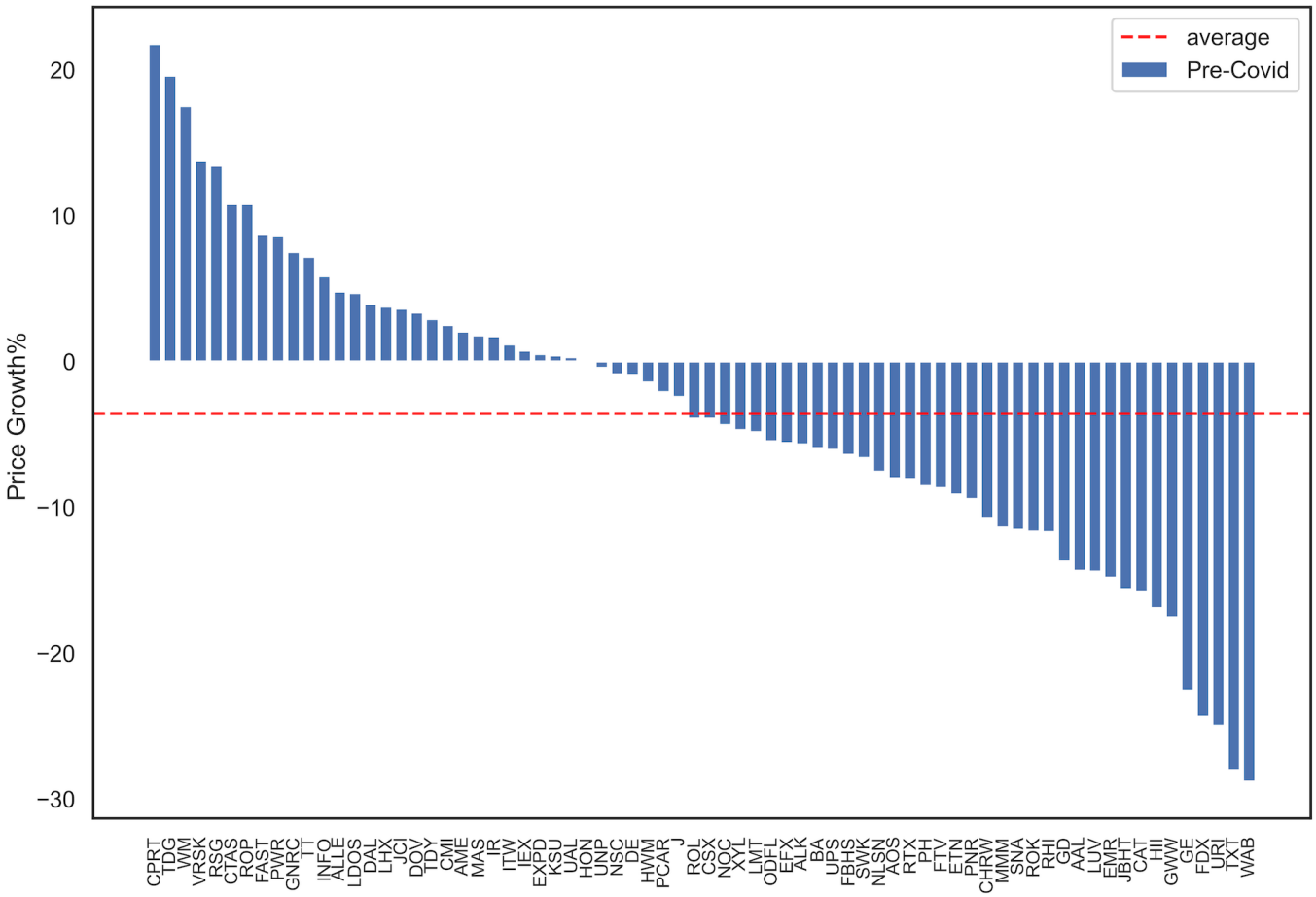

Figure 3.15: Industrials sector price growth during the Pre-Covid period October 2018 - October 2021; we have a negative growth with an average of -3.560% with **FDX** and **UPS** being on the negative side

Financials

The Financial sector is the only considerably Large sector in size that endured a loss in average price growth, yet owning to its already negative growth during the Pre-Covid period as in figure 3.17, the Covid period slightly aggravated this decrease to a -5.169% with the highest growth in attributed to investment banking companies as in figure 3.16.

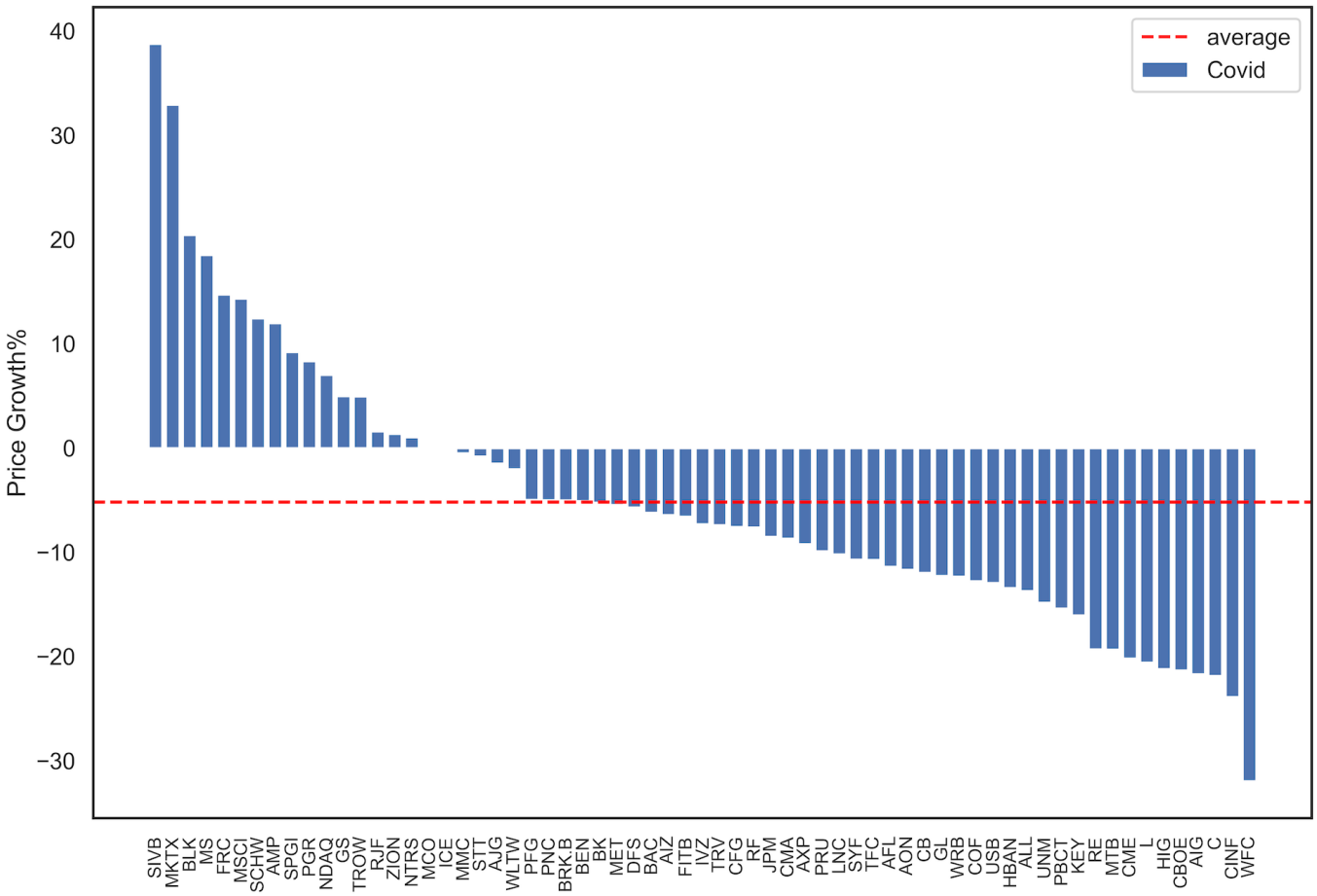

Figure 3.16: Financials sector price growth during Covid period March 2020-March 2021; we have a slight shift towards more negative growth with Investment banking companies **SIVB: SVB Financial Group, BLK: BlackRock, MS: Morgan Stanley and FRC:First Republic Bank** leading the growth with 38.7%,20.4%, 18.5% and 14.7 while insurance providing companies like **AIG:American International Group** dragging the growth

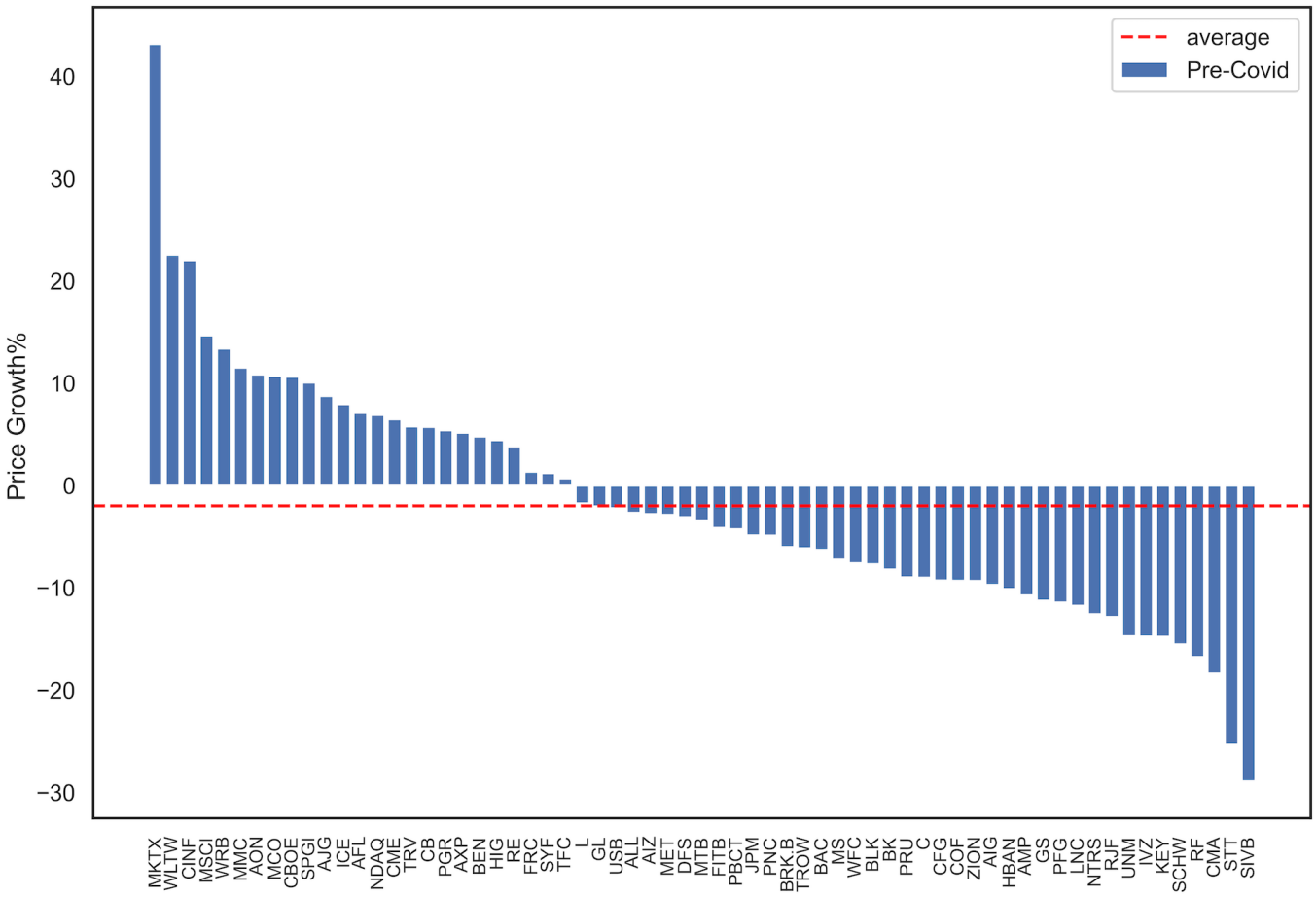

Figure 3.17: Industrials sector price growth during October 2018 - October 2021; we have an average negative growth of -1.971%

Consumer Discretionary

The Consumer Discretionary sector is one of the biggest sectors in terms of size and comprises of non-essential goods and services. During the Pre-Covid period we have a small balanced positive growth of around 1.9% as we can see in figure 3.19 during Pre-Covid period the positive growth was lead by Cosumer Durables& Apparel companies like **CMG:Chipotle Mexican Grill, SBUX:Starbucks** and **ETSY**. On the other hand, in figure 3.18 we see the growth is accelerated during Covid, with the same industry group leading the growth but with a shift in the sub-industry allowing the online leisure services to rise from the negative growth ranks to lead the growth with companies like **PENN: Penn National Gaming** and **AMZN:Amazon**. While **TSLA: Tesla Inc** contributed 166.6% growth to the average, its growth could not be attributed directly to pandemic, but to a series of technical reasons[7]. While

[7]More on Tesla's growth here

online leisure industries lead the growth as the consumers depend on it to alleviate negative psychological states from the pandemic isolation [10], other sub-industries suffered the major ones are the tour and cruise companies **CCL: Carnival Corp, NCLH:Norwegian Cruise Line Holdings Ltd, RCL: Royal Caribbean Cruises** and Fashion companies like **RL: Ralph Lauren, TPR:Tapestry , PVH: PVH Corp** and **TJX: TJX Companies**.

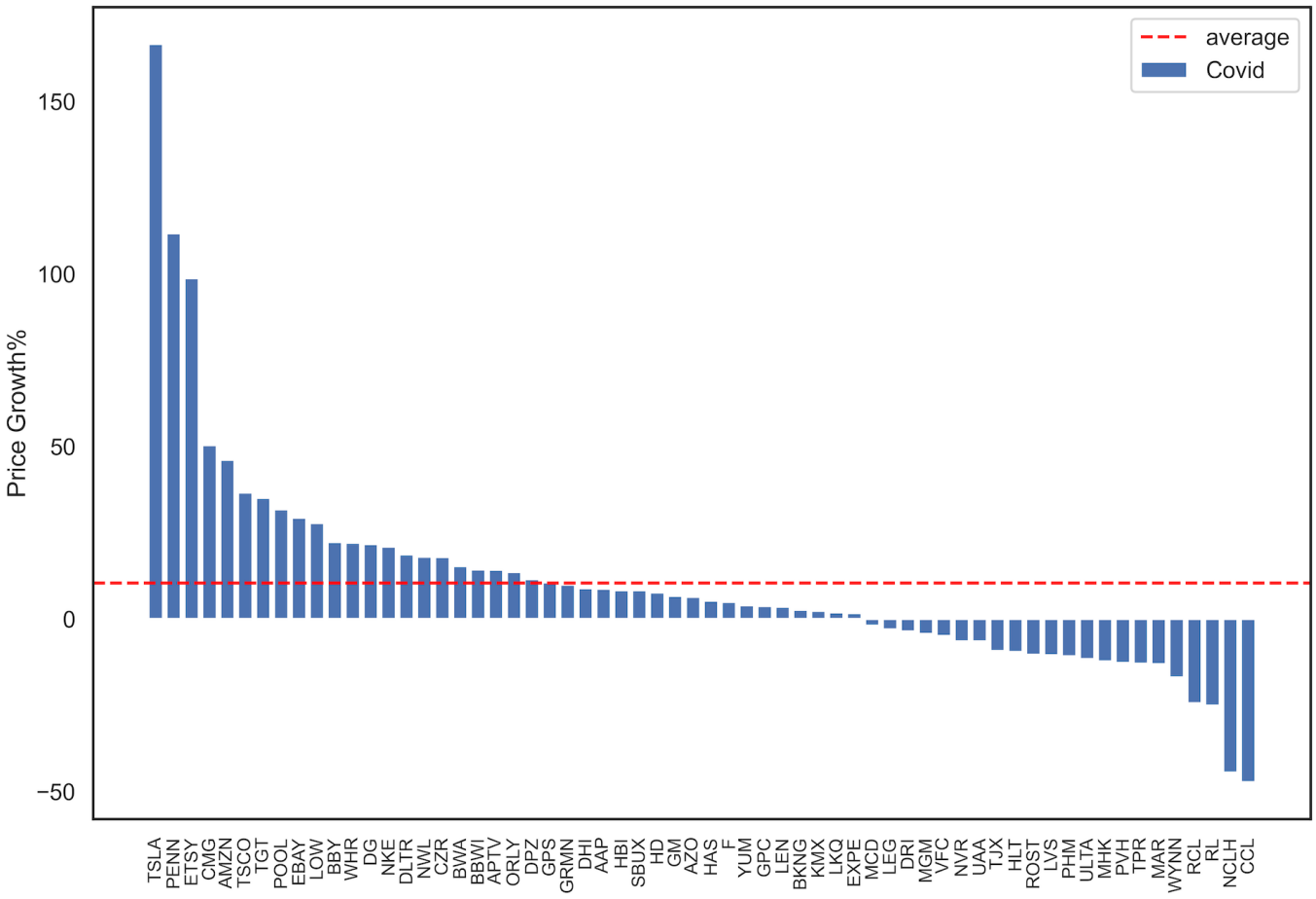

Figure 3.18: Consumer Discretionary sector price growth during the Covid period March 2020 - March 2021; we have a higher positive growth with an average of 10.492%. The companies leading the growth are: **TSLA: Tesla, PENN: Penn National Gaming, ETSY: Etsy, CMG: Chipotle Mexican Grill,** and **AMZN: Amazon**

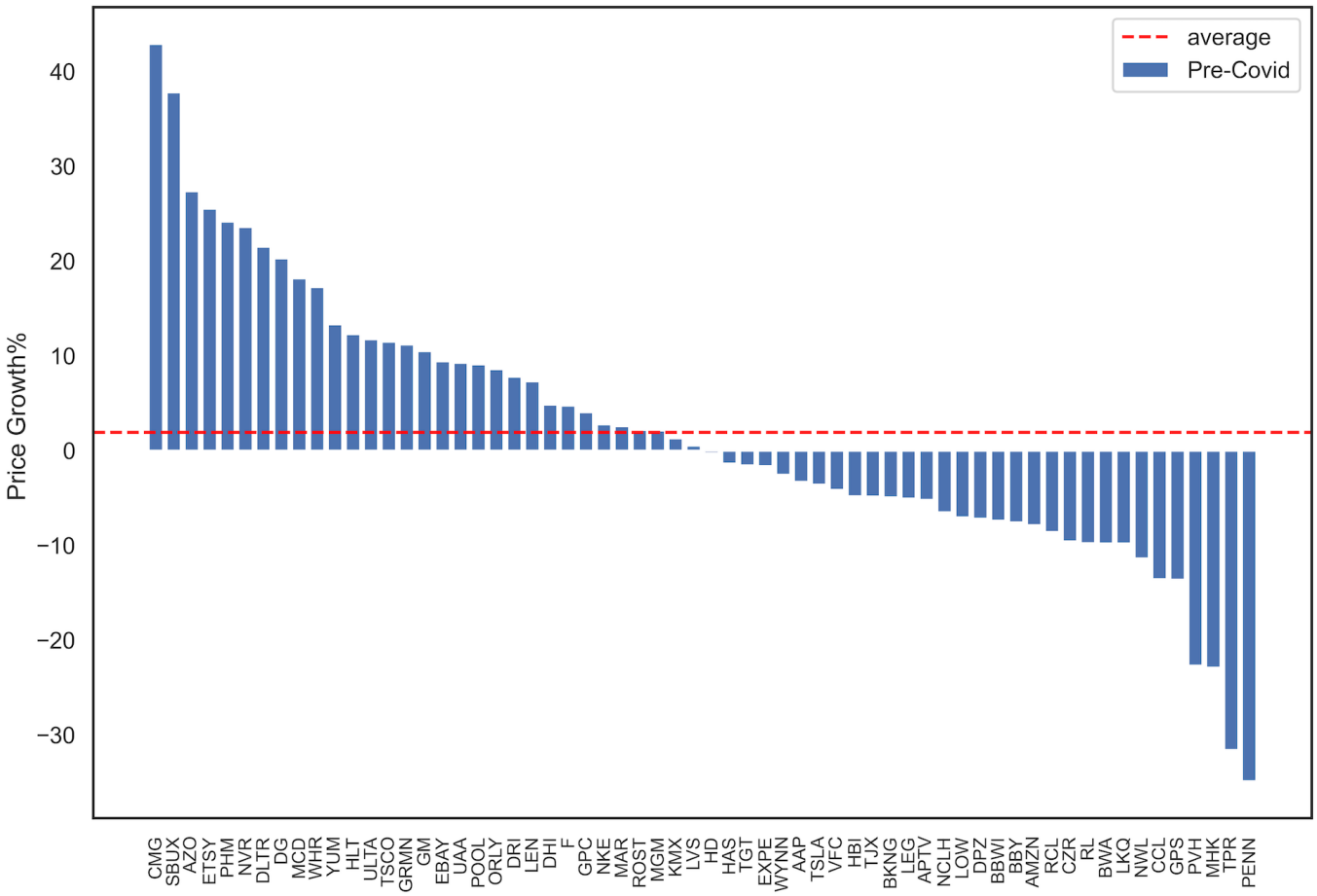

Figure 3.19: Consumer Discretionary sector price growth during the Pre-Covid period October 2018 - October 2021; we have an average growth of 1.963%, notice that **PENN: Penn National Gaming,** and **AMZN: Amazon** are on the negative growth side.

Consumer Staples

The consumer Staples sector small as 6% represents the essential goods for consumers. During the Covid period we see that it sustained a positive average growth, an explanation could be offered by [10] which argues that the consumer spending levels on the necessary products are elevated by fear of the pandemic. In 3.22 we see that we have a positive growth in both periods with a slightly less growth during Covid period.

Real Estate

The Real Estate sector took the biggest hit during the pandemic, with an average difference of -27.208% the dynamic of the price growth dynamic changed almost

entirely to be negative. In 3.25 we see a complete shift from a prosperous sector with almost completely positive growth to the complete opposite.

Utilities

As the pandemic caused a decline in the commercial and industrial demand for power the utilities sector suffered a great loss, with almost all of its companies performing similarly in terms of negative average growth.

Materials

The materials sector had the highest difference in average growth despite it being one of the smallest sectors. One of the major driving forces of this growth was **FCX: Freeport-McMoRan** a mining company whose own growth was driven by the increasing gold prices, used to store wealth during the pandemic.

Energy

The Energy sector adopted by the **GICS** has an outdated definition, mainly concerned with Coal, Oil&Gas production and services, leaving out renewable energy production and services to fall under other sectors. The Energy sector - with the restricted scope- was not a major player during the pandemic, in 3.34 we see that minimal change happened to the sector which agree with our findings in table 3.8.

Communication Services

The communication service sector small as it is with 4% has a difference average growth of 10.987%, with Video Game companies leading the growth and reversing their negative growth during the Pre-Covid like **TTWO: Take-Two Interactive and ATVI: Activision Blizzard**

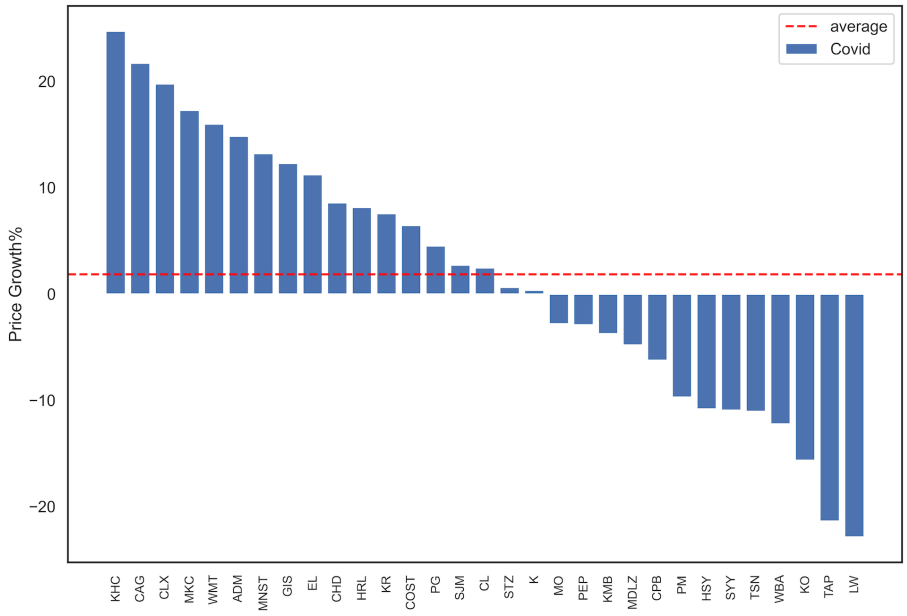

Figure 3.20: Consumer Stable sector price growth during the Covid period March 2020 - March 2021; we have a slightly lower positive growth with an average of 1.856%

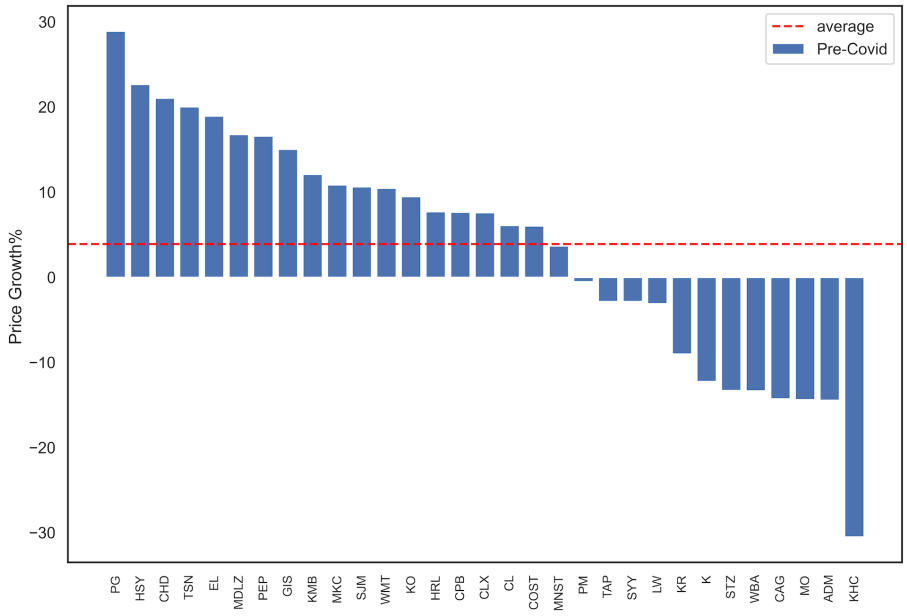

Figure 3.21: Consumer Stable services sector price growth during the Pre-Covid period October 2018 - October 2021; we have an average growth of 3.917%

Figure 3.22: Consumer Staples sector average price growth per company

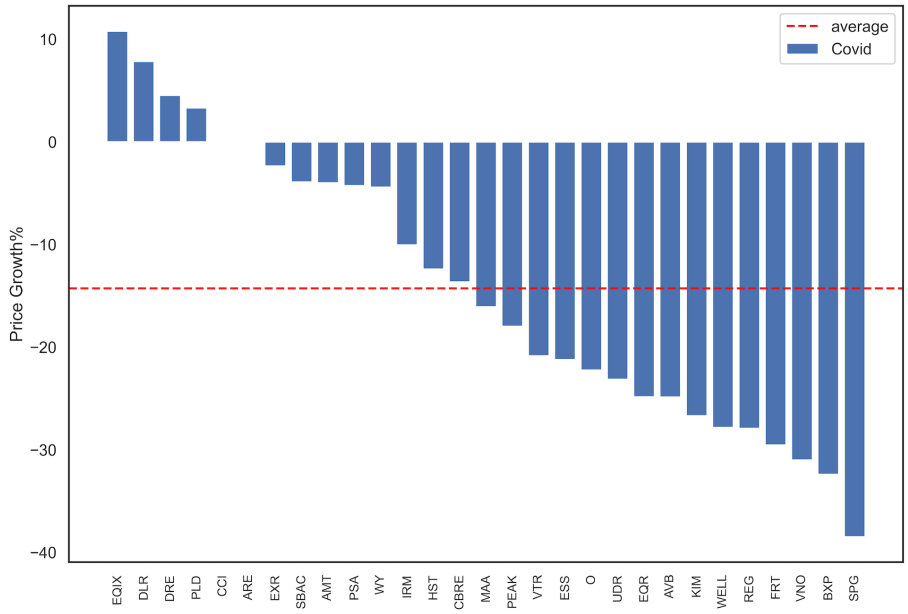

Figure 3.23: Real Estate sector price growth during the Covid period March 2020 - March 2021; we have an average growth of -14.263%, the highest growth belongs to **EQIX: Equinix ,DLR: Digital Realty Trust** with growth of 10.7% and 7.8%.

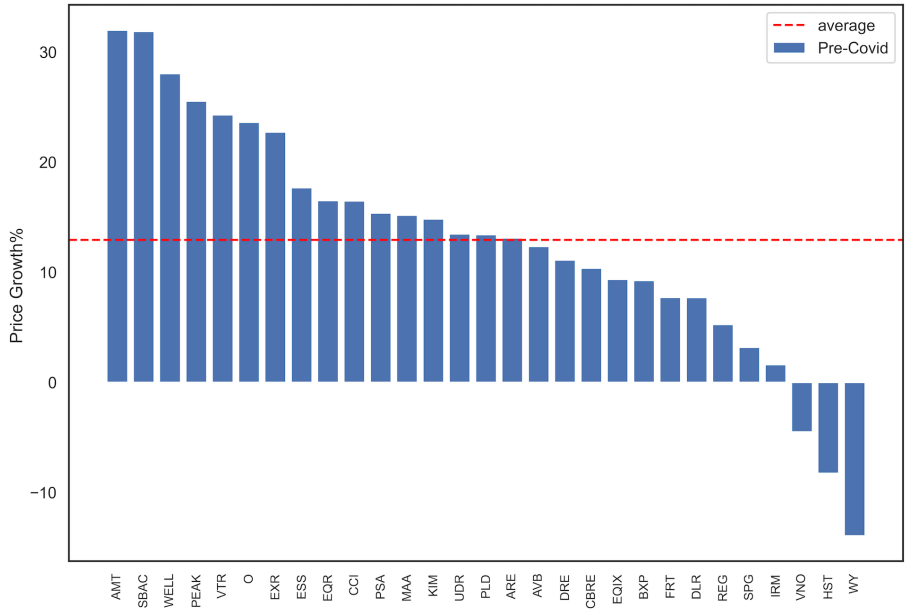

Figure 3.24: Real Estate sector price growth during Pre-Covid period October 2018 - October 2021; we have one of the highest average growth of 12.945%

Figure 3.25: Real Estate sector average price growth per company

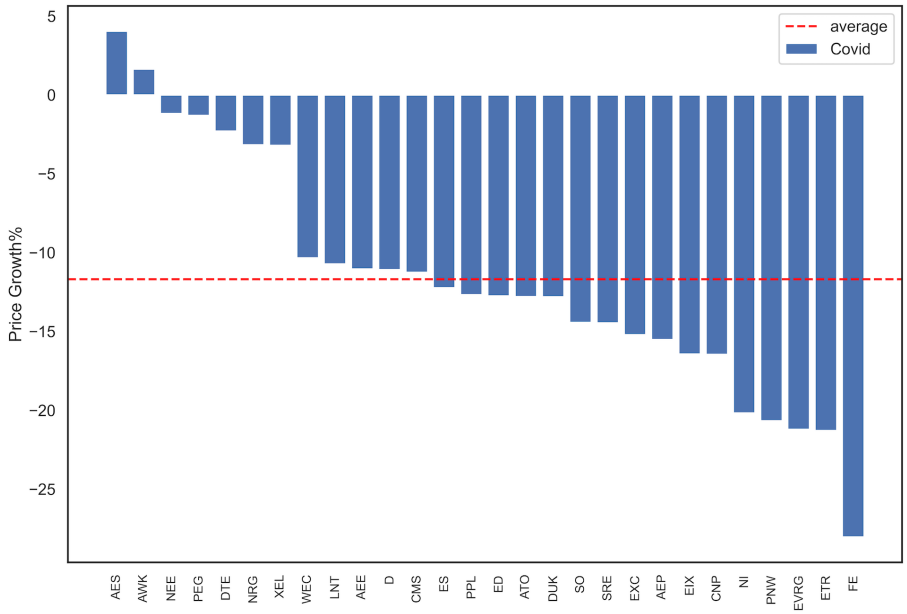

Figure 3.26: Utilities sector price growth during the Covid period March 2020 - March 2021; we see almost a complete shift towards negative growth with an average of -11.678% except for **AES:AES Corporation** with 4% growth and **AWK: American Water Works** with 1.6% growth

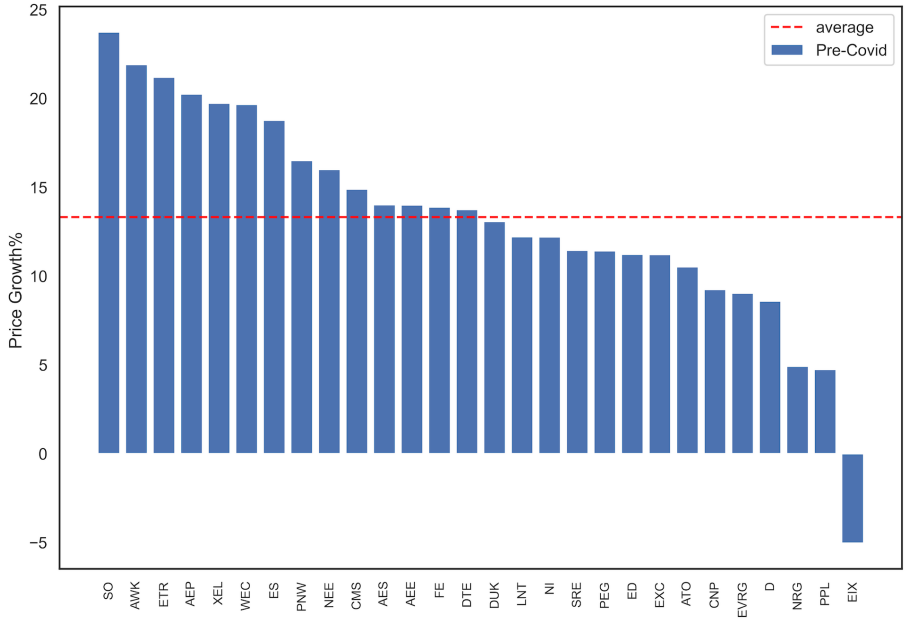

Figure 3.27: Utilities sector price growth during the Pre-Covid period October 2018 - October 2021; we see almost positive growth for all companies with an average of 13.335% except for **EIX: Edison International** with -5%

Figure 3.28: Utilities sector average price growth per company

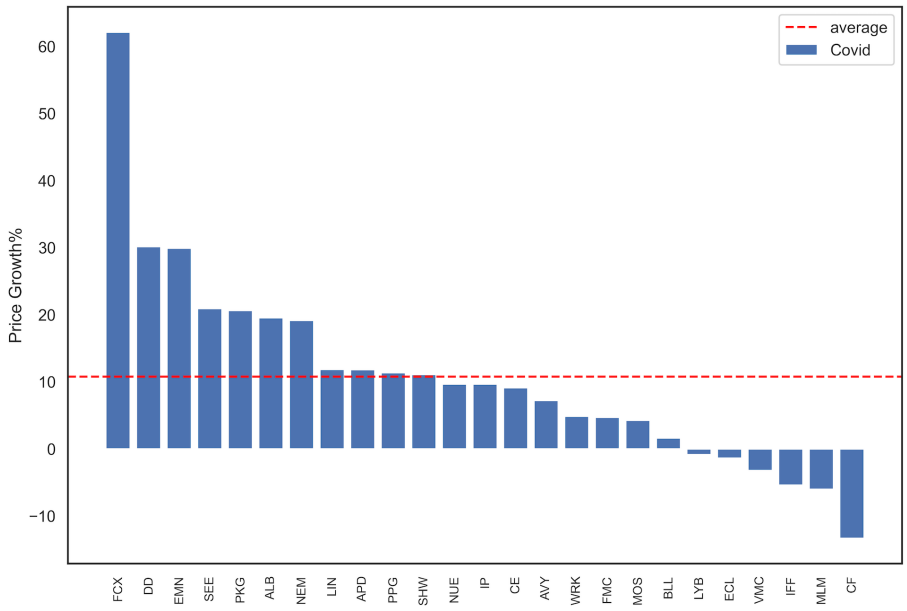

Figure 3.29: Materials sector price growth during the Covid period March 2020 - March 2021; we have a higher positive growth with an average of 10.793%

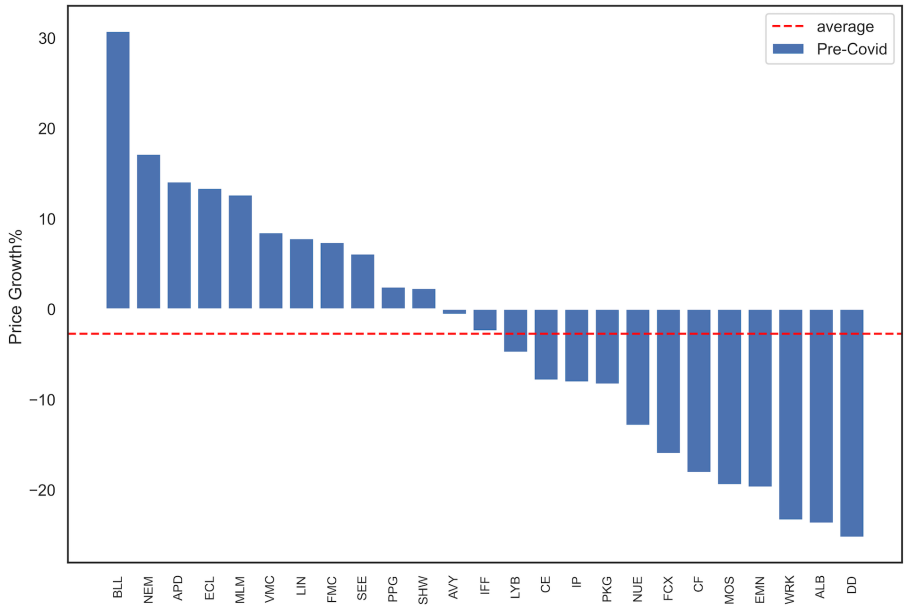

Figure 3.30: Materials sector price growth during the Pre-Covid period October 2018 - October 2021; we have an average growth of -2.713%

Figure 3.31: Materials sector average price growth per company

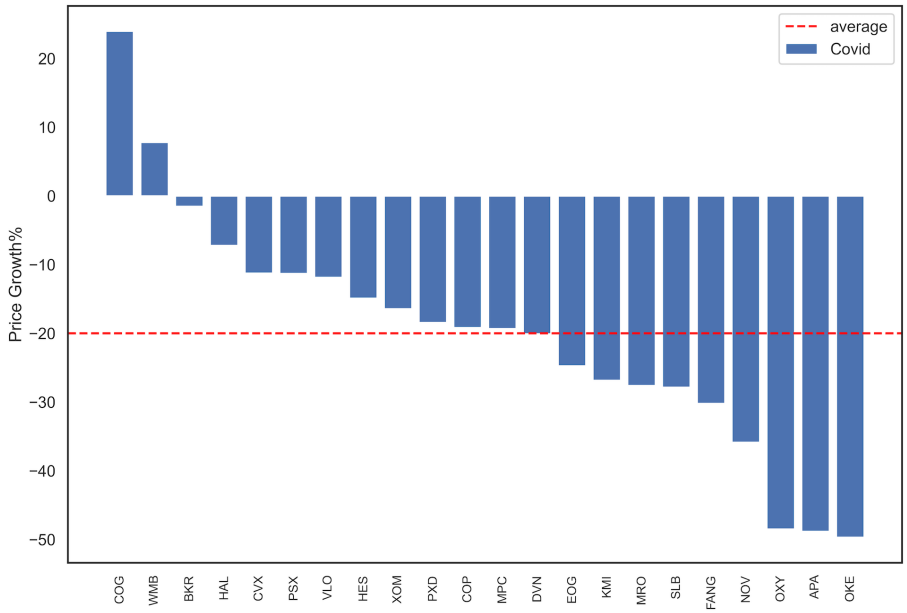

Figure 3.32: Energy sector price growth during the Covid period March 2020-March 2021; we have a slightly negative growth with an average of -19.987%. We also have no major players making noticeable difference

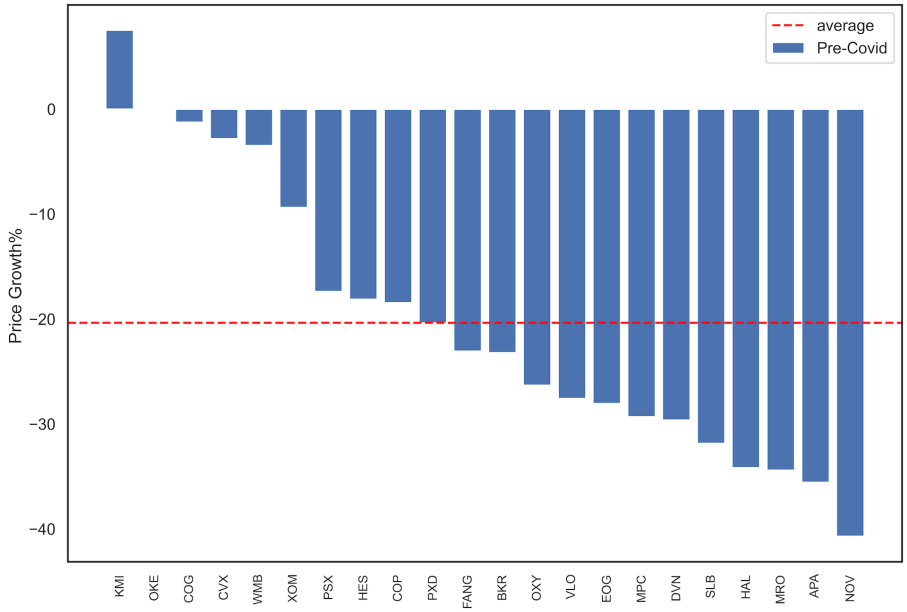

Figure 3.33: Energy sector price growth during the Pre-Covid period October 2018 - October 2021; we have the highest average negative growth of -20.341%

Figure 3.34: Energy sector average price growth per company

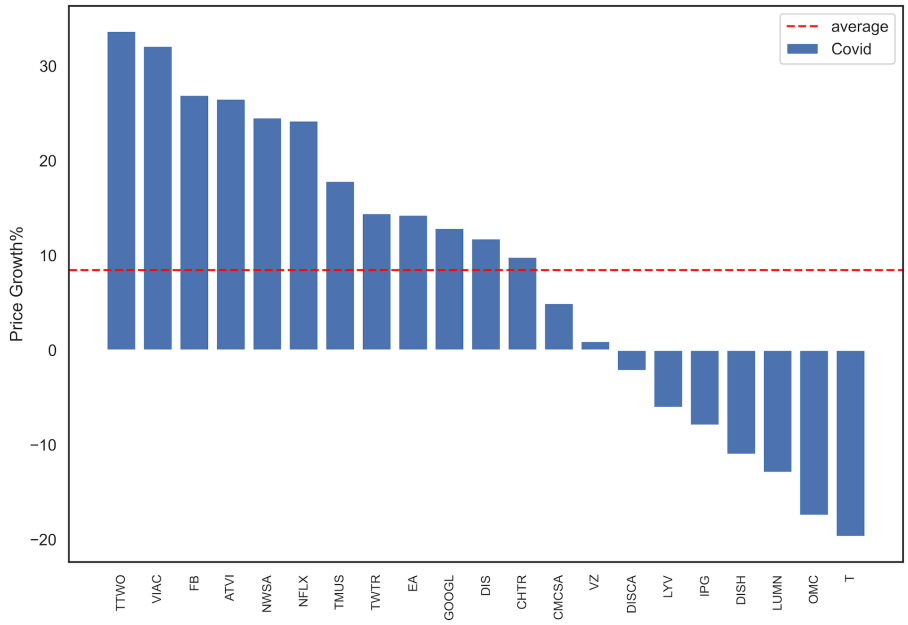

Figure 3.35: Communication services sector price growth during the Covid period March 2020 - March 2021; we have a shift towards positive growth with an average of 8.46%. We have **TTWO: Take-Two Interactive, VIAC: ViacomCBS, FB: FaceBook and ATVI: Activision Blizzard** leading this growth

1

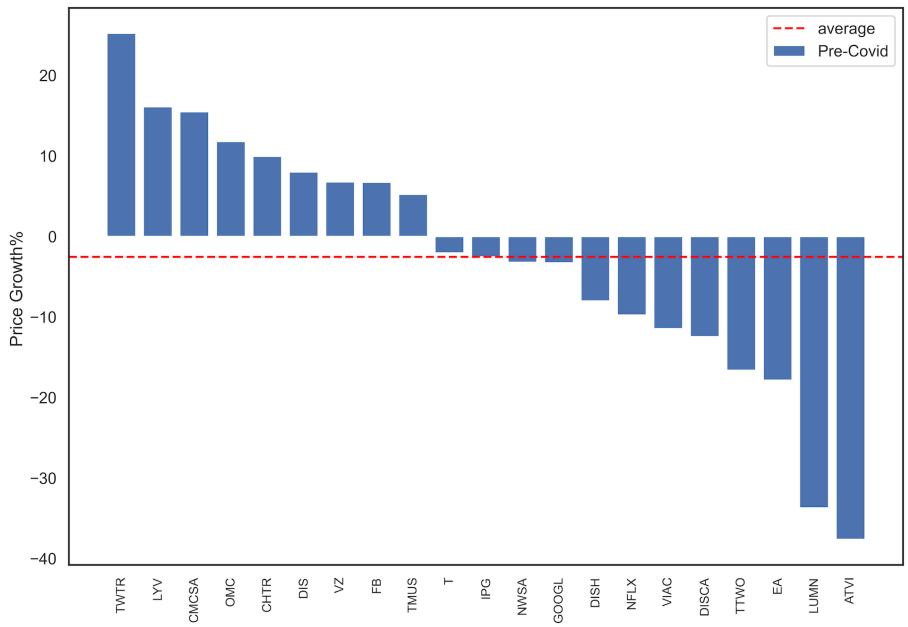

Figure 3.36: Communication services sector price growth during the Pre-Covid period October 2018 - October 2021; we have an average negative growth of -2.526%, note the initial position of **ATVI: Activision Blizzard and TTWO:Take-Two Interactive**

Figure 3.37: Communication sector average price growth per company

CHAPTER 4

Analysis of IBOV (BRZIL)

In 2020 Brazil had a gross domestic product (GDP) of 1.445 Trillion USD, accounting for 1.7% of the world's GDP, making it the largest economy in the south America [3]. To represent it, we select the IBOV stock index, constructed in 1964 it is an index of the most liquid stocks traded on the B3 Stock Exchange[1]. As of March 2022, the IBOV index accounts for around 80% of the number of trades and the financial volume of the B3 capital market [2]. In this chapter, we construct a minimal spanning tree using the IBOV companies selected according to section 2.1, analyze it using our network measures and investigate each sector performance and role.

4.1 Minimal Spanning Tree Construction

The number of companies used in this analysis is 77, a small number compared to the 490 used in our USA analysis. This small number gives us a complete different economic landscape as shown in figure 4.9 with the health care sector representing only 8% and the Information Technology sector representing only 3% whilst the utilities and materials sectors representing 14% and 12% respectively. This puts Brazil at a vulnerable position in front of Covid with a small Health care sector to absorb the pandemic and a smaller Technology sector to help with the ramifications.

Correlation Distribution

We proceed as we did in section 3.1 by following our methodology section, we construct the correlation matrix for **IBOV**. We find in agreement with our USA analysis that we have a shift towards a higher correlation coefficient with higher variance during the Covid period as can be seen in figure 4.1, yet the negative tale is less broader compared to the Pre-Covid period as can be seen in table 4.1.

[1] The companies are chosen according to the criteria provided **here**

	Min	Max	Av	Var	SK	Kur
Oct 2018-2019	-0.163	0.922	0.250	0.018	0.265	0.526
March 2020-2021	-0.104	0.976	0.531	0.020	-0.896	1.427

Table 4.1: Minimum, Maximum , Average, Variance, adjusted Fisher-Pearson coefficient of skewness and excess kurtosis of the correlation matrix for IBOV, Pre-Covid period (top) and Covid period (down).

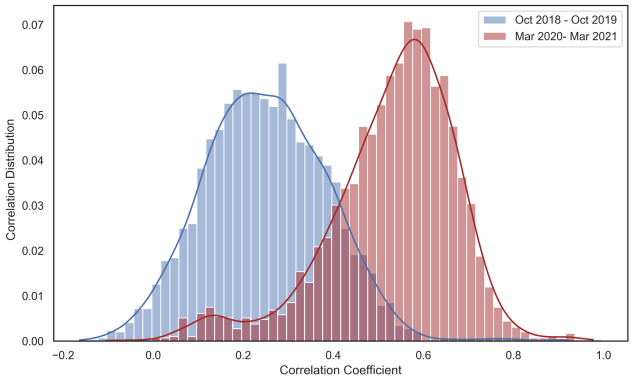

Figure 4.1: Density distribution of the correlation coefficients of the Pre-Covid period (Blue) and the Covid period (Red) the bin width used is 0.02 for IBOV

Construction of the Minimum Spaning Tree

Taking advantage of our smaller MSTs, we plot them in their hierarchical nature without the risk of overlapping, using the central nodes defined above we can see that in figure 4.2[2]. During the Pre-Covid period we have the Consumer Discretionary, Utilities and Financial sector occupying central roles in the tree, while the Health Care sector and Information Technology are occupying the least central roles and taking leaf positions. On the other hand, during the Covid period in figure 4.3 we have the Industrials and Health Care sectors taking a more central role, more so we can see the sectors forming disjoint communities contrary to what was found in the S&P500 MSTs.

[2]The symbols have been shortened to fit the graph

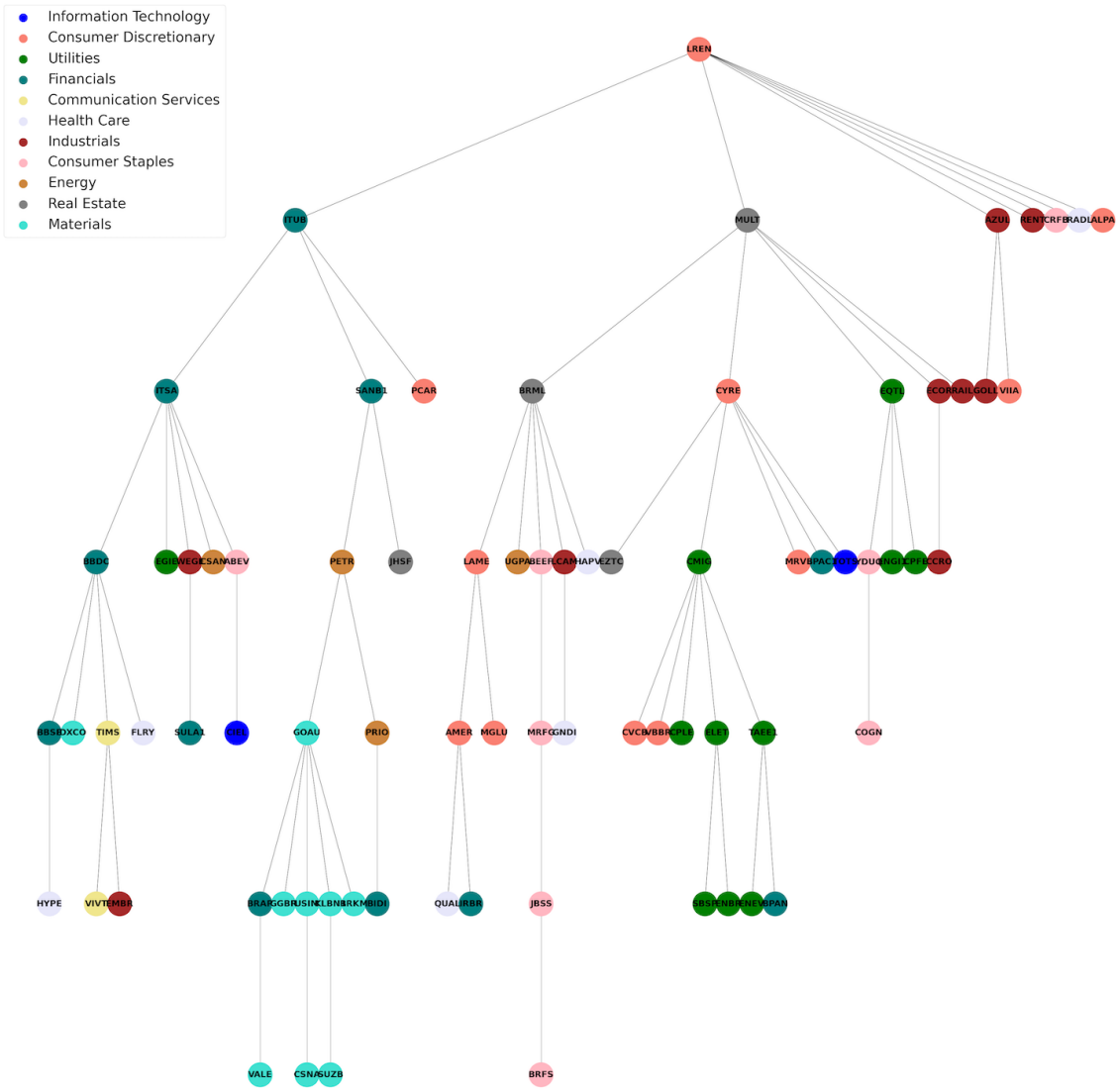

Figure 4.2: Full Network of IBOV during Pre-Covid period October 2018 - October 2019. The Tree is plotted in a hierarchical manner where colors of the nodes indicate their sector and the labels represent the trading symbol.

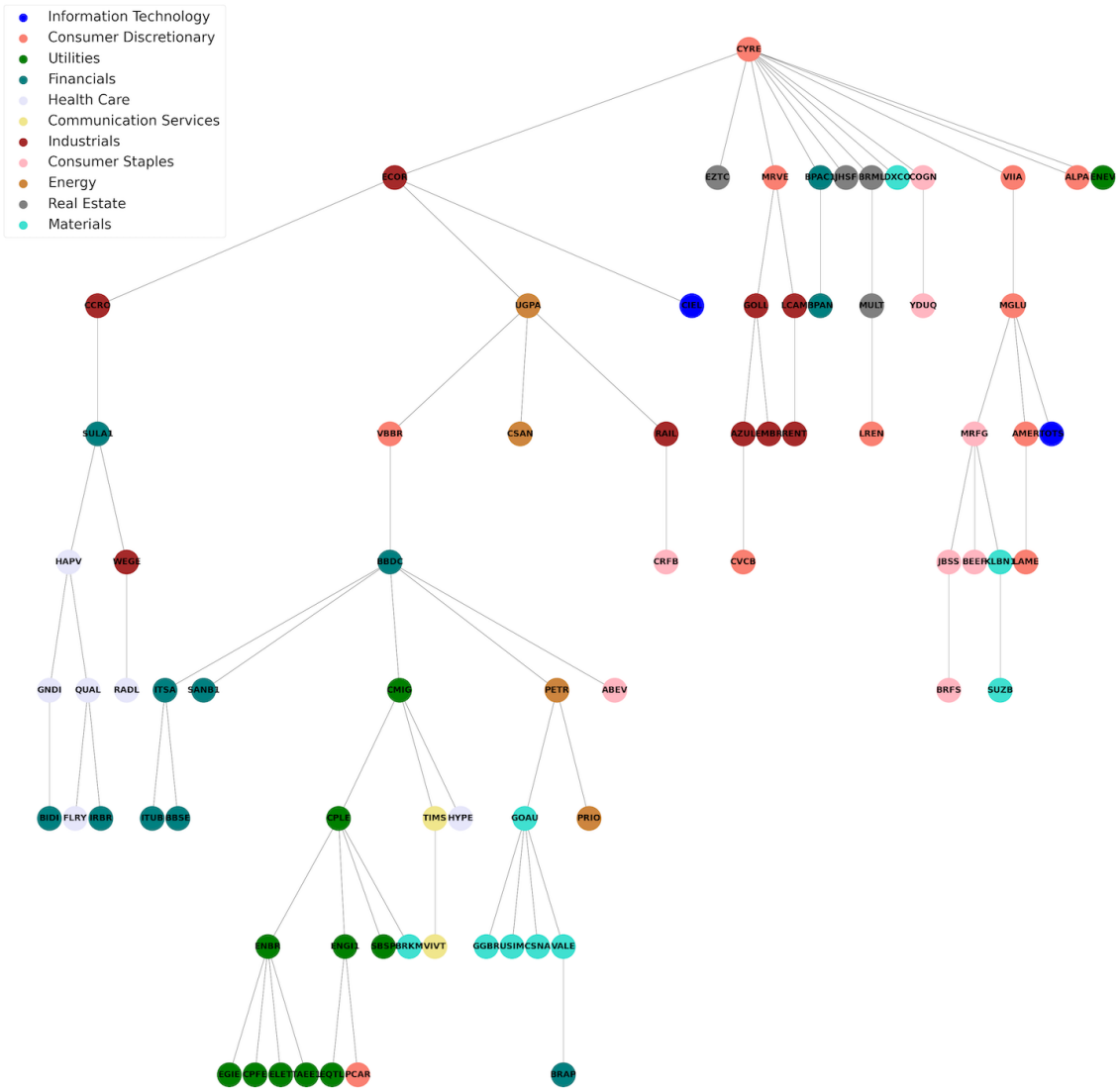

Figure 4.3: Full Network of IBOV during the Covid period March 2020 - March 2021. The Tree is plotted in a hierarchical manner where colors of the nodes indicate their sector and the labels represent the trading symbol.

4.2 Measures

Degree Distribution

The degree distribution follows a similar pattern to our previous analysis, with the prominent central node during the Covid period distinguishing itself with a degree of 11 as we can see in figure 4.4.

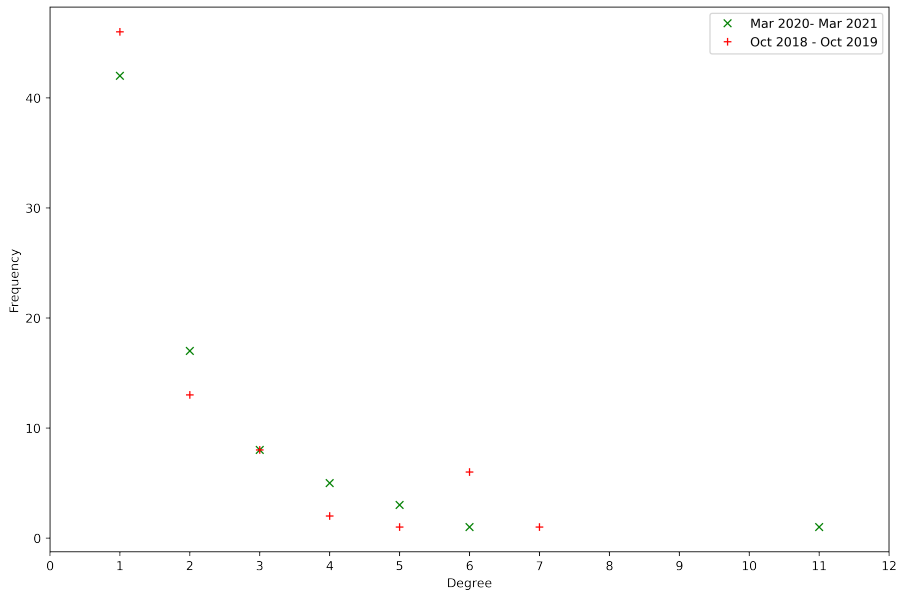

Figure 4.4: Degree Distribution of the nodes in the IBOV MSTs during the Pre-Covid and Covid preiod

	Node	Degree	Sector
Oct 2018 - 2019	**LREN3.SA**	7	Consumer Discretionary
	CYRE3.SA	6	Consumer Discretionary
	CMIG4.SA	6	Utilities
	MULT3.SA	6	Real Estate
	BRML3.SA	6	Real Estate
	GOAU4.SA	6	Materials
	ITSA4.SA	6	Financials
	BBDC3.SA	5	Financials
Mar 2020 - 2021	**CYRE3.SA**	11	Consumer Discretionary
	BBDC3.SA	6	Financials
	GOAU4.SA	5	Materials
	CPLE6.SA	5	Utilities
	ENBR3.SA	5	Utilities

Table 4.2: Nodes with the largest degree and their corresponding industry sector. We have listed nodes with degrees above 4 during the Pre-Covid period (top) and the Covid period (down)

Betweenness Centrality

The betweeness centrality in figure 4.5 shows that both periods have very similar bweetness centrality, with the Covid period having a slightly higher betweeness centrality from node number 10 to 20 before dropping to zero at node number 35.

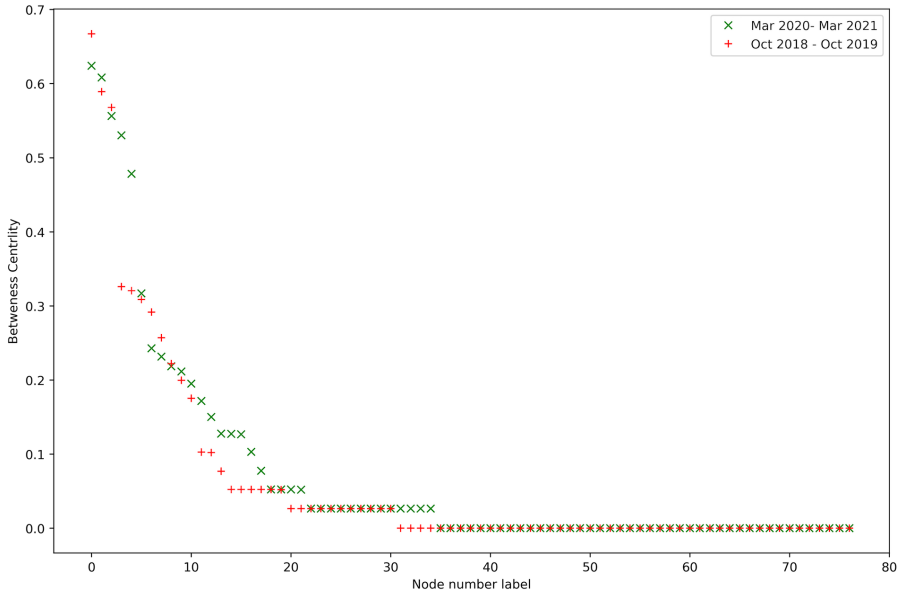

Figure 4.5: Betweness Centrality of the nodes in the IBOV MSTs during the Pre-Covid and Covid period, the nodes are numbered from 1 to 77

	Node	Betweenness Centrality	Sector
Oct 2018 - 2019	**MULT3.SA**	0.67	Real Estate
	LREN3.SA	0.59	Consumer Discretionary
	ITUB4.SA	0.57	Financials
	ITSA4.SA	0.33	Financials
	CYRE3.SA	0.32	Consumer Discretionary
Mar 2020 - 2021	**ECOR3.SA**	0.62	Industrials
	CYRE3.SA	0.61	Consumer Discretionary
	BBDC3.SA	0.56	Financials
	UGPA3.SA	0.53	Energy
	VBBR3.SA	0.48	Consumer Discretionary

Table 4.3: Nodes with the largest Betweenness Centrality and their corresponding industry sector during the Pre-Covid period (top) and the Covid period (down)

Strength Distribution

Since we have higher correlation distribution during the Covid period we expect to have a high strength distribution as we found in figure 3.6, in agreement we find that the Covid period has a higher strength in figure 4.6 with the central node **CYRE3.SA** taking the lead.

	Node	Strength	Sector
Oct 2018 - 2019	**LREN3.SA**	3.67	Consumer Discretionary
	ITSA4.SA	3.52	Financials
	MULT3.SA	3.49	Real Estate
	GOAU4.SA	3.34	Materials
	CMIG4.SA	3.2	Utilities
Mar 2020 - 2021	**CYRE3.SA**	8.38	Consumer Discretionary
	BBDC3.SA	4.79	Financials
	GOAU4.SA	4.16	Materials
	CPLE6.SA	3.88	Utilities
	ENBR3.SA	3.76	Utilities

Table 4.4: Nodes with largest Strength and their corresponding industry sector during the Pre-Covid period (top) and the Covid period (down)

Average Distance

Affirming our result in figure 3.7 we see again in figure 4.7 the same decreasing pattern of average distance during the Covid period showing a tightly connected MST.

	Node	Av. Distance	Sector
Oct 2018 - 2019	**LREN3.SA**	3.04	Consumer Discretionary
	MULT3.SA	3.06	Real Estate
	ITUB4.SA	3.25	Financials
	ITSA4.SA	3.49	Financials
	BRML3.SA	3.5	Real Estate
Mar 2020 - 2021	**ECOR3.SA**	2.64	Industrials
	CYRE3.SA	2.76	Consumer Discretionary
	UGPA3.SA	2.76	Energy
	VBBR3.SA	2.91	Consumer Discretionary
	CCRO3.SA	2.93	Industrials

Table 4.5: Nodes with smallest Average Distance and their corresponding industry sector during the Pre-Covid period (top) and the Covid period (down)

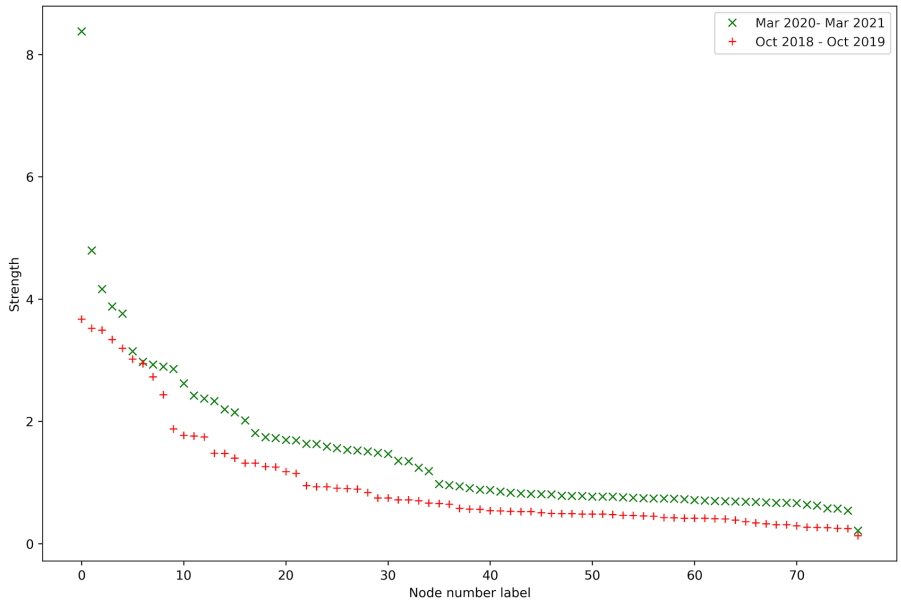

Figure 4.6: Strength Distribution of the nodes in the IBOV MSTs during Pre-Covid and Covid period, the nodes are numbered from 1 to 77

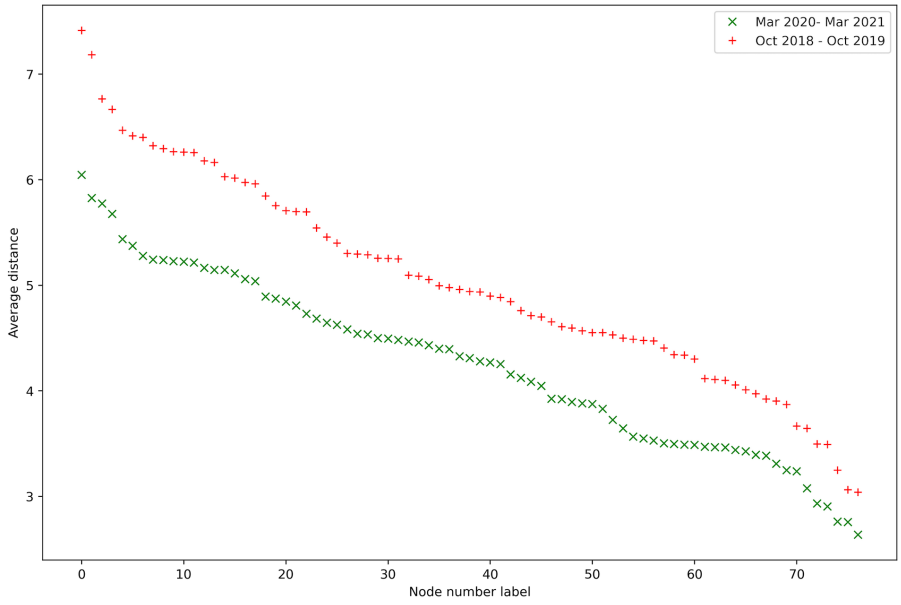

Figure 4.7: Average Distance Distribution of the nodes in the IBOV MSTs during the Pre-Covid and Covid period, the nodes are numbered from 1 to 77

Closeness Centrality

In figure 4.8 we have our expected pattern with the Covid period having higher Closeness centrality.

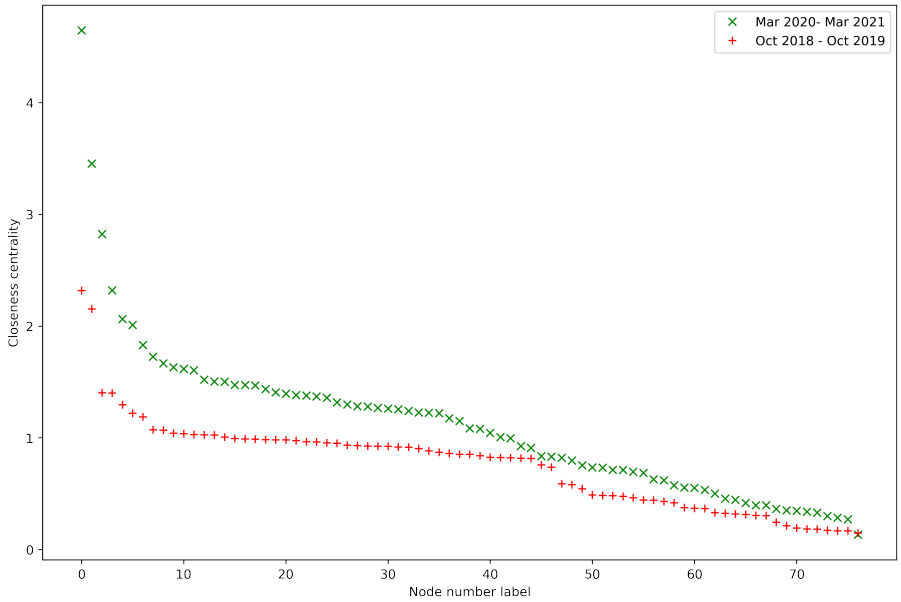

Figure 4.8: Closeness Centrality Distribution of the nodes in the IBOV MSTs during Pre-Covid and Covid period, the nodes are numbered from 1 to 77

	Node	Closeness Centrality	Sector
Oct 2018 - 2019	**LREN3.SA**	0.147	Consumer Discretionary
	BRML3.SA	0.168	Real Estate
	CYRE3.SA	0.169	Consumer Discretionary
	CMIG4.SA	0.172	Utilities
	GOAU4.SA	0.183	Materials
Mar 2020 - 2021	**CYRE3.SA**	0.133	Consumer Discretionary
	BBDC3.SA	0.271	Financials
	ENBR3.SA	0.285	Utilities
	CPLE6.SA	0.299	Utilities
	MGLU3.SA	0.331	Consumer Discretionary

Table 4.6: Nodes with smallest Closeness Centrality and their corresponding industry sector during the Pre-Covid period (top) and the Covid period (down)

4.3 Finding the Central node

Next we move on to the structure of our MSTs. We start by finding the central nodes, during the Covid period we had one clear central node with the a degree

of 11 belonging to the Consumer Discretionary sector **CYRE3.SA: CYRELA REALT**. Similarly we find that for the Pre-Covid period we also have a well defined center belonging to the same sector **LREN3.SA: LOJAS RENNER** with a degree of 7. In agreement with our USA analysis we find the we have a higher occupational layer in the Covid period as seen in table 4.7.

	Mean Occupation layer	Normalized Tree Length
Oct 2018 - 2019	$L_m = 3.454$	$L_n = 0.983$
Mar 2020 - 2021	$L_m = 4.246$	$L_n = 0.682$

Table 4.7: Mean Occupation Layer and Normalized Tree Length for IBOV during the Pre-Covid period (top) and the Covid period (down).

4.4 Analysis per sector

Before analyzing our sectors we begin by mapping the size of each one, as we can see in figure 4.9 we have an entirely different economic landscape spread over only 77 companies, with some sectors comprising of as little as two companies. We follow section 3.4 and summarize our main measures per sector, since the Brazilian Pre-Covid MST in figure 4.3 does not show a similar sector dispersion to its USA analgous in figure 3.3, we have no need to have a closer look into intra-sector inequality. In table 4.10 we see the sectors have decreasing average distance in agreement with figure 4.7 except for the Materials sector with a 55.873% increase. While we find strong variations in the average degree along all sectors, we have the highest increase of 83.3% taken by the Health Care sector moving it from leaf positions with zero betweeness to a betweeness of around 0.03. We can also notice a noticeable increase in the betweeness of the Industrials and Energy sectors.

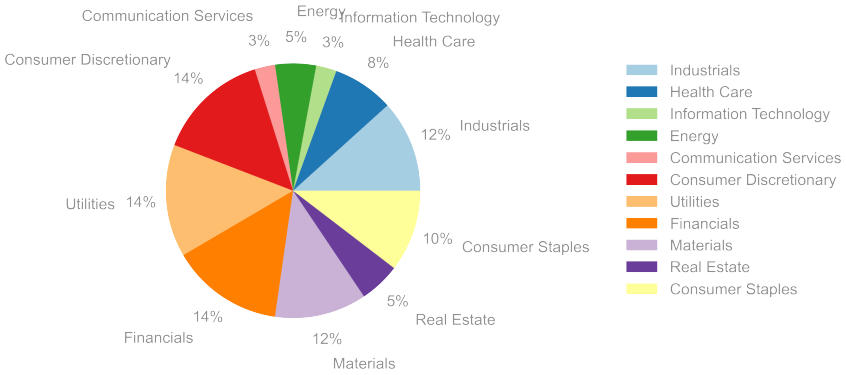

Figure 4.9: Brazil's relative sector size. The right color palette indicates the color of each sector. The size is constant throughout the two periods

Sector	Count	Av. Distance	Av. Degree	Av. Betweeness

Consumer Discretionary	11	3.246	2.636	0.158
Consumer Staples	8	3.837	1.625	0.022
Industrials	9	2.487	2.222	0.119
Financials	11	3.705	1.909	0.074
Real Estate	4	1.303	1.500	0.019
Materials	9	4.475	1.666	0.020
Utilities	11	2.231	2.090	0.063
Information Technology	2	3.737	1.000	0.000
Energy	4	2.103	2.250	0.174
Health Care	6	3.167	1.833	0.034
Communication Services	2	0.783	1.500	0.013

Table 4.8: Brazil Summary of measures per sector during the Covid period March 2020 - March 2021

Sector	Count	Av. Distance	Av. Degree	Av. Betweeness
Consumer Discretionary	11	3.454	2.454	0.098
Consumer Staples	8	4.497	1.625	0.025
Industrials	9	3.787	1.666	0.017
Financials	11	4.883	2.363	0.125
Real Estate	4	2.923	3.250	0.235
Materials	9	2.867	1.777	0.028
Utilities	11	3.402	2.090	0.041
Technology	2	6.583	1.000	0.000
Energy	4	4.097	1.750	0.070
Health Care	6	5.451	1.000	0.000
Communication Services	2	1.039	2.000	0.026

Table 4.9: Brazil Summary of measures per sector during the Pre-Covid period October 2018 - October 2019

Sector	% Av. Distance in	% Av. Degree in	% Av. Betweeness
Consumer Discretionary	-13.007	11.538	60.996
Consumer Staples	-14.621	0.000	-13.345
Industrials	-32.907	35.714	676.203
Financials	-24.186	-22.222	-41.276
Real Estate	-55.431	-57.143	-91.984
Materials	55.873	-6.250	-28.512
Utilities	-41.886	4.348	66.503
Information Technology	-43.227	0.000	Undefined
Energy	-48.691	28.571	147.525
Health Care	-40.371	83.333	∞
Communication Services	-24.041	-25.000	-49.664

Table 4.10: Brazil Percentage of the difference in measures per sector.

CHAPTER 5

Conclusion

In our study, we investigate the effects of the Covid pandemic on two major stock market indices the S&P500 and IBOV. We analyze the price correlation distribution and use the ultra-metric distance developed by Mantenga [18] to construct a minimal spanning tree of both indices before and after the pandemic. During the pandemic, we find the normalized tree length decreases as the stocks become more correlated in agreement with [24], while the tree goes under topological restructuring and its mean occupational layer increases in contrast with financial crashes [23]. Generally, we find that during the pandemic we have slightly higher betweenness centrality, strength, and closeness centrality. While we have a lower average distance and similar degree distribution except for a well-defined central node. In our Brazil analysis, we find that during the pandemic each sector formed disjoint communities; representing different branches of the MST, contrary to the USA analysis where the separation between the sectors is reduced, and the MST has a higher degree of homogeneity.
We notice that in the case of the USA as a consequence of more developed sectors and diverse industries we have intra-sector price growth inhomogeneities with the Health Care sector as an example, resulting in less separation between the sectors as shown in figure 3.3 and a big variation in the average distance between sector as shown in table 3.10.
We quantify a centrality shift between sectors in Brazil with the hubs originally belonging to the financial, real estate, and consumer discretionary sectors while during the pandemic we have instead the Energy, Industrial, and the consumer discretionary sector assume more central roles and become hubs with the central node in both periods belonging to the consumer discretionary sector. However, in the USA analysis, the hubs do not change much during the pandemic with the Financials, Industrials, Information Technology, and Real Estate sectors forming the original hubs, while during the pandemic we notice that only the Real Estate hub disappear and a new hub emerges from the Utilities sector with the center in both periods belonging to the industrial sector.
To conclude, the USA and Brazil stock indices exhibit similar changes to a regular market crash [7] and [24], however, both go under different topological restructuring. We identify the central nodes, hubs, and newly formed ties in the pandemic, which could be used by policymakers to help stabilize financial markets in future similar events and by investors to diversify and optimize their portfolios.

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Appendices

APPENDIX A

Appendix A: Symbols

A.1 S&P500 companies

Table A.1: S&P Companies, Symbols and Sectors

Symbol	Name	Sector
MMM	3M	Industrials
AOS	A. O. Smith	Industrials
ABT	Abbott Laboratories	Health Care
ABBV	AbbVie	Health Care
ABMD	Abiomed	Health Care
ACN	Accenture	Information Technology
ATVI	Activision Blizzard	Communication Services
ADM	ADM	Consumer Staples
ADBE	Adobe	Information Technology
AAP	Advance Auto Parts	Consumer Discretionary
AMD	Advanced Micro Devices	Information Technology
AES	AES Corp	Utilities
AFL	Aflac	Financials
A	Agilent Technologies	Health Care
APD	Air Products & Chemicals	Materials
AKAM	Akamai Technologies	Information Technology
ALK	Alaska Air Group	Industrials
ALB	Albemarle Corporation	Materials
ARE	Alexandria Real Estate Equities	Real Estate
ALGN	Align Technology	Health Care
ALLE	Allegion	Industrials
LNT	Alliant Energy	Utilities
ALL	Allstate Corp	Financials
GOOGL	Alphabet (Class A)	Communication Services
MO	Altria Group	Consumer Staples
AMZN	Amazon	Consumer Discretionary
AEE	Ameren Corp	Utilities
AAL	American Airlines Group	Industrials
AEP	American Electric Power	Utilities
AXP	American Express	Financials

AIG	American International Group	Financials
AMT	American Tower	Real Estate
AWK	American Water Works	Utilities
AMP	Ameriprise Financial	Financials
ABC	AmerisourceBergen	Health Care
AME	Ametek	Industrials
AMGN	Amgen	Health Care
APH	Amphenol	Information Technology
ADI	Analog Devices	Information Technology
ANSS	Ansys	Information Technology
ANTM	Anthem	Health Care
AON	Aon	Financials
APA	APA Corporation	Energy
AAPL	Apple	Information Technology
AMAT	Applied Materials	Information Technology
APTV	Aptiv	Consumer Discretionary
ANET	Arista Networks	Information Technology
AJG	Arthur J. Gallagher & Co.	Financials
AIZ	Assurant	Financials
T	AT&T	Communication Services
ATO	Atmos Energy	Utilities
ADSK	Autodesk	Information Technology
ADP	Automatic Data Processing	Information Technology
AZO	AutoZone	Consumer Discretionary
AVB	AvalonBay Communities	Real Estate
AVY	Avery Dennison	Materials
BKR	Baker Hughes	Energy
BLL	Ball Corp	Materials
BAC	Bank of America	Financials
BBWI	Bath & Body Works Inc.	Consumer Discretionary
BAX	Baxter International	Health Care
BDX	Becton Dickinson	Health Care
BRK.B	Berkshire Hathaway	Financials
BBY	Best Buy	Consumer Discretionary
BIO	Bio-Rad Laboratories	Health Care
TECH	Bio-Techne	Health Care
BIIB	Biogen	Health Care
BLK	BlackRock	Financials
BK	BNY Mellon	Financials
BA	Boeing	Industrials
BKNG	Booking Holdings	Consumer Discretionary
BWA	BorgWarner	Consumer Discretionary
BXP	Boston Properties	Real Estate
BSX	Boston Scientific	Health Care
BMY	Bristol Myers Squibb	Health Care
AVGO	Broadcom	Information Technology
BR	Broadridge Financial Solutions	Information Technology
CHRW	C. H. Robinson	Industrials
CDNS	Cadence Design Systems	Information Technology
CZR	Caesars Entertainment	Consumer Discretionary

CPB	Campbell Soup	Consumer Staples
COF	Capital One Financial	Financials
CAH	Cardinal Health	Health Care
KMX	CarMax	Consumer Discretionary
CCL	Carnival Corporation	Consumer Discretionary
CTLT	Catalent	Health Care
CAT	Caterpillar	Industrials
CBOE	Cboe Global Markets	Financials
CBRE	CBRE	Real Estate
CDW	CDW	Information Technology
CE	Celanese	Materials
CNC	Centene Corporation	Health Care
CNP	CenterPoint Energy	Utilities
CERN	Cerner	Health Care
CF	CF Industries	Materials
CRL	Charles River Laboratories	Health Care
SCHW	Charles Schwab Corporation	Financials
CHTR	Charter Communications	Communication Services
CVX	Chevron Corporation	Energy
CMG	Chipotle Mexican Grill	Consumer Discretionary
CB	Chubb	Financials
CHD	Church & Dwight	Consumer Staples
CI	Cigna	Health Care
CINF	Cincinnati Financial	Financials
CTAS	Cintas Corporation	Industrials
CSCO	Cisco Systems	Information Technology
C	Citigroup	Financials
CFG	Citizens Financial Group	Financials
CTXS	Citrix Systems	Information Technology
CLX	Clorox	Consumer Staples
CME	CME Group	Financials
CMS	CMS Energy	Utilities
KO	Coca-Cola Company	Consumer Staples
CTSH	Cognizant Technology Solutions	Information Technology
CL	Colgate-Palmolive	Consumer Staples
CMCSA	Comcast	Communication Services
CMA	Comerica	Financials
CAG	Conagra Brands	Consumer Staples
COP	ConocoPhillips	Energy
ED	Consolidated Edison	Utilities
STZ	Constellation Brands	Consumer Staples
CPRT	Copart	Industrials
GLW	Corning	Information Technology
COST	Costco	Consumer Staples
CCI	Crown Castle	Real Estate
CSX	CSX	Industrials
CMI	Cummins	Industrials
CVS	CVS Health	Health Care
DHI	D. R. Horton	Consumer Discretionary
DHR	Danaher Corporation	Health Care

DRI	Darden Restaurants	Consumer Discretionary
DVA	DaVita	Health Care
DE	Deere & Co.	Industrials
DAL	Delta Air Lines	Industrials
XRAY	Dentsply Sirona	Health Care
DVN	Devon Energy	Energy
DXCM	DexCom	Health Care
FANG	Diamondback Energy	Energy
DLR	Digital Realty Trust	Real Estate
DFS	Discover Financial Services	Financials
DISCA	Discovery (Series A)	Communication Services
DISH	Dish Network	Communication Services
DG	Dollar General	Consumer Discretionary
DLTR	Dollar Tree	Consumer Discretionary
D	Dominion Energy	Utilities
DPZ	Domino's Pizza	Consumer Discretionary
DOV	Dover Corporation	Industrials
DTE	DTE Energy	Utilities
DUK	Duke Energy	Utilities
DRE	Duke Realty Corp	Real Estate
DD	DuPont	Materials
DXC	DXC Technology	Information Technology
EMN	Eastman Chemical	Materials
ETN	Eaton Corporation	Industrials
EBAY	eBay	Consumer Discretionary
ECL	Ecolab	Materials
EIX	Edison International	Utilities
EW	Edwards Lifesciences	Health Care
EA	Electronic Arts	Communication Services
LLY	Eli Lilly & Co	Health Care
EMR	Emerson Electric Company	Industrials
ENPH	Enphase Energy	Information Technology
ETR	Entergy	Utilities
EOG	EOG Resources	Energy
EFX	Equifax	Industrials
EQIX	Equinix	Real Estate
EQR	Equity Residential	Real Estate
ESS	Essex Property Trust	Real Estate
EL	Estauder Companies	Consumer Staples
ETSY	Etsy	Consumer Discretionary
RE	Everest Re	Financials
EVRG	Evergy	Utilities
ES	Eversource Energy	Utilities
EXC	Exelon	Utilities
EXPE	Expedia Group	Consumer Discretionary
EXPD	Expeditors	Industrials
EXR	Extra Space Storage	Real Estate
XOM	ExxonMobil	Energy
FFIV	F5 Networks	Information Technology
FB	Facebook	Communication Services

FAST	Fastenal	Industrials
FRT	Federal Realty Investment Trust	Real Estate
FDX	FedEx	Industrials
FIS	Fidelity National Information Services	Information Technology
FITB	Fifth Third Bancorp	Financials
FRC	First Republic Bank	Financials
FE	FirstEnergy	Utilities
FISV	Fiserv	Information Technology
FLT	Fleetcor	Information Technology
FMC	FMC Corporation	Materials
F	Ford	Consumer Discretionary
FTNT	Fortinet	Information Technology
FTV	Fortive	Industrials
FBHS	Fortune Brands Home & Security	Industrials
BEN	Franklin Resources	Financials
FCX	Freeport-McMoRan	Materials
GPS	Gap	Consumer Discretionary
GRMN	Garmin	Consumer Discretionary
IT	Gartner	Information Technology
GNRC	Generac Holdings	Industrials
GD	General Dynamics	Industrials
GE	General Electric	Industrials
GIS	General Mills	Consumer Staples
GM	General Motors	Consumer Discretionary
GPC	Genuine Parts	Consumer Discretionary
GILD	Gilead Sciences	Health Care
GPN	Global Payments	Information Technology
GL	Globe Life	Financials
GS	Goldman Sachs	Financials
HAL	Halliburton	Energy
HBI	Hanesbrands	Consumer Discretionary
HAS	Hasbro	Consumer Discretionary
HCA	HCA Healthcare	Health Care
PEAK	Healthpeak Properties	Real Estate
HSIC	Henry Schein	Health Care
HES	Hess Corporation	Energy
HPE	Hewlett Packard Enterprise	Information Technology
HLT	Hilton Worldwide	Consumer Discretionary
HOLX	Hologic	Health Care
HD	Home Depot	Consumer Discretionary
HON	Honeywell	Industrials
HRL	Hormel	Consumer Staples
HST	Host Hotels & Resorts	Real Estate
HWM	Howmet Aerospace	Industrials
HPQ	HP	Information Technology
HUM	Humana	Health Care
HBAN	Huntington Bancshares	Financials
HII	Huntington Ingalls Industries	Industrials
IBM	IBM	Information Technology
IEX	IDEX Corporation	Industrials

IDXX	Idexx Laboratories	Health Care
INFO	IHS Markit	Industrials
ITW	Illinois Tool Works	Industrials
ILMN	Illumina	Health Care
INCY	Incyte	Health Care
IR	Ingersoll Rand	Industrials
INTC	Intel	Information Technology
ICE	Intercontinental Exchange	Financials
IFF	International Flavors & Fragrances	Materials
IP	International Paper	Materials
IPG	Interpublic Group	Communication Services
INTU	Intuit	Information Technology
ISRG	Intuitive Surgical	Health Care
IVZ	Invesco	Financials
IPGP	IPG Photonics	Information Technology
IQV	IQVIA	Health Care
IRM	Iron Mountain	Real Estate
JBHT	J. B. Hunt	Industrials
JKHY	Jack Henry & Associates	Information Technology
J	Jacobs Engineering Group	Industrials
SJM	JM Smucker	Consumer Staples
JNJ	Johnson & Johnson	Health Care
JCI	Johnson Controls	Industrials
JPM	JPMorgan Chase	Financials
JNPR	Juniper Networks	Information Technology
KSU	Kansas City Southern	Industrials
K	Kellogg's	Consumer Staples
KEY	KeyCorp	Financials
KEYS	Keysight Technologies	Information Technology
KMB	Kimberly-Clark	Consumer Staples
KIM	Kimco Realty	Real Estate
KMI	Kinder Morgan	Energy
KLAC	KLA Corporation	Information Technology
KHC	Kraft Heinz	Consumer Staples
KR	Kroger	Consumer Staples
LHX	L3Harris Technologies	Industrials
LH	LabCorp	Health Care
LRCX	Lam Research	Information Technology
LW	Lamb Weston	Consumer Staples
LVS	Las Vegas Sands	Consumer Discretionary
LEG	Leggett & Platt	Consumer Discretionary
LDOS	Leidos	Industrials
LEN	Lennar	Consumer Discretionary
LNC	Lincoln National	Financials
LIN	Linde	Materials
LYV	Live Nation Entertainment	Communication Services
LKQ	LKQ Corporation	Consumer Discretionary
LMT	Lockheed Martin	Industrials
L	Loews Corporation	Financials
LOW	Lowe's	Consumer Discretionary

LUMN	Lumen Technologies	Communication Services
LYB	LyondellBasell	Materials
MTB	M&T Bank	Financials
MRO	Marathon Oil	Energy
MPC	Marathon Petroleum	Energy
MKTX	MarketAxess	Financials
MAR	Marriott International	Consumer Discretionary
MMC	Marsh & McLennan	Financials
MLM	Martin Marietta Materials	Materials
MAS	Masco	Industrials
MA	Mastercard	Information Technology
MKC	McCormick & Company	Consumer Staples
MCD	McDonald's	Consumer Discretionary
MCK	McKesson Corporation	Health Care
MDT	Medtronic	Health Care
MRK	Merck & Co.	Health Care
MET	MetLife	Financials
MTD	Mettler Toledo	Health Care
MGM	MGM Resorts International	Consumer Discretionary
MCHP	Microchip Technology	Information Technology
MU	Micron Technology	Information Technology
MSFT	Microsoft	Information Technology
MAA	Mid-America Apartments	Real Estate
MHK	Mohawk Industries	Consumer Discretionary
TAP	Molson Coors Beverage Company	Consumer Staples
MDLZ	Mondelez International	Consumer Staples
MPWR	Monolithic Power Systems	Information Technology
MNST	Monster Beverage	Consumer Staples
MCO	Moody's Corporation	Financials
MS	Morgan Stanley	Financials
MSI	Motorola Solutions	Information Technology
MSCI	MSCI	Financials
NDAQ	Nasdaq	Financials
NTAP	NetApp	Information Technology
NFLX	Netflix	Communication Services
NWL	Newell Brands	Consumer Discretionary
NEM	Newmont	Materials
NWSA	News Corp (Class A)	Communication Services
NEE	NextEra Energy	Utilities
NLSN	Nielsen Holdings	Industrials
NKE	Nike	Consumer Discretionary
NI	NiSource	Utilities
NSC	Norfolk Southern	Industrials
NTRS	Northern Trust	Financials
NOC	Northrop Grumman	Industrials
NLOK	NortonLifeLock	Information Technology
NCLH	Norwegian Cruise Line Holdings	Consumer Discretionary
NRG	NRG Energy	Utilities
NUE	Nucor	Materials
NVDA	Nvidia	Information Technology

NVR	NVR	Consumer Discretionary
NXPI	NXP	Information Technology
ORLY	O'Reilly Automotive	Consumer Discretionary
OXY	Occidental Petroleum	Energy
ODFL	Old Dominion Freight Line	Industrials
OMC	Omnicom Group	Communication Services
OKE	Oneok	Energy
ORCL	Oracle	Information Technology
PCAR	Paccar	Industrials
PKG	Packaging Corporation of America	Materials
PH	Parker-Hannifin	Industrials
PAYX	Paychex	Information Technology
PAYC	Paycom	Information Technology
PYPL	PayPal	Information Technology
PENN	Penn National Gaming	Consumer Discretionary
PNR	Pentair	Industrials
PBCT	People's United Financial	Financials
PEP	PepsiCo	Consumer Staples
PKI	PerkinElmer	Health Care
PFE	Pfizer	Health Care
PM	Philip Morris International	Consumer Staples
PSX	Phillips 66	Energy
PNW	Pinnacle West Capital	Utilities
PXD	Pioneer Natural Resources	Energy
PNC	PNC Financial Services	Financials
POOL	Pool Corporation	Consumer Discretionary
PPG	PPG Industries	Materials
PPL	PPL	Utilities
PFG	Principal Financial Group	Financials
PG	Procter & Gamble	Consumer Staples
PGR	Progressive Corporation	Financials
PLD	Prologis	Real Estate
PRU	Prudential Financial	Financials
PTC	PTC	Information Technology
PEG	Public Service Enterprise Group	Utilities
PSA	Public Storage	Real Estate
PHM	PulteGroup	Consumer Discretionary
PVH	PVH	Consumer Discretionary
QRVO	Qorvo	Information Technology
QCOM	Qualcomm	Information Technology
PWR	Quanta Services	Industrials
DGX	Quest Diagnostics	Health Care
RL	Ralph Lauren Corporation	Consumer Discretionary
RJF	Raymond James Financial	Financials
RTX	Raytheon Technologies	Industrials
O	Realty Income Corporation	Real Estate
REG	Regency Centers	Real Estate
REGN	Regeneron Pharmaceuticals	Health Care
RF	Regions Financial Corporation	Financials
RSG	Republic Services	Industrials

RMD	ResMed	Health Care
RHI	Robert Half International	Industrials
ROK	Rockwell Automation	Industrials
ROL	Rollins	Industrials
ROP	Roper Technologies	Industrials
ROST	Ross Stores	Consumer Discretionary
RCL	Royal Caribbean Group	Consumer Discretionary
SPGI	S&P Global	Financials
CRM	Salesforce	Information Technology
SBAC	SBA Communications	Real Estate
SLB	Schlumberger	Energy
STX	Seagate Technology	Information Technology
SEE	Sealed Air	Materials
SRE	Sempra Energy	Utilities
NOW	ServiceNow	Information Technology
SHW	Sherwin-Williams	Materials
SPG	Simon Property Group	Real Estate
SWKS	Skyworks Solutions	Information Technology
SNA	Snap-on	Industrials
SO	Southern Company	Utilities
LUV	Southwest Airlines	Industrials
SWK	Stanley Black & Decker	Industrials
SBUX	Starbucks	Consumer Discretionary
STT	State Street Corporation	Financials
STE	Steris	Health Care
SYK	Stryker Corporation	Health Care
SIVB	SVB Financial	Financials
SYF	Synchrony Financial	Financials
SNPS	Synopsys	Information Technology
SYY	Sysco	Consumer Staples
TMUS	T-Mobile US	Communication Services
TROW	T. Rowe Price	Financials
TTWO	Take-Two Interactive	Communication Services
TPR	Tapestry	Consumer Discretionary
TGT	Target Corporation	Consumer Discretionary
TEL	TE Connectivity	Information Technology
TDY	Teledyne Technologies	Industrials
TFX	Teleflex	Health Care
TER	Teradyne	Information Technology
TSLA	Tesla	Consumer Discretionary
TXN	Texas Instruments	Information Technology
TXT	Textron	Industrials
COO	The Cooper Companies	Health Care
HIG	The Hartford	Financials
HSY	The Hershey Company	Consumer Staples
MOS	The Mosaic Company	Materials
TRV	The Travelers Companies	Financials
DIS	The Walt Disney Company	Communication Services
TMO	Thermo Fisher Scientific	Health Care
TJX	TJX Companies	Consumer Discretionary

TSCO	Tractor Supply Company	Consumer Discretionary
TT	Trane Technologies	Industrials
TDG	TransDigm Group	Industrials
TRMB	Trimble	Information Technology
TFC	Truist Financial	Financials
TWTR	Twitter	Communication Services
TYL	Tyler Technologies	Information Technology
TSN	Tyson Foods	Consumer Staples
USB	U.S. Bancorp	Financials
UDR	UDR	Real Estate
ULTA	Ulta Beauty	Consumer Discretionary
UAA	Under Armour (Class A)	Consumer Discretionary
UNP	Union Pacific	Industrials
UAL	United Airlines	Industrials
UPS	United Parcel Service	Industrials
URI	United Rentals	Industrials
UNH	UnitedHealth Group	Health Care
UHS	Universal Health Services	Health Care
VLO	Valero Energy	Energy
VTR	Ventas	Real Estate
VRSN	Verisign	Information Technology
VRSK	Verisk Analytics	Industrials
VZ	Verizon Communications	Communication Services
VRTX	Vertex Pharmaceuticals	Health Care
VFC	VF Corporation	Consumer Discretionary
VIAC	ViacomCBS	Communication Services
V	Visa	Information Technology
VNO	Vornado Realty Trust	Real Estate
VMC	Vulcan Materials	Materials
WRB	W. R. Berkley Corporation	Financials
GWW	W. W. Grainger	Industrials
WAB	Wabtec	Industrials
WBA	Walgreens Boots Alliance	Consumer Staples
WMT	Walmart	Consumer Staples
WM	Waste Management	Industrials
WAT	Waters Corporation	Health Care
WEC	WEC Energy Group	Utilities
WFC	Wells Fargo	Financials
WELL	Welltower	Real Estate
WST	West Pharmaceutical Services	Health Care
WDC	Western Digital	Information Technology
WU	Western Union	Information Technology
WRK	WestRock	Materials
WY	Weyerhaeuser	Real Estate
WHR	Whirlpool Corporation	Consumer Discretionary
WMB	Williams Companies	Energy
WLTW	Willis Towers Watson	Financials
WYNN	Wynn Resorts	Consumer Discretionary
XEL	Xcel Energy	Utilities
XLNX	Xilinx	Information Technology

XYL	Xylem	Industrials
YUM	Yum! Brands	Consumer Discretionary
ZBRA	Zebra Technologies	Information Technology
ZBH	Zimmer Biomet	Health Care
ZION	Zions Bancorp	Financials
ZTS	Zoetis	Health Care
COG	Coterra Energy Inc.	Energy
NOV	NOV Inc.	Energy
PRGO	Perrigo Company plc	Health Care
UNM	Unum Group	Financials

A.2 IBOV companies

Table A.2: IBOV Companies, Symbols and Sectors

Symbol	Name	Sector
ALPA4.SA	ALPARGATAS	Consumer Discretionary
ABEV3.SA	AMBEV S/A	Consumer Staples
AMER3.SA	AMERICANAS	Consumer Discretionary
AZUL4.SA	AZUL	Industrials
BIDI4.SA	BANCO INTER	Financials
BPAN4.SA	BANCO PAN	Financials
BBSE3.SA	BBSEGURIDADE	Financials
BRML3.SA	BR MALLS PAR	Real Estate
BBDC3.SA	BRADESCO	Financials
BRAP4.SA	BRADESPAR	Financials
BRKM5.SA	BRASKEM	Materials
BRFS3.SA	BRF SA	Consumer Staples
BPAC11.SA	BTGP BANCO	Financials
CRFB3.SA	CARREFOUR BR	Consumer Staples
CCRO3.SA	CCR SA	Industrials
CMIG4.SA	CEMIG	Utilities
CIEL3.SA	CIELO	Technology
COGN3.SA	COGNA ON	Consumer Staples
CPLE6.SA	COPEL	Utilities
CSAN3.SA	COSAN	Energy
CPFE3.SA	CPFL ENERGIA	Utilities
CVCB3.SA	CVC BRASIL	Consumer Discretionary
CYRE3.SA	CYRELA REALT	Consumer Discretionary
DXCO3.SA	DEXCO	Materials
ECOR3.SA	ECORODOVIAS	Industrials
ELET3.SA	ELETROBRAS	Utilities
EMBR3.SA	EMBRAER	Industrials
ENBR3.SA	ENERGIAS BR	Utilities
ENGI11.SA	ENERGISA	Utilities
ENEV3.SA	ENEVA	Utilities
EGIE3.SA	ENGIE BRASIL	Utilities
EQTL3.SA	EQUATORIAL	Utilities
EZTC3.SA	EZTEC	Real Estate
FLRY3.SA	FLEURY	Health Care
GGBR4.SA	GERDAU	Materials
GOAU4.SA	GERDAU MET	Materials
GOLL4.SA	GOL	Industrials
HAPV3.SA	HAPVIDA	Health Care
HYPE3.SA	HYPERA	Health Care
GNDI3.SA	INTERMEDICA	Health Care
IRBR3.SA	IRBBRASIL RE	Financials
ITSA4.SA	ITAUSA	Financials
ITUB4.SA	ITAUUNIBANCO	Financials
JBSS3.SA	JBS	Consumer Staples

JHSF3.SA	JHSF PART	Real Estate
KLBN11.SA	KLABIN S/A	Materials
RENT3.SA	LOCALIZA	Industrials
LCAM3.SA	LOCAMERICA	Industrials
LAME4.SA	LOJAS AMERIC	Consumer Discretionary
LREN3.SA	LOJAS RENNER	Consumer Discretionary
MGLU3.SA	MAGAZ LUIZA	Consumer Discretionary
MRFG3.SA	MARFRIG	Consumer Staples
BEEF3.SA	MINERVA	Consumer Staples
MRVE3.SA	MRV	Consumer Discretionary
MULT3.SA	MULTIPLAN	Real Estate
PCAR3.SA	P.ACUCAR-CBD	Consumer Discretionary
PETR3.SA	PETROBRAS	Energy
PRIO3.SA	PETRORIO	Energy
QUAL3.SA	QUALICORP	Health Care
RADL3.SA	RAIADROGASIL	Health Care
RAIL3.SA	RUMO S.A.	Industrials
SBSP3.SA	SABESP	Utilities
SANB11.SA	SANTANDER BR	Financials
CSNA3.SA	SID NACIONAL	Materials
SULA11.SA	SUL AMERICA	Financials
SUZB3.SA	SUZANO S.A.	Materials
TAEE11.SA	TAESA	Utilities
VIVT3.SA	TELEF BRASIL	Communication Services
TIMS3.SA	TIM	Communication Services
TOTS3.SA	TOTVS	Technology
UGPA3.SA	ULTRAPAR	Energy
USIM5.SA	USIMINAS	Materials
VALE3.SA	VALE	Materials
VIIA3.SA	VIA	Consumer Discretionary
VBBR3.SA	VIBRA	Consumer Discretionary
WEGE3.SA	WEG	Industrials
YDUQ3.SA	YDUQS PART	Consumer Staples