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TESTING PURCHASING POWER PARITY: A LOOK TO THE EVIDENCE FROM SIX DIFFERENT TERRITORIES. FROM PERIOD 1970-2021

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Degree in Economics

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Abstract

The theory of the Purchasing Power Parity states that “the exchange rate between the currencies of two countries is equal to the ratio between the price levels of those countries so that a unit of currency of one country will have the same purchasing power in a foreign country” (Alan M. Taylor, and Mark P. Taylor, 2004, p.135).

The purpose of this work is to study the validity of the different versions of the Purchasing Power Parity (PPP) for a sample of six different currencies covering the period from 1970 to 2021 using monthly data for long-span real consumer price indexes. I have employed two different methods to study the existence of Relative and Absolute Purchasing Power Parities.

The first one is a Unit Root test to check if the Relative version of the PPP holds, by testing if the difference between each country’s CPI series with respect to the USA’s CPI series are stationary. The second one is a Cointegration test to validate the Absolute version of the PPP. Additionally, I have complemented the study by obtaining the Impulse Response Functions to keep track of the response given by each country’s CPI to external changes in the USA’s CPI.

Index

14. Appendix

List of Tables

List of Graphs

List of R Code

1. Introduction

The main purpose of this work is to analyze six different currencies and use econometric methods to test the validity of the hypothesis that Purchasing Power Parity holds. The different currencies are the U.S.A. dollar, Japanese yen, U.K. pound, Mexican peso, German mark, and Colombian peso, taking the U.S.A. dollar as the foreign country and the rest as the home or domestic country. It goes without saying that the study of the purchasing power parity has generated a lot of debate ever since the theory was originated, the principal aspiration widely studied is to find stochastic trend between the exchange rates applying different econometric tests.

There is a tendency to believe that the theoretical framework behind the PPP is based on the Law of One Price, which states that “in competitive markets free of transportation costs and official barriers to trade, identical goods sold in different countries must sell for the same price when their prices are expressed in terms of the same currency” (Krugman, P. R., Obstfeld, M., & Melitz, M. J., 2017, p.450)

There are many applications for the PPP, it is an indicator used to compare economic productivity, assess socio economic issues, it aids to gather data and generate economic predictions, and a tool to help with the understanding of the different welfare levels and well-being of countries. Having ratios and measures that improve our understanding of economic performance contribute facilitating the application of adequate policies in countries. In this work I take five different currencies versus the U.S.A dollar to study if the PPP hypotheses are accomplished focusing over the period starting January 1970 to January 2021. I have decided to analyze currencies from countries around the world to compare how tendencies may vary depending on the territory studied.

I have divided the analysis of the PPP within the two variants. On the first place I have studied whether Relative PPP was held by using Unit Roots tests to check for stationarity, and on the second place I have validated the existence of Absolute PPP in the cases where the Relative was not accepted. This second study has been done with a Cointegration test. I have implemented the Augmented Dickey-Fuller in both levels and first difference and including seasonally varying means, to incorporate them two different functions have been generated on RStudio.

On the first place, the ADF test has been conducted on the differences between each CPI series with respect to the US.A. dollar, including the trigonometric representation of deterministic seasonality, and therefore I have reached to the conclusion that the relative version of the PPP holds for Japan and the UK. I rejected null hypothesis that relative PPP does not hold for these two countries, in other words, null hypothesis is accepted for Mexico, Germany and Colombia. Once stationarity has been studied for all CPI series I proceeded to run the two-step Engle-Granger cointegration test and also the Johansen procedure to test if Absolute PPP is held and if the currencies might have a long-run relationship.

Regarding results obtained, Germany and Mexico have at least one cointegrating relationship with the USA. Finally, I have conducted an additional study for the countries where the Relative version of the PPP is not held. I have obtained the Impulse Response Functions for a 20-month period of time to understand the dynamic of each country and analyze how each of them reacts to an endogenous shock to the USA's CPI series, and vice versa.

The organization of the work is the following: Part 1 is an introduction to what the PPP is and a moderate glimpse through the historical and theoretical background behind it, Part 2 sets out the different versions of the PPP. Part 3 explains the econometric procedure followed, all tests conducted throughout the study, and its mathematical fundaments, Part 4 introduces data used and exhibits some descriptive statistics; Part 5 show results obtained from the study. Lastly, Part 6 reveals conclusions reached and summarizes findings. Additionally, at the end of the work there is an Appendix where complementary explanations of tests and concepts have been collected.

2. Theoretical background

2.1 Law Of One Price

As to start, it is important to present the Law of One Price. This Law reinforces the idea that in each market identical goods converted into a common currency should have the same price. It is generally believed that the origins for the PPP come from this law, and as similar as their definition might seem the truth is there are some restrictions that differentiate them.

The Law of One Price (LOOP), is governed by the principle of arbitraging, and it strictly states that it is only applicable in markets where no transportation costs are considerate neither as barriers to international trade. Following Krugman, P. R., Obstfeld, M., & Melitz, M. J.(2017) a formal definition can be given with the following example:

> Letting P_{US}^{i} be the dollar price of good i when sold in the United States, P_{E}^{i} the corresponding euro price in Europe. Then the law of one price implies that the dollar price of good i is the same wherever it is sold.
>
> $$P_{US}^{i} = (E_{\$/€}) * (P_{E}^{i})$$
>
> Equivalently, the dollar/euro exchange rate is the ratio of good i's U.S. and European money prices,
>
> $$E_{\$/€} = (P_{US}^{i}) \,/\, (P_{E}^{i})$$
>
> (pp.450-451)

2.2 History of the PPP

The fundaments of the PPP have been studied since the XVIth century by various British economists. Theoretically it has influenced many people and provided them with the basis for other theories about international price determination. Among them, David Ricardo who first originated the theory of the Comparative Advantage, Balassa and Samuelson (1964), but it wasn't until the early 20th century when Swedish economist Gustav Cassel, postulated the theory and made it one of the most valuable works of the economic science mainstream of exchange rates.

Reflecting on economic developments during World War I, Cassel wrote:

> At every moment the real parity between two countries is represented by this quotient between the purchasing power of the money in the one country and the other. I propose to call this parity "the purchasing power parity". As long as anything like free movement of merchandise and a somewhat comprehensive trade between the two countries takes place, the actual rate of exchange cannot deviate very much from this purchasing power parity. (Cassel,1928, p.413)

Cassel defined the Absolute PPP version which determines that "the relative prices (in different currencies and locations) of a common basket of goods will be equalized when quoted in the same currency" (Papell, D. H., & Prodan, R., 2003, p.1). His main goal was to determine the PPP as a guideline for normative policy prescription, and to determine new gold parities for countries, which was one of the major issues in the 1920's. Cassel himself thought of the PPP to only be an approximation of the real exchange rates, he initially introduced the concept of an equilibrium exchange rate, which from that moment on would result on various deviations from the first definition in both the short and long run.

Following Krugman, P. R., Obstfeld, M., & Melitz, M. J.(2017), its formal definition can be explained as followed:

> Letting P_{US} be the dollar price of a reference commodity basket sold in the United States and P_E the euro price of the same basket in Europe.
>
> Then PPP predicts a dollar/euro exchange rate of
>
> $$E_{\$/€}= P_{US}/ P_E$$
>
> If we rearrange the equation, it is possible to obtain an alternative interpretation of PPP
>
> $$P_{US} = (E_{\$/€}) * (P_E)$$

> The left side of the equation is the dollar price of the reference commodity basket in the United States; the right side is the dollar price of the reference basket when purchased in Europe (it measures the purchasing power of a dollar when exchanged for euros and spent in Europe). These two prices are the same if PPP holds, and when, at going exchange rates, every currency's domestic purchasing power is always the same as its foreign purchasing power. (p.451)

Both are pretty similar at first sight, but the LOOP is referred to individual goods whereas the PPP takes into account the general price levels of countries. Subsequently, it can be said that the Law of One Price is a strong version of the PPP. Supporters of the PPP contend that its validity, especially in the long run, does not necessitate the exact application of the LOOP. Even if the law of one price does not apply for each specific good, prices and exchange rates should not deviate too far from the PPP-predicted relationship. When goods and services in one nation become temporarily more costly than in others, demand for that country's currency and products declines, bringing the exchange rate and domestic prices back into line with PPP. Therefore, "the PPP states that even when the LOOP does not hold, economic forces behind it will help eventually to equalize a currency's purchasing power in all countries" (Krugman, P. R., Obstfeld, M., & Melitz, M. J.,2017, p.452)

Through the years there have been different methods taken up and measures created to analyze deviations from the PPP and determine whether currencies are at their "correct" level against a specific currency. In order to facilitate the comparison of prices of identical goods and the understanding to people, different indexes have been created, as an example there is the Big Mac Index, Tall Latte Index, KFC Index, among others. They are all based on the theory of the Purchasing Power Parity with the aim to analyze if the currency exchange rate moves towards an equalizing price of goods and services in the long run and then obtain a conclusion upon the hypothesis of the PPP.

2.3 Big Mac Index

The Big Mac Index was created by *The Economist* newspaper in year 1986, based on the theory of the purchasing power parity. *The Economist* uses the Mc Donald's Big Mac^{TM} Index because, generally speaking it is an example of the only homogeneous product which you can get globally, it is a way to assess and compare using "like with like" in that respect. The so-called, "Burgereconomics, were not intended to be a precise gauge of currency misalignment, merely a tool to make exchange-rate theory more digestible" (The Economist, 2022). Having the Big Mac Index allows to imagine what the exchange rate should be for different local prices and make approximations about how much a currency is under- or over-valued relative to another.

Surprisingly the Big Mac Index achieved a global popularity, and other variants of the index were created, an outstanding one is the Tall latte Index, also created by *The Economist*, based on the price of Starbucks coffee whose average price in America happened to be the same as the average price of a Big Mac in year 2004, $2.80. Both indexes have the same core and show same conclusions for the vast majority of currencies. Regarding some of the currencies studied in this work some current comparisons found using the Interactive Currency Comparison Tool from *The Economist* are the following:

> According to the Big Max Index and setting the US dollar as base currency, the British pound was 17.1% undervalued against the US dollar. The Euro was 14.7% undervalued, the Japanese yen was 41.7% undervalued and the Colombian peso was 43.5% undervalued against the US dollar in January 2022.

An important fact to take into account is that the Big Mac Index lacks in certain aspects, it doesn't include labor costs, as a consequence, this explains how "historically the adjustment has tended to raise currencies' valuations against the dollar, so that emerging-market currencies tend to look reasonably more priced". (The Economist, 2022). This whole notion of PPP comes above because when we look at GDP numbers in different currencies it might be confusing as it doesn't take into consideration the purchasing power of the currency. It is necessary to use a PPP conversion which tells us precisely an accurate comparison of how many goods and services we can purchase at a country.

3. Main variants of the PPP

The purchasing power parity has two main variants depending on the assumptions that must be complied:

3.1 Absolute PPP

Absolute purchasing power parity holds when the purchasing power of a unit of currency is exactly equal in the domestic economy and in a foreign economy once it is converted into foreign currency at the market exchange rate. (Alan M.Taylor & Mark P. Taylor, 2004, pp.137-138).

It is the strict version, and it requires the construction of identical baskets of goods and services to proceed with the comparison of indexes for a long period of time. Following Isard P. (1995), the formal definition of the absolute PPP states that the exchange rate between the currencies of two countries should equal the ratio of the price levels of the two countries. Specifically,

$$S = \frac{P}{P^*}$$

> where S is the nominal exchange rate measured in units of currency A per unit currency B, P is the price level in country A, and $P*$ is the price level in country B. (p.58)

The Absolute PPP predicts that a decrease in purchasing power of a currency (reflected as an increase of internal prices) leads to a depreciation of the currency.

It is meaningful to say that if Absolute PPP holds and there is a positive correlation within the residuals this it could explain the presence of permanent deviations from the PPP in the short run. As a consequence, it is decisive to study stationarity on the residuals, if they turn out to be non-stationary all the results from the regression won't be valid, and it will also determine the reason why prices and exchange rates deviate from the equilibrium.

3.2 Relative PPP

Relative purchasing power parity is a less strict version, Isard (1995) explained relative PPP hypothesis states that exchange rate variations should bear a constant proportionate relationship to the ratio of national price levels; in particular,

$$S = k\frac{P}{P^*}$$

where k is a constant parameter. The logarithmic transformations of both equations have the form

$$s = \alpha + p - p^*$$

Where s, p, p^* are the logarithms of S, P, P^* and $\alpha = 0$ under absolute PPP. Under either variant of PPP, a change in the ratio of price levels implies an equiproportionate change rate, such that:

$$\Delta s = \Delta p - \Delta p^*$$

Where Δs demonstrates that the percentual increase in the nominal exchange rate is equivalent to the difference between price levels in both countries. (pp.58-59)

Just like with the previous version of the PPP it is necessary for the residuals to be stationary, as the whole formulation of the hypothesis takes into consideration that all variables are stationary, so their first difference, the residuals, and the statistical analysis should be too.

Following Krugman, P. R., Obstfeld, M., & Melitz, M. J., (2017), in practice there is a firm belief that Absolute PPP makes no sense, except for the case when the two baskets compared are exactly the same. Therefore, the hypothesis of relative PPP is useful and convenient because it may be valid even when absolute PPP is not held. Provided the factors causing deviations from absolute PPP are more or less stable over time, percentage changes in relative price levels can still approximate percentage changes in exchange rates. (p.453)

4. Deviations and arguments against the PPP

Empiric evidence systematically rejects the theory of the PPP due to multiple counterarguments that lead to permanent deviations from the purchasing power parity originated by changes in price levels. Some examples are transportation costs, barriers to trade, non-tradable goods, market power and other international differences. There are arguments against the validity of the PPP especially in the short run, but this might not necessarily stand in the long run. According to Isard (1977) the PPP hypothesis would be valid for the general price levels of any two countries under the following conditions:

A) If each tradable good comply with the Law of one Price, exhibiting identical prices when transformed into a common currency in each country,
B) If factor price equalization and identical production functions brought the prices of non-tradable goods into equality internationally, and
C) If each good received identical weights in the aggregate price indices of the two countries. (p.60)

Following the idea explained by Krugman, P. R., Obstfeld, M., & Melitz, M. J., (2017), an advantageous way of explaining how exchange rates and monetary factors react in the long run is to combine a framework of money supply and demand with the hypothesis of the PPP. The monetary approach to the exchange rate is thought to be a long run but not short run theory as it does not allow for the price rigidities that seem important in explaining short run macroeconomic developments, the monetary approach proceeds as if prices can adjust right away to maintain PPP.

Some reasons to understand why the PPP might not hold in the short run are that for some goods after being converted into the same currency, prices from products might show divergences from nominal exchange rates, consequently, condition *A* from above does not comply. Moreover, transportation costs and barriers to trade and taxes reduce demand elasticities and make exchange rates and prices more likely to deviate from the PPP. Theoretically these deviations are expected to be transitory, also meaning that the exchange rates should be stationary and not incorporate any unit root and be mean reverting when facing external shocks.

Nonetheless, it is important to stand out that the study of the PPP is relevant as it acts as a guide for Central Banks to fix official exchange rates, “its aim is to suggest exchange rates and stablish trade relations” (Cassel, 1918). It is also of great help to determine “equilibrium” exchange rates.

5. Description of the methodology

With the purpose of evaluating the hypotheses, I have done empirical analysis over monthly data of six different consumer price indexes covering the period from January 1970 to January 2021. In the first part of the analysis, I studied whether the relative hypothesis of the PPP holds by checking stationarity on the difference between each country's CPI series with respect to the USA's CPI, to do that, I have employed the Augmented Dickey-Fuller test (ADF). If the results of the ADF test show that there is a Unit Root, then it is not possible to reject null hypothesis that the Relative version of the PPP does not hold.

On the second place, for the Absolute version of the PPP to be held, I have conducted a Cointegration test to study if there is a long run relationship between each CPI series with the USA. I have employed the Engle-Granger procedure that runs an ADF test on the residuals, and also the Johansen procedure to determine the possible amount of cointegration relationships that each country might have with the USA. Lastly, I have created a VAR model to obtain the Impulse Response Function for the countries where relative PPP is not held.

5.1. Unit Root and Stationarity tests

It is necessary to start the study by checking whether the difference between each CPI series with respect to the USA are non-stationary. To do that I have run the Augmented Dickey-Fuller test (ADF) in both levels and transformed into first difference. It is important to stand out that the ADF test has been conducted taking into consideration deterministic seasonality and setting a lag length no shorter than 12 as working with monthly data. Later on, I will dig deeper into the importance of introducing dummy variables to deal with deterministic seasonality in this study.

In addition, two more tests have been run, a unit root test the Phillips-Perron test (PP), and a stationary test, the Kwiatkowski-Phillips- Schmidt-Shin test (KPSS), both only demonstrate that neglecting seasonal means might lead to distorted decisions under null hypothesis. Therefore, the results compared to the ADF test will demonstrate to be incoherent and non consistent.

5.1.1. Unit Root test: Dickey-Fuller test (ADF)

The Dickey-Fuller test (DF, Dickey and Fuller, 1979) is a unit root test, based on the form of an AR(1) aimed to statistically detect the presence of stochastic trend in time series of the variables through a hypothesis contrast. It is based in the form of the first order auto regressive process from Box, Jenkins (1970):

$$Y_t = \phi Y_{t-1} + u_t \qquad t = 1, \dots, T$$

where ϕ is the autoregression parameter, u_t the non-systematic component, a white noise, then :

$$(Y_t) \sim I(0) if\ \phi < 1$$

$$(Y_t) \sim I(1)\ if\ \phi = 1\ \ (a\ random\ walk)$$

In order to identify if the presence of stochastic trend exists in a time series, the first regressor of the AR (1) process will tend to be 1 or very close to 1, due to the property of reversion to the mean. If the process was stationary, then this regressor would be less than 1 or very close to 0. Consequently, we can distinguish the cases where there is stochastic trend or not, depending on the number given to the first regressor of the process.

The process implies specifying both hypotheses:

H0: $\phi = 1$,There is a unit root, therefore the process is non-stationary
H1: $\phi < 1$,There is not a unit root, and the process is stationary

Mathematically, following Pfaff, B. (2008),

Starting from the model of an AR (1):

$$Y_t = \phi Y_{t-1} + u_t \text{ where } u_t \sim Normal\ (0, \sigma^2)$$

Subtracting the independent variable Y_{t-1} from both sides we obtain,

$$Y_t - Y_{t-1} = \phi Y_{t-1} - Y_{t-1} + u_t$$

$$\Delta Y_t = (\phi - 1) Y_{t-1} + ut$$

The resulting model can also be written as

$$\Delta Y_t = \beta Y_{t-1} + u_t$$

Where $\beta = \phi_1 - 1$. Now the hypotheses will be:

H0: $\beta = 0$, then $\phi = 1$, there is a unit root, and the process is non-stationary
H1: $\beta < 0$, then $\phi < 1$, there is not a unit root, and the process is stationary

It is possible to calculate the DF statistic that follows the Dickey-Fuller distribution, as:

$$t_{DF} = \frac{\hat{\phi}_1 - 1}{SE_{\hat{\phi}_1}}$$

Where $\hat{\phi}_1$ is a least square estimate of ϕ_1 and $s_{\hat{\phi}_1}$the standard error estimate.

In my study I have applied the Augmented Dickey-Fuller Test (ADF test), which can handle larger and more complex set of time series models in this work. Now the model is transformed into:

$$\Delta Y_t = \alpha + \beta Y_{t-1} + u_t$$

Containing a constant in the random-walk process to capture the nonzero mean under the alternative, a unit root exists in a time series when $\alpha = 1$. This variation of the test implies adding lagged values of the dependent variable:

$$Y_t = \alpha + \beta Y_{t-1} + \phi_1 \Delta Y_{t-1} + \phi_2 \Delta Y_{t-2} \ldots + \phi_p \Delta Y_{t-p} \ldots + u_t$$

The lag length should be chosen so that the residuals are not correlated, Ng and Perron (1995) suggest first setting an upper bound $P_{\max}$, estimate the ADF regression subject to $P = P_{\max}$. If the absolute value of the t-statistic of the last lagged difference is greater than 1-6 then proceed to set $P = P_{\max}$, if not reduce the lag length. Another suggestion by Schwert (1989) is setting $P\max = \left[12\left(\frac{n}{100}\right)^{\frac{1}{4}}\right]$, the option is to follow the AIC procedure (Akaike's Information criteria), which is the one used in this study, or the BIC procedure (Bayesian Information criteria).

Both hypotheses remain the same:

H0: $\phi = 1, \beta = 0$ → there is a unit root, the process is non-stationary
H1: $\phi < 1, \beta < 0$ → the time series is stationary or trend-stationary

In conclusion, the criteria to follow for rejecting null hypothesis is the following:

If the t-statistic < ADF critical value → Not reject H0
If the t-statistic > ADF critical value → Reject H0

As mentioned on the introductory part of the Methodology process, taking into consideration the presence of deterministic seasonality is indispensable when working with seasonal data. Demetrescu, M., and Hassler, U. (2007) explain that "whenever deterministic seasonality is ignored, the distribution of the Dickey- Fuller test is shifted to the left. Dickey-Fuller test without seasonal dummies is oversized and has little power at the same time, due to the need of lag augmentation". (p.385)

In their paper *Effect of neglected deterministic seasonality on unit root tests*, Demetrescu, M., and Hassler, U., validate the fact that seasonal dummy variables should be included to increase the power of the Dickey-Fuller and Augmented Dickey-Fuller tests (DF, ADF). Whenever they are not considered, the power of the tests is reduced, and distortions become less predictable.

In this study, the ADF test has been done considering the trigonometric representation of deterministic seasonality in the regression in both levels and first difference. Results obtained for both KPSS and PP test do not include it, and demonstrate the presence of asymptotic distortion that might lead to error.

There are two equivalent models that allow to deal with deterministic seasonality, either with the presence of fictitious variables or with the use of the trigonometric representation of deterministic seasonality. Mathematically, the suggested correction for the ADF test including the trigonometric representation of deterministic seasonality following Ghysels, E., and Osborn, D.R. (2001) is the following:

$$\mathit{\Delta} y_t = \alpha_0 + \sum_{j=1}^{5} \alpha_j \cos\left(\frac{2\mathit{\Pi}_j}{12} t\right) + \sum_{j=1}^{5} \beta_j \sin\left(\frac{2\mathit{\Pi}_j}{12} t\right) + \alpha_6 \cos(\Pi \mathrm{t})$$

Where $t = 1,2,\dots,T$. The equivalent model including fictitious seasonal variables (dummies), following del Barrio Castro, T.(n.d.) is:

$$\mathit{\Delta} y_t = \sum_{j=1}^{12} D_{jt} \cdot C_j + \beta t + \varphi y_{t-1} + \sum_{j=1}^{p} \gamma_j \mathit{\Delta} y_{t-j} + e_t$$

where $\mathrm{s} = 1,2,\dots,S$. In this case $S = 12$ as working with monthly data. Parameter D_{jt} is defined by:

$$D_t^k = \begin{cases} 1 & if \;\; k = 1 + int\ [(t-1) \bmod S] \\ 0 & if \;\; k \neq 1 \; + int\ [(t-1) \bmod S] \end{cases}$$

Where mod is an operator that returns the rest of the division between a and b, therefore $1 + int\ [(t-1) \bmod S]$ allows to calculate for each observation t, to which observation it belongs. (pp.1-3). Both models have an exact and unique one-to-one relationship for each coefficient of the seasonal dummy variables.

Whenever using the trigonometric representation of deterministic seasonality, the elements that appear in the so-called T matrix of order $S \times S$ are tightly related with the transformations used in HEGY contrast. In case of willing to determine whether seasonality is stationary or not there are specific tests than should be conducted rather than Unit Root test, these are called Tests for Seasonal Unit Roots, the most well-known and applied is the HEGY test, Hylleberg, Engle, Granger and Yoo test.

The HEGY test provides evidence for testing null hypothesis seasonal unit roots. A paper by Richard J. Smith, A.M. Robert Taylor and Tomás del Barrio Castro (2009), *Regression-based seasonal unit roots,* contributes characterizing the theorem of the subhypotheses regarding seasonal unit root hypothesis, and provides a specific formulation of the alternative hypotheses by modelling a regression-based seasonal unit root test.

According to Ghysels, Lee and Noh (1994) when applying Augmented Dickey-Fuller test (ADF), with and without constant and trend and also including seasonal dummies, “the usual Dickey-Fuller t-statistics can still be used to test the hypothesis of a unit root at the zero frequency, even in the presence of unit roots at other seasonal frequencies to the extent that lagged terms of dependent variables are appropriately augmented. (p.420)

As they explain, to conduct the ADF test for a unit root at the zero frequency in presence of seasonal unit roots, the time series can be written as:

$$\Delta x_t = \phi x_{t-1} + \phi_2 x_{t-1} + \phi_3 x_{t-2} + \phi_4 x_{t-3} + u_t$$

Where: $\phi_1 = \alpha - 1$ and $\phi_2 = \phi_3 = \phi_4 = -\alpha$. It is demonstrated that the limiting distribution of the t-statistic is the same as discussed in DF (1979), but it is suggested that the normalized-bias statistic should be divided by four.

The main point explained by Demetrescu, M., & Hassler, U. (2007) is that including lagged differences helps to ensure the invariance of the asymptotic null distribution of the t-statistic. However, when including lagged differences but not seasonal dummies the distribution of the ADF test statistics will be furthermore distorted.

I have used RStudio program to conduct all tests on my empirical analysis, and I have opted to develop the model which includes the trigonometric representation of deterministic seasonality. As explained both models have a one-to-one relationship, which means in practice, same results would be obtained when introducing seasonal dummies. To do that, two functions have been generated, whose correspondent code can be found on the RCode list.

5.2 Cointegration test

After having tested stationarity for all-time series, I have proceeded to check if the Absolute version of the Purchasing Power Parity holds conducting a cointegration test. Testing for cointegration is a conventional method taken on empirical research on the study of the PPP hypothesis, as it allows to demonstrate if there is a long run relationship that ties the variables studied. Cointegration can: “explain the existence of an equilibrium or a stationary relationship among two or more time-series, each of which is individually non-stationary”. (Anindya Banerjee, Juan Dolado, John W. Galbraith, and David F.Hendry, 1993, pp.136-137).

One of the issues to be dealt with when testing hypotheses about the coefficients using standard statistical inference is that it could lead to spurious results. Granger started studying this with the aim of developing more realistic econometric models to avoid spurious regressions. Granger’s solution can be illustrated by the following regression equation:

$$y_t = \alpha + \beta x_t + u_t$$

Where y_t is the dependent variable, x_t the exogenous regressor, and u_t a white noise. Granger argues that in order to be meaningful, an equation has to be consistent “in the sense that a simulation of the explanatory right-hand side should produce the major properties of the variable being explained” (Granger,1981, pp.121-130)

Furthermore, Granger also introduced the idea of degree of integration of a variable, that is if a variable as z_t can be transformed into stationary by differencing it d number of times, then the degree of integration will be of order d or $I(d)$.[1]It is essential to select the adequate lag length, in this case as I am working with monthly data lag length should not be shorter than 12. Including too many lags can result in reducing the effectiveness of the tests and leading to accept Null Hypothesis, and on the other hand, choosing too few lags will also make the estimation less precise. For my tests I have followed the Akaike Schwarz criteria for the appropriate lag selection in RStudio.

There are different methods for testing for cointegration, in this work I will apply the Engle-Granger Two-Step method and the Johansen procedure.

5.2.1 Engle-Granger Two-Step Method

Engle and Granger (1987) proposed a Two-Step Method, considering there is a single cointegrating vector. Following Pfaff (2008):

- First step is to determine a regression of the variables in the set of $I(1)$

  $$y_t = \alpha_1 x_{t,1} + \alpha_2 x_{t,2} + \ldots + \alpha_k x_{t,k} + z_t \quad for\ t = 1, \ldots, T$$

  Where z_t is the error term.

  It is necessary to determine the order of integration of the variables, as the cointegration test can only be applied when they are integrated of the same order. By applying the standard OLS estimation model, the residuals are estimated, and a Unit Root test must be run on the residuals to test for stationarity. “Once the null hypothesis of a unit root in the series $\hat{z}_t$ has been rejected the second step follows” (p.76)

- Second step involves specifying an Error-Correction Model (ECM), in which two variables with order of integration $I(1)$ are considered. The values from the estimated coefficients obtained will determine the speed of adjustment and if the system diverges from the long run equilibrium path. Following Granger causality “in case of two cointegrated $I(1)$ variables, Granger causality must exist in at least one direction” (Granger, 1987, p.77)

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[1] In pursuance of carrying out the cointegration test it is necessary that the order of integration of the variables is the same in the long run. By looking again at the regression before, if we assume that both $x_t \sim I(1)$ and $y_t \sim I(1)$, then generally the linear combination $y_t - \beta x_t \sim I(1)$ too, nonetheless there is an exception that states that if $u_t \sim I(0)$, then $y_t - \beta x_t \sim I(0)$, and variables x_t and y_t are cointegrated.

5.2.2 Johansen Procedure

Johansen (1988) and Johansen (1991) developed a method to test for cointegrating relationships between non-stationary time series. There are two different tests to follow, the Maximum Eigenvalue test, and the Trace test. Johansen's procedure starts with a vector autoregression (VAR) of order p:

$$y_t = \mu + A_1 y_{-t-1} + \ldots + A_p y_{t-p} + \varepsilon_t$$

Where y_t is an $nx1$ vector of variables integrated of order $I(1)$, and ε_t an $nx1$ vector of innovations. Hjalmarsson, E., Österholm, P. (2007) explain that this VAR model can also be written as:

$$\Delta y_t = \mu + \Pi y_{t-1} + \sum_{i=1}^{P^{-1}} \Gamma_i \Delta y_{t-i} + \varepsilon_t$$

Where $\Pi = \sum_{i=1}^{J_P} A_i - I \ and \ \Gamma_{i=} \sum_{j=i+1}^{p} A_j$

If the coefficient matrix has reduced rank $r < n$, then there will exist nxr matrices $\alpha \ and \ \beta$ each one with a rank r such that $\Pi = \alpha\beta'$ and $\beta' yt$ is stationary. The number of cointegrating relationships is denoted by r, and the elements of α are the adjustment parameters in the VECM and β the cointegrating vector.(pp. 4-6). Following Hjalmarsson, E., Österholm, P. (2007) two likelihood ratio tests are derived:

5.2.2.1 Trace test

This test evaluates a restricted VECM against an unrestricted VECM with r number of cointegrating vectors. The Trace test null hypothesis will be r cointegrating vectors against alternative of n cointegrating vectors. The test statistic is given by:

$$J_{trace} = -T \sum_{i=r+1}^{n} \ln\left(1 - \hat{\lambda}_i\right)$$

(p.5)

5.2.2.2 Maximal Eigenvalue test

This other variant of the test, tests null hypothesis of r cointegrating vectors against alternative hypothesis of $r + 1$ cointegrating vectors.

$$J_{max} = -T \ln\left(1 - \hat{\lambda}_{r+1}\right)$$

Neither of both tests follow a chi square distribution, and the statistic depends on the dimension and deterministic trend specified. One positive outcome

> that the Johansen procedure (Hjalmarsson, E., Österholm, P.,2007; Johansen,1995) states is that, it is not essential to determine the order of integration of each variable previous to conducting the test, therefore it allows to run the test among variables that might have a different order of integration based on the fact that this one will reveal itself through the cointegrating vector. (p.5)

In this work Cointegration has been studied for the countries where relative version of the PPP is not held, in other words, to test if the Absolute hypothesis of the Purchasing Power Parity holds in the long run. This will demonstrate if the variables, each country's CPI series, have a long run relationship with the CPI taken as the domestic country, the USA. I have followed both tests of the Johansen procedure, to compare different results.

To sum up, I have employed the ADF test to study for stationarity on the difference between each CPI series with respect to the USA and obtained the order of integration of each one of them. Furthermore, I have followed Engle-Granger procedure to test stationarity on the residuals. With that purpose I have employed the Ljung-Box test that determines if the residuals are a white noise, which means they do not exhibit serial correlation, if they are uncorrelated its mean is equal to zero. If the residuals do not follow the process of a white noise an Augmented Dickey-Fuller test or any other standard unit root test such as the PP test should be implemented.

As results confirmed that residuals are a white noise, I have continued to follow the Johansen procedure to compare both results obtained and determine the number of possible cointegration relationships that each series might have with the USA.

5.3 Spectral Analysis

I have complemented the study of seasonality with the Spectral analysis for time series. Theoretically the Spectral analysis' main application is to decompose stationary time series in terms of its cyclic components, it "involves the calculation of waves or oscillations in a set of sequenced data" (J.N. Rayner, 2005, pp.14861-14864). It was originally applied in engineering, and its use in standard statistical analysis has risen ever since then. It is a necessary tool for seasonal adjustment in time series.

The graphical representation of the spectrum of a process allows to express it as a sum of periodic sequences, more specifically trigonometric sequences. The Statistical Fourier Analysis (D.S.G. Pollock, 2008) states that:

> It is always possible to approximate an arbitrary function defined over a finite interval of the real line, to any desired degree of accuracy, by a weighted sum of sine and cosine functions of harmonically increasing frequencies. The accuracy of approximation increases with the number of functions with the sum. (Pollock D.S.G., 2008, p.1)

For a sample of T observations, the mathematical expression following D.S.G. Pollock (2008) can be written as:

$$y_t = \sum_{j=0}^{n} \{\alpha_1 \cos(w_j t) + \beta_j \sin(w_j t)\}$$

Where $w_j = \frac{2\Pi}{T}$. This expression is called the Fourier decomposition. When T is even, $n = \frac{T}{2}$. And the equation from before becomes:

$$y_t = \alpha_0 + \sum_{j=1}^{n-1} \{\alpha_1 \cos(w_j t) + \beta_j \sin(w_j t)\} + \alpha_n (-1)^t$$

In this case the frequencies of the trigonometric functions range from $w_1 = \frac{2\Pi}{T}$ to $w_n = \pi$. On the other hand, when T is an odd number, $n = \frac{(T-1)}{2}$, and the equation becomes:

$$y_t = \alpha_0 + \sum_{j=0}^{n} \{\alpha_1 \cos(w_j t) + \beta_j \sin(w_j t)\}$$

Where the range goes from $w_1 = \frac{2\Pi}{T}$ to $w_n = \frac{\Pi(T-1)}{T}$. (p.1)

In order to proceed to the Spectral Representation of a stationary process, it is necessary to allow the value of n in the expression before to tend to infinity. Following Hamilton, James D. (1994, 6.1-6.2), the results can be expressed as an integral over the range $(-\Pi, \Pi]$, leading to the Spectral Representation of the process y_t is the following:

$$y(t) = \int_{-\Pi}^{\Pi} e^{iwt}\, dZ\,(w)$$

The Autocovariances and Spectral Density function can be obtained using the Fourier transformation Hamilton, James D. (1994, pp.152-163), resulting on $\gamma_\tau = \int_\omega \; e^{iwt} f(w) dw$ and $\gamma_0 = \int_w f(\omega)\, dw$. The inverse of the Autocovariances results on a function comparable to the so-called Periodogram, represented as:

$$f(w) = \frac{1}{2\Pi} \left\{ \gamma_0 + 2 \sum_{T=1}^{\infty} \gamma_\tau \cos(\omega\tau) \right\}$$

In my study I have considered a finite sample of data, the Spectral analysis done complements the previous explanation on the need of including seasonal dummies to test for stationarity. In the spectrums that will be shown forward on, the difference between each CPI series and the USA are represented.

Stationarity is hidden in the spectrums as the asymptote of the density at zero frequency is very dominant. Density at zero frequency corresponds to the trend. Whenever examining the spectrum of an adjusted series to detect stationarity one must focus on the peaks observed as they represent the remaining seasonal component.

Usually, when working with monthly data a standardization process is carried out to be able to do an appropriate graphical representation. The units in which the frequencies are expressed will be cycles per year and the interval where the frequencies move is not $(-\Pi,\Pi]$ anymore, but $\left[0,\frac{s}{2}\right]$ where s takes the value of 12 in this case, then the maximum frequency observed in cycles per year for monthly data is 6 cycles per year.

The conclusion reached then, is that when series are trending it is necessary to check for stationarity on the series, and also difference them to be capable of nullifying the effect of zero frequency domination. Consequently, the CPI series studied demonstrate to have strong seasonality, and therefore the ADF test including the trigonometric representation of deterministic seasonality will be effective.

5.4 Impulse Response Function

In addition to the results from the Cointegration analysis I have reinforced the conclusions reached with an alternative analysis, the Impulse Response Function. “The IRF employs vector autoregressive models (VAR) and its aim is to describe the evolution of the variables in a model over a determined period of time in reaction to a shock in a given moment” (Alloza M., n.d.). Following Hamilton, James D. (1994) explanation, a VAR model can be written in vector $MA(\infty)$ form as:

$$y_t = \mu + \varepsilon_t + \psi_1\varepsilon_{t-1} + \psi_2\varepsilon_{t-2} + \ldots$$

Where the matrix Ψ_s has the interpretation:

$$\Psi_s = \frac{\partial y_{t+s}}{\partial \varepsilon_t'}$$

And it identifies the consequences of a one-time impulse in the variable’s innovation at date $t\left(\varepsilon_{j_t}\right)$ for the value of the ith variable at time $t+s\left(y_{i,t+s}\right)$, holding all other innovations at all dates constant, describing the response of $y_{i,t+s}$. (p.318-320)

I have obtained the Impulse Response Function for the countries where the Relative PPP is not held, for both the USA and each country. Results will be compared with the ones obtained from the cointegration test to predict a possible long run relationship between the CPI series.

6. Data description

Data has been downloaded from the Federal Reserve Economic Data (FRED) from the Federal Reserve Bank of St. Louis. All data are consumer price indexes from the total items in each country, monthly organized, taking year 2015 as the base year (2015=100) and covering the period form January 1970 to January 2021.

[Insert Table 1]

In Table 1 it is possible to see a summary of the main descriptive statistics of all indexes studied. All data is expressed in logarithms, so the mean value represents the average discrepancy over this period between the CPI and its value in January 1970. First column shows the mean of each time series, and the second column shows the standard deviation, a measure of how dispersed data is in relation to the mean. Japan's standard deviation is the lowest among the rest of countries, meaning data is not very spread out, excepting Colombia's CPI which is the one that fluctuates the most and barely drives away from zero. Regarding south American countries, it is difficult to find long time series of data due to the peculiarities each country has, its history and economical situation. Depending on the policies applied, data collection has been a very challenging issue over these territories, managing director at Mercaplan Central America and Caribbean explains, "Traditionally, in home face-to-face has been the most-used data collection method" (Martin French. J, 2015). Consequently, it is hard to find data and therefore proceed to do studies or predictions for these countries.

Having explained the reasons above, it is understandable that Colombia's CPI does not show much fluctuation due to the lack of data collection, its standard deviation is barely zero whereas the rest of countries demonstrate to have a similar standard deviation. This drawback will also affect obtaining appropriate results for Colombia's currency on the upcoming tests made. Next two columns summarize the maximum and minimum values reached by each CPI during this period. It stands out that in all cases the difference between the maximum point reached and the minimum there is at least one point difference. Last two columns show the values of the 1st and 3rd quartiles, these measures divide the sample taken in equal parts, 1st quartile indicates that 25% of the observations takes a value lower or equal to this one, and same for the 3rd quartile for 75% of the observations. It stands out the values obtained for Colombia, following the explanation from before, 75% of the data takes values lower than 0. Similar case for Mexico where 75% will take values lower than 4.6. On the contrary, in the case of the USA most data are higher than 4.

[Insert Graph 1]

Graph 1 shows the consumer price indexes of each country, from year 1970 to 2021, taking year 2015 as base year, 2015=100. Following the explanations from before it is clearer to see that Colombia's CPI does not deviate much during the period covered, it practically stays constant near zero. The USA has the highest growth at consumer price index level, but in general the rest of countries demonstrate to have a positive increasing trend.

7. Results

First step is to analyze stationarity, I have implemented three different tests to check whether the differences between each CPI series with respect to the USA are stationary or non-stationary. I have run the ADF test including both a constant and a trend term[2], including trigonometric representation of deterministic seasonality and lag length not shorter than 12, to determine if they are $I(0)$ or $I(1)$ on zero frequency, and to deal with seasonality. Taking the USA dollar as the domestic currency and the rest of the countries as the foreign currencies. On the opposite side, I have also run the KPSS and PP tests all in levels and first difference to demonstrate the distorted conclusions reached whenever deterministic seasonality is neglected.

[Insert Table 2]

In Table 2 it is possible to see the results obtained from the ADF test that will determine if the CPI series are stationary. It is shown that null hypothesis for non-stationarity can be rejected for the cases of the USA and Japan at a 10%, 5% and 1% significance level, and for the UK at a 10%. The rest of the CPI series contain a unit root.

[Insert Table 3]

Table 3 shows results obtained from the KPSS and PP tests that have been done regarding the explanation done before. These tests do not take into consideration deterministic seasonality. The first line shows the decision taken on whether the Null hypothesis is rejected or not, the second line shows the lag order chosen, following AIC criteria to run the test[3], and last line shows the statistic values from the test, the p-value. Having that in mind, it is clear that the results obtained are noncoherent and consistent with the ADF test done before. According the KPSS all CPI series are non-stationary, and regarding the PP test after transforming them into first difference we obtain same results, and they all become stationary series. Consequently, the ADF test will be the only one providing coherent results and considering the trigonometric representation of deterministic seasonality, from now on all the tests will be done with the same procedure as the one used for the ADF test.

[Insert Table 4]

The results in Table 4 demonstrate if Relative hypothesis of the PPP holds. The ADF test has been conducted for the difference between each country's CPI and the USA, except for Colombia that from this moment on won't be taken into account due to the missing values that don't provide coherent and consistent results. It can be seen that only for the case of Japan and the UK, at a certain extent, the relative version of the PPP is accomplished. In the case of the UK

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[2] The model used is ADF with constant and trend: $\Delta y_t = \alpha + \gamma y_{t-1} + \lambda_t + u_t$

[3] An important issue is the number of lag selection to run the tests. There is a complication when choosing this number of lags. I have used RStudio to analyze all data and run Unit Root and Stationary tests, in RStudio there is an automatic lag selection when running the tests. I have followed the lag length based in the Akaike Schwarz criteria.

the t-statistic has a value of -3.20, null hypothesis is rejected at a 10% significance, therefore it is stationary, and it does not have unit roots. For the case of Japan, the t-statistic is -4.30 and null hypothesis is rejected at a 1% significance level. For the last two remaining countries the relative version does not hold, and I will study cointegration for these last two countries.

[Insert Table 5]

The previous results from the unit root test have provided the necessary information to determine the order of integration of each CPI difference with respect to the USA. The results seen in Table 5 show in which cases Cointegration test will be carried out. The UK and Japan have an order of integration *I(0)*, whereas the rest of countries all have an order *I(1).* For these last countries except for Colombia, Cointegration will be studied.

[Insert Graph 2]

In addition, I have obtained the autocorrelation and partial autocorrelation functions and I have complemented the results from the ADF test with the Spectral analysis. In Graph 2 it is represented: the difference between the USA's CPI, the spectrum on the right side, and below the differenced series to nullify the effect of zero frequency domination, for each country. The spectrum in all four cases represents the presence of seasonality in each series, this remark is essential to understand the need of including seasonal dummies in the conduction of the ADF test. The domination of the zero frequency is clear and its transformation after first difference shows a variation on its graphical representation too. The peaks in each spectral representation leads to the conclusion that there still is a seasonal component remaining.

The following step on my analysis is examining if the Absolute purchasing power parity hypothesis holds in the long run by studying Cointegration between the USA and the two countries that have same order of integration *I(1),* where the relative version is not held. I have conducted a cointegration test using the two-step Engle Granger method and the Johansen procedure to compare results. For the Absolute PPP hypothesis to hold first condition is that residuals from the equilibrium regression[4] must be stationary, for this reason Ljung-Box test[5] has been conducted.

[Insert Table 6]

The results obtained from this test shown in Table 6, demonstrate that all residuals estimated from the model are independently distributed, they do not exhibit serial correlation. Null hypothesis: that they are uncorrelated, is accepted in all cases at a 5% significance level. It is proven that the residuals follow the model of a white noise, they are uncorrelated variables. Regarding the results obtained from the ADF test, the countries where the relative version of the PPP is not held are Colombia, Mexico, and Germany, in order to compare results, Johansen procedure has been caried out.

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[4] In this case referred as the difference between each CPI series and the USA

[5] Ljung-Box Pierce test mathematical fundaments developed at the Appendix

[Insert Table 7]

I proceed to run Johansen cointegration test. In Table 7 both Null and Alternative hypotheses are displayed for the 2 different methods of the test, on the left side Trace test and on the right side Maximal Eigenvalue test.

[Insert Table 8]

On the first place, in Table 8 results from the Trace and Maximal Eigenvalue test for the case of Germany are shown. When $r = 0$, the t-statistic for the Trace test is 162.67 and this value is greater than all values at 10%, 5%, and 1% significance levels, so null hypothesis is rejected, and it can be concluded that there is at least one cointegrating relationship. Moving on to the second row that shows $r = 1$, the t-statistic is no longer greater than any values, so null hypothesis cannot be rejected. Same conclusion is reached with the Maximal Eigen value test, the t-statistic variates a little, it is 158.98. In conclusion, Germany has at least one cointegrating relationship with the USA.

Continuing with the case of Mexico, when $r = 0$ t-statistic for the Trace test is 129.44, null hypothesis is rejected at all significance levels, so there is at least one cointegrating relationship with the USA. Regarding the case when $r = 1$, null hypothesis can no longer be rejected. Same happens in the case of Maximal Eigenvalue test. In conclusion, results from the 2 different methods lead to the same reasoning, there is at least one cointegrating relationship between Germany and Mexico with the USA.

[Insert Graph 3]

In addition, I have also carried out one more analysis to support the conclusion reached with the cointegration test, the Impulse Response Function (IRF). The study of the IRF contributes to the understanding of the dynamic relationship between the different variables in a model and their response to an endogenous impact or shock given at a certain point in time. I have obtained the IRF setting a maximum lag of 2 and following the AIC information criteria. I have also set a 20-month limit for the functions to be determined. Graph 3 shows the different responses of CPI series from Mexico and Germany to one Standard Deviation shock (innovation) to the USA and the other way round for Mexico and Germany.

It can be seen that one S.D. shock to Germany initially increases USA's CPI series. This positive response can be seen from period 1 to 5 once there it hits its steady state value from where it remains constant from period 5 to 20. In the case of Mexico, when one S.D. shock affects Mexico, the response is quite similar. During the first 2 periods it increases, but the rise is further less than for the case of Germany and once in period 2 it remains constant for the rest of the period.

Turning now to the response given by Germany, when one S.D. shock to the USA, at first it causes a positive effect, but the dominant negative effect stands out during the rest of the period. Finally, a similar reasoning with the response given by Mexico. A standard shock affecting the USA will cause a negative response function from Mexico. It gradually declines and takes negative values throughout the whole period covered.

These findings obtained help to understand how the CPI's are likely to react in front of a shock to USA's CPI series in the short and long run. In both cases I have found that at least there is at least one possible cointegrating relationship that might explain why they react similar to the USA at the beginning, but both response functions gradually deviate from the one given by the USA in the long run.

8. Conclusions

The Theory of the PPP has generated a huge debate of study for the last decades. Its study has also improved with the help of new technologies, as there are new data sets that allow to do long term analysis of time series.
The purpose of this work is to study the validity of the different hypotheses of the Purchasing Power Parity. I have started dividing my study into two main analyses: checking for stationarity through Unit Root tests, and conducting Cointegration tests to determine if the CPI series have a long run relationship.

My study has started by conducting a Unit Root test explaining the importance of including seasonal deterministic components in the analysis, to validate the Relative version of the PPP. A specific function has been generated on RStudio to include the trigonometric representation of deterministic seasonality on the Augmented Dickey-Fuller test conducted. Results show that the hypothesis for Relative PPP is accepted only for the cases of Japan, and the UK, at a certain extent.

I have proceeded to test for Cointegration using the two different methods to check if the Absolute PPP is held for the cases where the Relative version does not. I have created a VAR model for each country to run both tests. Regarding the results obtained from the Johansen procedure, it is concluded that both Germany and Mexico have at least one cointegrating relationship with the USA.

Lastly, I have complemented the results collected by obtaining the Impulse Response Functions for Mexico, Germany, and USA. They demonstrate their behavior to a S.D. shock for a 20-month period. It can be confirmed that Germany does react in a similar way to the USA, whether Mexico deviates much more even though both had similar results on the Cointegration test.

Results have led me reach to the conclusion that Relative version of the PPP holds for countries Japan and the UK, and the Absolute version does hold for the case of Germany, but Mexico demonstrates to deviate from the USA. In general, it cannot be definitely confirmed that neither of the two versions are held for all territories studied, but there is evidence that each version is accepted at a certain extent in determined countries.

9. References

Alloza, M. (2014). *A very short note on computing impulse response functions*. https://bit.ly/37nCjhM

Banerjee, A., Galbraith, J. W., Dolado, J., & Hendry, D. (Eds.) (1993). Cointegration. En *Co-Integration, Error Correction And The Econometric Analysis Of Non-Stationary Data (Advanced Texts in Econometrics)* (1.st ed., pp. 135-158). Clarendon Press. https://doi.org/10.1093/0198288107.001.0001

Cassel, G. (1918). Abnormal Deviations in International Exchanges. *The Economic Journal*, *28*(112), 413-415. https://doi.org/10.2307/2223329

Demetrescu, M., & Hassler, U. (2007). Effect of neglected deterministic seasonality on unit root tests. *Statistical Papers*, *48*(3), 385–402. https://doi.org/10.1007/s00362-006-0343-6

Engle, R. F. (1976). Interpreting Spectral Analyses in Terms of Time-Domain Models. En S. V. Berg (Ed.), *Annals of Economic and Social Measurement, 5(1),* 89-109. NBER. https://bit.ly/3yt1hHA

Engle, R. F., & Granger, C. W. J. (1987). Co-Integration and Error Correction: Representation, Estimation, and Testing. *Econometrica*, *55*(2), 251–276. https://doi.org/10.2307/1913236

Ghysels, E., Lee, H. S., & Noh, J. (1994). Testing for unit roots in seasonal time series Some theoretical extensions and a Monte Carlo investigation. *Journal of Econometrics*, *62*(2), 415–442. https://doi.org/10.1016/0304-4076(94)90030-2

Ghysels, E., & Osborn, D. (2001). Deterministic Seasonality. In *The Econometric Analysis of Seasonal Time Series* ,19-41. Cambridge University Press. https://bit.ly/3FsKOVG

Granger, C. (1981). Some properties of time series data and their use in econometric model specification. *Journal of Econometrics*, *16*(1), 121–130. https://doi.org/10.1016/0304-4076(81)90079-8

Hamilton, J. D. (1994). Spectral Analysis. En *Time Series Analysis* (pp. 152–160). Princeton University Press.

Hamilton, J. D. (1994). Vector Autoregressions. En *Time Series Analysis* (pp. 318–335). Princeton University Press.

Hjalmarsson, E., & Österholm, P. (2007, junio). *Testing for Cointegration Using the Johansen Methodology when Variables are Near-Integrated*. International Monetary Fund. https://bit.ly/3KXDjXU

Isard, P. (1995). *Exchange Rate Economics (Cambridge Surveys of Economic Literature)*. Cambridge University Press. https://bit.ly/37o385t

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, *12* (2-3), 231-254. https://doi.org/10.1016/0165-1889(88)90041-3

Krugman, P. R., Obstfeld, M., & Melitz, M. J. (2017). *International Economics. Theory and Policy* (Eleventh Edition). Pearson.

Ng, S., & Perron, P. (1995). Unit Root Tests in ARMA Models with Data-Dependent Methods for the Selection of the Truncation Lag. *Journal of the American Statistical Association*, *90*(429), 268-281. https://doi.org/10.1080/01621459.1995.10476510

Österholm, P., & Hjalmarsson, E. (2007). Testing for Cointegration Using the Johansen Methodology When Variables Are Near-Integrated. *IMF Working Papers*, *07*(141), 1-6. https://doi.org/10.5089/9781451867053.001

Papell, D. H., & Prodan, R. (2003, November). *Long run Purchasing Power Parity: Cassel or Balassa-Samuelson?* University of Houston. https://bit.ly/3kSrveR

Pardoe, D. I., Simon, D. L., & Young, D. D. (2018). *10.2 - Autocorrelation and Time Series Methods* | *STAT 462*. STAT 462! Applied Regression Analysis. Retrieved from January 24th 2022, from https://bit.ly/3MOKzGO

Pfaff, B. (2008). Multivariate Analysis of Stationary Time Series. *Analysis of Integrated and Cointegrated Time Series with R*, 23-48. Springer Publishing. https://bit.ly/3P5OhxF

Pollock, D. S. G. (2009b). *STATISTICAL FOURIER ANALYSIS*. https://bit.ly/3kZaq2G

Prins, J. (2012, April). *6.4.4.8.1. Box-Ljung Test*. Engineering Statistics Handbook. https://bit.ly/3LZQCs1

R. Pakko, M., & S. Pollard, P. (2003, December). Burgernomics: A Big Mac^{TM} Guide to Purchasing Power Parity. *Economic Research Federal Reserve Bank of St.Louis*, *85*(6). Retrieved from November 1st 2021, from https://bit.ly/38a6lWN

Taylor, A. M., & Taylor, M. P. (2004). The Purchasing Power Parity Debate. *Journal of Economic Perspectives*, *18*(4), 135-158. https://doi.org/10.1257/0895330042632744

The Economist. (2017, January 14). *Our Big Mac index of global currencies reflects the dollar’s strength*. https://econ.st/3l7vC73

The Economist. (2022, February 2). *The Big Mac Index*. https://econ.st/39Kx6l8

Zivot, E. (2013, April 22). *Trend-Cycle Decompositions and Unit Root Tests*. University of Washington. https://bit.ly/3P2vRxH

Zivot, E., & Wang, J. (2003). Unit Root tests. In *Modeling Financial Time Series with S-PLUS* (pp. 105–127). Springer Publishing, New York, NY. https://doi.org/10.1007/978-0-387-21763-5_4

10. Bibliography

Arltová, M., & Fedorová, D. (2016, September). *Selection of Unit Root Test on the Basis of Length of the Time Series and Value of AR(1) Parameter* (Nº3). Statistika: Statistics and Economy Journal. https://bit.ly/3vSiA3i

Dotis-Georgiou, A. (2019, May 6). *Autocorrelation in Time Series Data*. Dzone.Com. Retrieved from December 15th 2021, from https://bit.ly/38cR2wv

González Gómez, D. (2001). Análisis Espectral: Consideraciones teóricas y Aplicabilidad. *Economía & Sociedad*, *6*(16), 45-60. https://bit.ly/38fNYQ9

Lorduy Herrera, O. (2014). La paridad del poder adquisitivo: nueva evidencia para Colombia y Latinoamérica. *Revista CIFE: Lecturas de Economía Social*, *16*(25), 123-152. https://doi.org/10.15332/s2248.4914.2014.0025.04

Loukopoulos, G., & Antonopoulos, D. (2015). Purchasing Power Parity: A Unit Root, Cointegration and VAR Analysis in Emerging and Advanced Countries. *Business and Economic Research*, *5*(1), 262–279. https://doi.org/10.5296/ber.v5i1.7177

Morales Juárez, R. (2005, August). *Paridad de Poder de Compra y Ley del Unico Precio en 34 ciudades de México (1988–2004)*. https://bit.ly/3wvlcU3

Morin, D. (2009). Fourier Analysis. In *Waves (Draft)* (Vol. 1, pp. 1-43). Harvard University.

Österholm, P., & Hjalmarsson, E. (2007). Testing for Cointegration Using the Johansen Methodology When Variables Are Near-Integrated. *IMF Working Papers*, *07*(141), 1-6. https://doi.org/10.5089/9781451867053.001

Papell, D. H., & Prodan, R. (2003, November). *Long run Purchasing Power Parity: Cassel or Balassa-Samuelson?* University of Houston. https://bit.ly/3vSwN0g

Pardoe, D. I., Simon, D. L., & Young, D. D. (2018). *10.2 - Autocorrelation and Time Series Methods | STAT 462*. STAT 462! Applied Regression Analysis. Retrieved from January 24th 2022, from https://bit.ly/3P4e2hR

Peacock, J. A. (2013). *Fourier Analysis* (Session: 2013/14). The University of Edinburgh. https://bit.ly/3sqaGfv

Pizarro Rios, J. V. (1992, July). Contrastes de cointegración sobre la paridad del poder de compra: una aplicación a los datos de la economía peruana. *PUCP ECONOMIA*, *15*(29). https://bit.ly/37nFLJg

Pollock, D. S. G. (2008, October). *Statistical Fourier Analysis: Clarifications and Interpretations* (N.° 08/36). University of Leicester. https://doi.org/10.2202/1941-1928.1004

Pollock, D. S. G. (2009a). *BIVARIATE SPECTRAL ANALYSIS*. https://bit.ly/38ptc0x

The Purchasing Power Parity Debate (Vol. 18, 4.ed.). (2004). https://bit.ly/3900CTE

Ranjan Behera, S. (2019, March 1). *Purchasing Power Parity tests in cointegrated panels: Evidence from newly industrialized countries*. *Journal of Economic Development*. 44(1):69-76 https://bit.ly/3vPbCft

Rayner, J. (2001). Spectral Analysis. *International Encyclopedia of the Social & Behavioral Sciences*, 14861–14864. https://doi.org/10.1016/b0-08-043076-7/02514-6

Rodríguez-Benavides, D., Climent-Hernández, J. A., & Hoyos-Reyes, L. F. (2018). Purchasing Power Parity Principle in Latin American Countries. *Revista Mexicana de Economía y Finanzas*, *13*(3), 461-477. https://doi.org/10.21919/remef.v13i3.330

Salcedo Muñoz, V. (2020). Teoría de la Paridad de Poder Adquisitivo (PPA): Aportes de Gustav Cassel sobre el tipo de cambio de equilibrio. *Revista Venezolana de Gerencia*, 25(92):1837-1849. https://bit.ly/3sipQT

11. Tables

Table 1

Descriptive Statistics. Consumer Price Indexes

Country	*Mean*	*Standard Deviation*	*Maximum*	*Minimum*	*1st Qu*	*3rd Qu*
USA	4,878	0,555	5,569	3,635	4,582	5,359
Japan	4,628	0,303	4,628	3,429	4,394	4,584
UK	3,920	0,717	4,695	2,096	3,620	4,450
Mexico	1,604	3,271	4,834	-4.269	-1,833	4,323
Colombia	-Inf	NA	2,207	-Inf	-1,400	0,582
Germany	4,211	0,354	4,669	3,377	4,011	4,669

Table 2

ADF test including trend and drift

Critical values for test statistic	***1%***	***5%***	***10%***
τ_3	-3.96	-3.41	-3.12
Country	**Value of t-ratio**		
USA	-4.559		
UK	-3.063		
Japan	-4.059		
Mexico	-0.960		
Germany	-1.895		
Colombia	NA		

Table 3

Unit Root and Stationary tests

- *Phillips-Perron test (PP) in levels and first differences*

	USA	*UK*	*Japan*	*Mexico*	*Colombia*	*Germany*
Null Hypothesis	Not Reject	Not Reject	Not Reject	Not Reject	Reject	Not Reject
Lag order	6	6	6	6		6
p-value	0.981	0.957	0.865	0.99		0.927

	USA	*UK*	*Japan*	*Mexico*	*Colombia*	*Germany*
Null Hypothesis	Reject	Reject	Reject	Reject	Reject	Reject
Lag order	6	6	6	6		6
p-value	0.01	0.01	0.01	0.01		0.01

- *Kwiatkowski-Phillips- Schmidt-Shin test (KPSS) in levels and first differences*

	USA	*UK*	*Japan*	*Mexico*	*Colombia*	*Germany*
Null Hypothesis	Reject	Reject	Reject	Reject	Reject	Reject
Lag	6	6	6	6		6
p-value	0.01	0.01	0.01	0.01		0.01

	USA	*UK*	*Japan*	*Mexico*	*Colombia*	*Germany*
Null Hypothesis	Reject	Reject	Reject	Reject	Reject	Reject
Lag	6	6	6	6		6
p-value	0.01	0.01	0.01	0.01		0.01

Table 4

ADF test including trigonometric representation of deterministic seasonality for differenced series with the USA

Critical values for test statistic	***1%***	***5%***	***10%***
τ_3	-3.96	-3.41	-3.12
Country	**Value of test-statistic**		
UK	-3.200		
Japan	-4.306		
Mexico	-1.284		
Germany	-1.940		

Table 5

Order of Integration I(d) - based on the ADF test results

Countries	*I (d)*
UK	I (0)
Japan	I (0)
Mexico	I (1)
Colombia	I (1)
Germany	I (1)

Table 6

Ljung-Box test

- *On the residuals including constant and trend*

Df lags	USA-GERMANY		USA-MEXICO	
	X-squ	*p-value*	*X-squ*	*p-value*
Lag 1	0.247	0.618	0.308	0.578
Lag 2	0.298	0.861	0.580	0.748
Lag 3	0.301	0.959	0.922	0.820
Lag 4	0.306	0.989	0.992	0.911
Lag 5	0.485	0.992	1.107	0.953
Lag 6	0.486	0.998	1.170	0.978
Lag 7	0.498	0.999	1.214	0.990
Lag 8	0.924	0.998	1.226	0.996
Lag 9	1.093	0.999	1.656	0.995
Lag 10	1.126	0.999	1.667	0.998
Lag 11	1.497	0.999	2.973	0.991
Lag 12	3.616	0.989	5.047	0.956

- *On the residuals including trigonometric representation of deterministic seasonality*

Df lags	USA-GERMANY		USA-MEXICO	
	X-squ	*p-value*	*X-squ*	*p-value*
Lag 1	0.147	0.701	0.106	0.744
Lag 2	0.234	0.889	0.171	0.917
Lag 3	0.252	0.968	0.209	0.976
Lag 4	0.252	0.992	0.210	0.994
Lag 5	0.340	0.996	0.215	0.998
Lag 6	0.417	0.998	0.238	0.999
Lag 7	0.417	0.999	0.239	1
Lag 8	0.485	0.999	0.239	1
Lag 9	0.525	1	0.601	0.999
Lag 10	0.531	1	0.676	1
Lag 11	0.913	1	1.510	0.999
Lag 12	1.404	0.999	2.134	0.999

Table 7

Null and Alternative Hypotheses for Johansen procedure

Trace test		Eigenvalue test	
H_0	H_1	H_0	H_1
r=0	r >0	r=0	r=1
r=1	r >1	r=1	r=2

Table 8

Trace and Maximal Eigenvalue test statistics and critical values

Germany Trace test statistic and critical values

	Test statistic	*10%*	*5%*	*1%*
r=1	3.69	7.52	9.24	12.97
r=0	162.67	17.85	19.96	24.60

Germany Maximal Eigenvalue test statistic and critical values

	Test statistic	*10%*	*5%*	*1%*
r=1	3.69	7.52	9.24	12.97
r=0	158.98	13.75	15.67	20.20

Mexico Trace test statistic and critical values

	Test statistic	*10%*	*5%*	*1%*
r=1	2.84	7.52	9.24	12.97
r=0	129.44	17.85	19.96	24.60

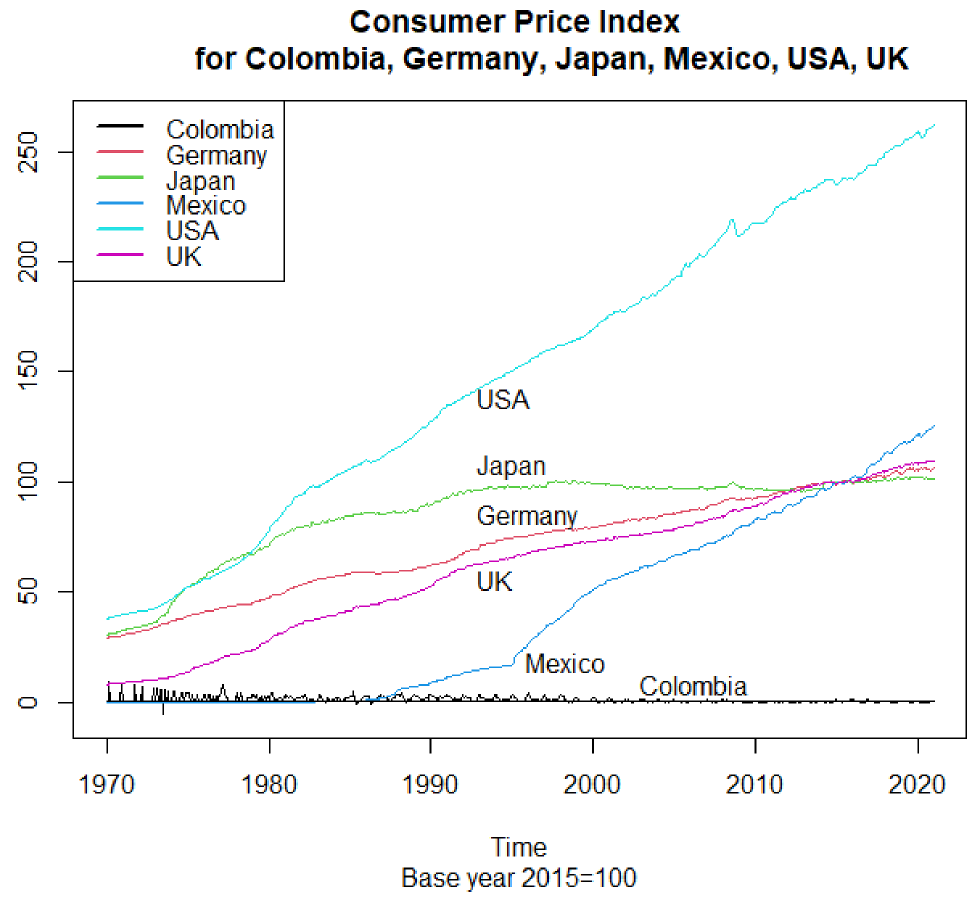
Mexico Maximal Eigenvalue test statistic and critical values

	Test statistic	*10%*	*5%*	*1%*
r=1	2.84	7.52	9.24	12.97
r=0	126.60	13.75	15.67	20.20

12. Graphs

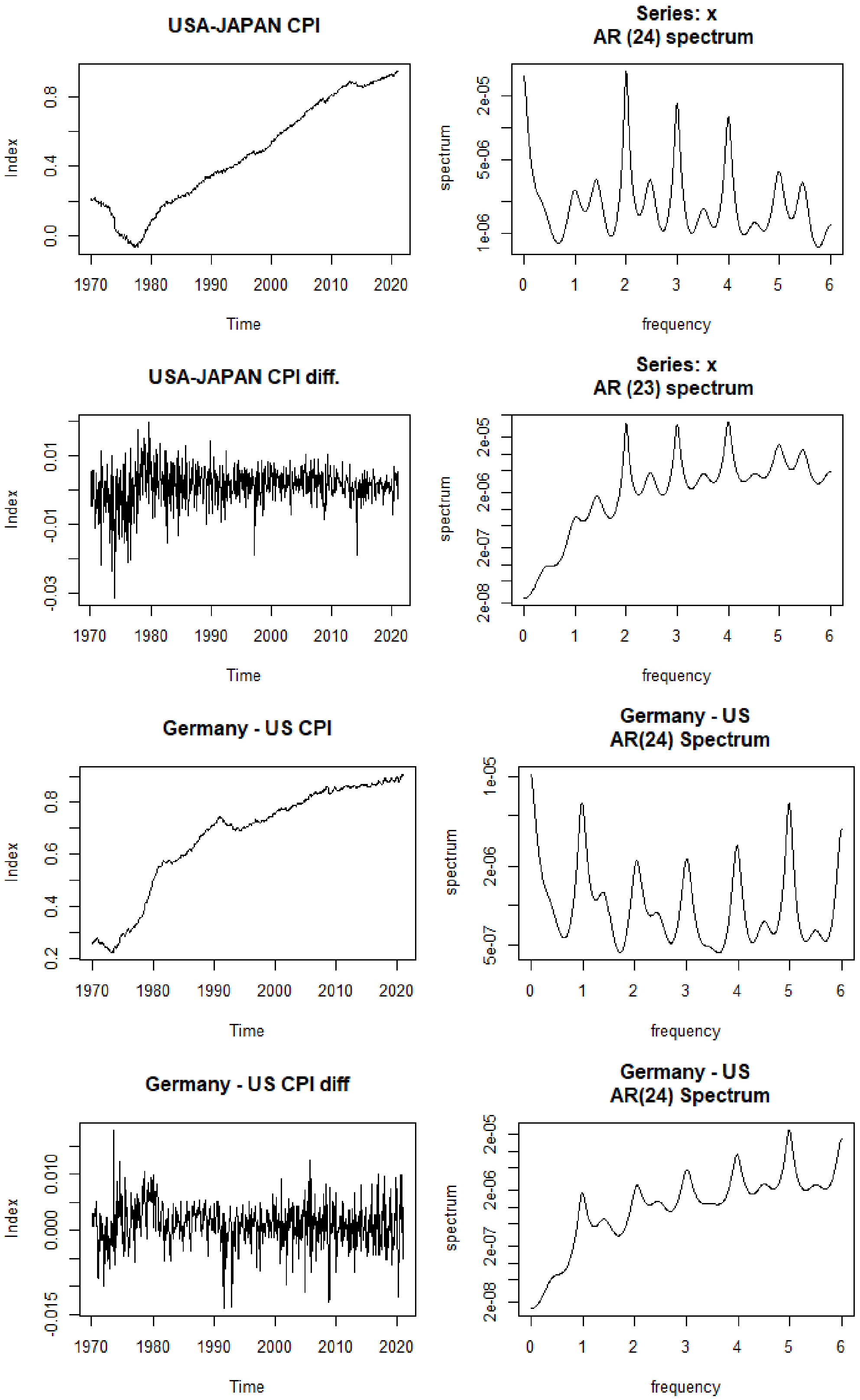
Graph 1

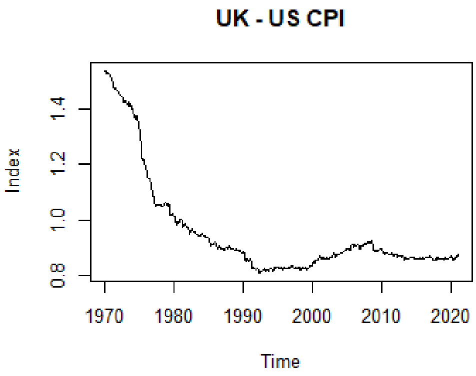
Consumer Price Indexes

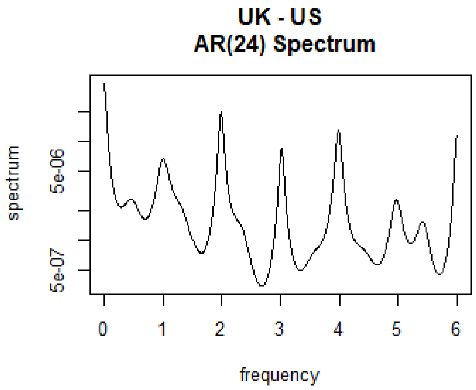


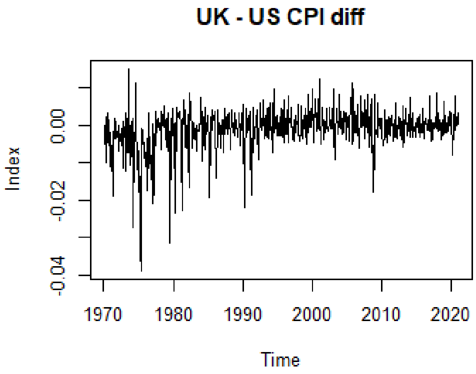
Graph 2

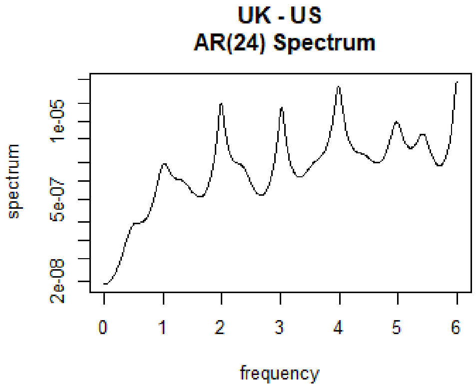
Spectrum for each country's CPI, and the USA, and differenced series

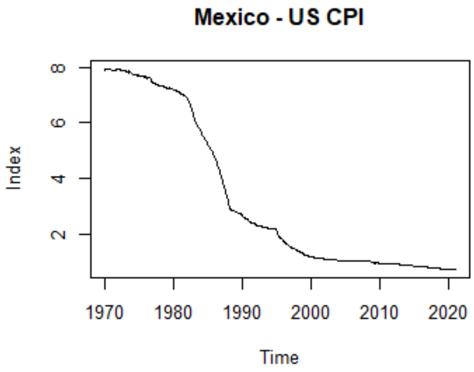


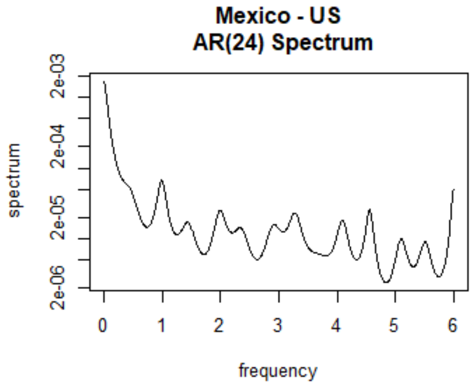

UK - US CPI
Index
0.8
1.0
1.2
1.4
1970
1980
1990
2000
2010
2020
Time

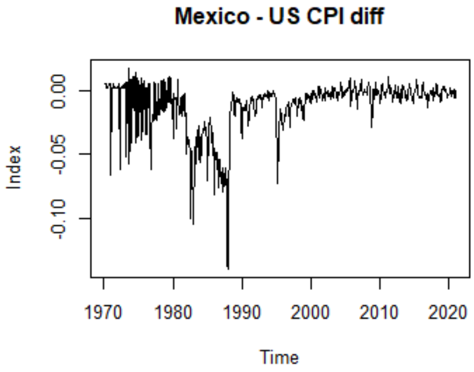

UK - US
AR(24) Spectrum
spectrum
5e-07
5e-06
0
1
2
3
4
5
6
frequency

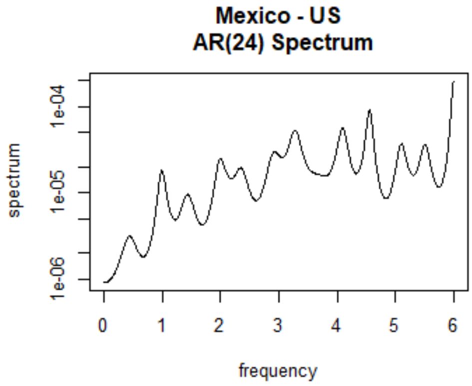

UK - US CPI diff
Index
-0.04
-0.02
0.00
1970
1980
1990
2000
2010
2020
Time


UK - US
AR(24) Spectrum
spectrum
2e-08
5e-07
1e-05
0
1
2
3
4
5
6
frequency


Mexico - US CPI
Index
2
4
6
8
1970
1980
1990
2000
2010
2020
Time

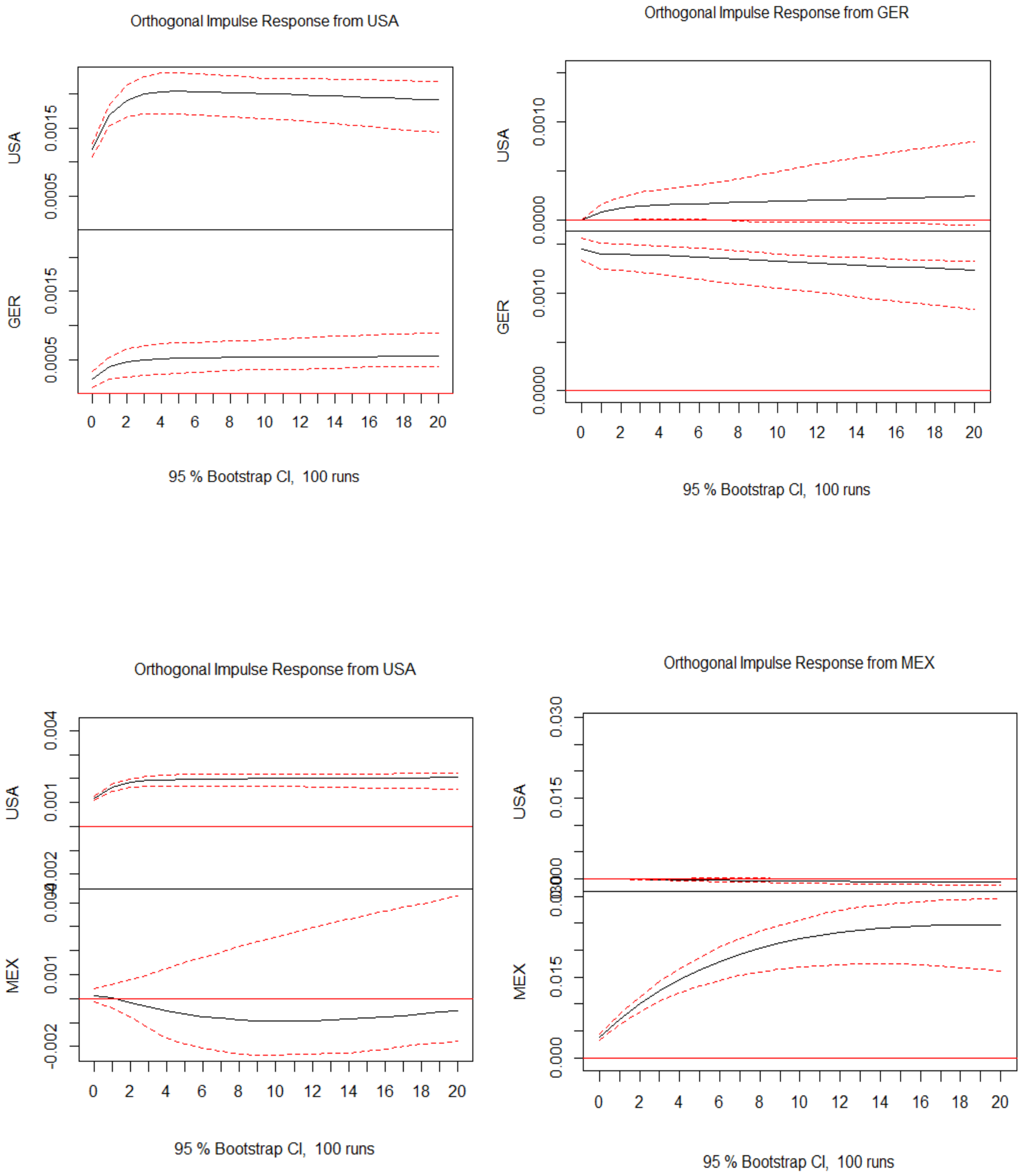

Mexico - US
AR(24) Spectrum
spectrum
2e-06
2e-05
2e-04
2e-03
0
1
2
3
4
5
6
frequency


Mexico - US CPI diff
Index
-0.10
-0.05
0.00
1970
1980
1990
2000
2010
2020
Time


Mexico - US
AR(24) Spectrum
spectrum
1e-06
1e-05
1e-04
0
1
2
3
4
5
6
frequency

Graph 3

Impulse Response Functions (IRF) for Mexico, Germany, and USA



13. RCode

13.1 *ADF test with constant and trend*

```r
# Function to run ADF test with constant and trend
# dat: series to analyze
# k:  number of lags used
adf_ct <- function(dat,k) {
  dx<-dat[2:nrow(dat),]-dat[1:(nrow(dat)-1),]
  dx<-matrix(data=dx,nrow=(nrow(dat)-1),ncol=1)
  end1 <- dx[(k+1):nrow(dx)]
  end1<-matrix(data=end1,nrow=(nrow(dx)-k),ncol=1)
  exo1 <- cbind(dat[(k+1):(nrow(dat)-1)],dx[k:(nrow(dx)-1),])
  for (j in 2:k){
    exo1 <- cbind(exo1,dx[(k+1-j):(nrow(dx)-j)])
  }
  exo1 <- cbind(exo1,array(1,nrow(end1)),seq(1,nrow(end1)))
  inver <- solve(t(exo1)%*%exo1)
  beta <- inver%*%t(exo1)%*%end1
  res <- end1-exo1%*%beta
  s2u <- (t(res)%*%res)/(nrow(end1)-ncol(exo1))
  t_rat=beta[1]/((s2u*inver[1,1])^0.5)

  t_rat_1=array(t_rat,nrow(res))
  results<-cbind(t_rat_1,res)
  return(results)}

# results are given in a Tx2 matrix
# T: number of observations of the residuals
# First column shows t-ratio repeated T times
# Second column shows residuals
```

13.2 *ADF test including trigonometric representation of deterministic seasonality*

```r
# Function to run ADF test including trigonometric representation of
# deterministic seasonality
adf_ct_seas <- function(dat,k) {
  dx<-dat[2:nrow(dat),]-dat[1:(nrow(dat)-1),]
  dx<-matrix(data=dx,nrow=(nrow(dat)-1),ncol=1)
  end1 <- dx[(k+1):nrow(dx)]
  end1<-matrix(data=end1,nrow=(nrow(dx)-k),ncol=1)
  exo1 <- cbind(dat[(k+1):(nrow(dat)-1)],dx[k:(nrow(dx)-1),])
  for (j in 2:k){
    exo1 <- cbind(exo1,dx[(k+1-j):(nrow(dx)-j)])
  }
  t=seq(1,nrow(end1))
  det_seas=cbind(cos(0*t),cos(pi*t))
  for (j in 1:5) {det_seas=cbind(det_seas,cos(2*pi*t*j/12),sin(2*pi*t*j/12))}

  exo1 <- cbind(exo1,det_seas,seq(1,nrow(end1)))
  inver <- solve(t(exo1)%*%exo1)
  beta <- inver%*%t(exo1)%*%end1
  res <- end1-exo1%*%beta
  s2u <- (t(res)%*%res)/(nrow(end1)-ncol(exo1))
  t_rat=beta[1]/((s2u*inver[1,1])^0.5)

  t_rat_1=array(t_rat,nrow(res))
  results<-cbind(t_rat_1,res)
  return(results)}
```

14. Appendix

14.1 Autocorrelation and Partial Autocorrelation functions (ACF and PACF)

In addition to the econometric tests explained before I have also studied both the Autocorrelation and Partial Autocorrelation functions generated by the difference between each country's CPI series and the US. Theoretically, the coefficient of autocorrelation between two values in a time series is called the autocorrelation function (ACF), and it represents the degree of similarity between them. The ACF allows to measure the linear relationship between an observation made at time t and previous observations in time, and the degree of linear association existing. Mathematically, the ACF for a time series y_t is the following:

$$\rho_j = corr\left(X_j, X_{j-k}\right) = \frac{\operatorname{cov}\left(x_j, x_{j-k}\right)}{\sqrt{v\left(x_j\right)}\sqrt{v\left(x_{j-k}\right)}}$$

Where k represents the time gap considered, also known as lag. If we take $k=1$, then it represents the correlation between values one period of time apart. (Pardoe, D. I., Simon, D. L., & Young, D. D., 2018, 10.2 Autocorrelation and Time Series Methods)

In this study, as I am working with monthly data, the number of lags chosen to deal with autocorrelation must be multiple of 12, for all cases the number of lags chosen is 36. Having obtained the respective results, graphically it is claimed that each series now is stationary. Moreover, it is possible to transform time series taking out the linear influence of the random variables that are in between of the studied ones, and therefore obtain the Partial Autocorrelation Function (PACF).

The PACF helps to identify the order of an autoregressive model and determine the autocorrelation between two variables separated by k number of periods when the dependence generated by the lags in between them is not taken into consideration. (Pardoe, D. I., Simon, D. L., & Young, D. D., 2018, 10.2 Autocorrelation and Time Series Methods). It is defined as it follows:

$$\Pi_j = corr\left(X_j, X_{j-k} / X_{j-1} X_{j-2} \ldots X_{j-k+1}\right)$$

$$\Pi_j = \frac{cov(X_j - \hat{X}_j, X_{j-k} - \hat{X}_{j-k})}{\sqrt{v(X_j - \hat{X}_j)}\sqrt{v\left(x_{j-k} - \hat{x}_{j-14}\right)}}$$

14.2 Ljung-Box test

The Ljung-Box test is a statistical test of whether a group of autocorrelation coefficients of a time series are different from zero, therefore they are independent. It tests randomness based on the number of lags chosen. If the observations are not independent one can be correlated with another one, that "k" units of time after will be called autocorrelation. As it is explained by Prins, J. (2012), the hypotheses are:

H0: the residuals are independently distributed
H1: the residuals are not independently distributed; they exhibit serial correlation

The test statistic is defined as:

$$Q = n \cdot (n+2) \cdot \sum_{k=1}^{m} \left(\frac{\hat{p}_k^2}{n-k} \right) \sim X^2(m)$$

Where n is the sample size, $\hat{\rho}_k$ the sample autocorrelation at lag k and m the length of the lags used in the test. (6.4.4.8.1. Box-Ljung Test)

The test statistic follows a chi-square distribution, the critical region for rejection of the null hypothesis is:

$$Q > X^2_{1-\alpha,h}$$

Where h is the degrees of freedom and $X^2_{1-\propto,h}$ is the $(1-\alpha)$ quantile of the chi-square distribution. (6.4.4.8.1. Box-Ljung Test)

14.3 Unit Root test: Phillips-Perron test (PP)

Both the ADF and PP tests are asymptotically equivalent but might differ in certain samples because of the different procedures on how they deal with serial correlation and heteroskedasticity in the errors of the test regression. Due to the problem of lag selection in the regression model explained in the previous part, Phillips and Perron (1998) handled this problem using the Dickey-Fuller test with non-parametrically modified test statistics. Following del Barrio Castro, T.(n.d.), the test regression is:

$$\varDelta Y = \beta' D_t + \varPi Y_{t-1} + u_t \qquad u_t \sim I(0)$$

The PP test is intended to correct serial correlation and heteroskedasticity in the errors of the regression by modifying the test statistics, these are given by:

$$Z_t = \left(\frac{\hat{\sigma}^2}{\hat{\lambda}^2}\right)^{\frac{1}{2}} \cdot t_{\varPi=1} - \frac{1}{2}\left(\frac{\hat{\lambda}^2 - \hat{\sigma}^2}{\hat{\lambda}^2}\right) \cdot \left(\frac{T \cdot SE(\hat{\varPi})}{\hat{\sigma}2}\right)$$

$$z_{\varPi} = T_{\hat{\pi}} - \frac{1}{2} \cdot \frac{T^2 \cdot SE(\pi)}{\hat{\sigma}^2}\left(\hat{\lambda}^2 - \hat{\sigma} z\right)$$

Terms $\hat{\sigma}^2$ and $\hat{\lambda}^2$ are consistent estimates of the variance parameters:

$$\sigma^2 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} E[u_t^2]$$

$$\lambda^2 = \lim_{T\to\infty} \sum_{t=1}^{T} E[T^{-1} S_T^2]$$

Where $S_T = \sum_{t=1}^{T} u_t$

Sample variance of the least squares residuals $\hat{u}_t$ is a consistent estimate of $\hat{\sigma}^2$, and the Newey-West long run variance estimate of u_t using $\hat{u}_t$ is a consistent estimate of λ^2. The hypotheses remain the same being:
H0: $\varPi = 0$, consequently, the PP Z_t and $Z_{\varPi}$ statistics have the same asymptotic distributions as the ADF t-statistic and normalized bias statistics. (del Barrio Castro, T., n.d. Part 2. Phillips-Perron Unit Root Tests)

One benefit this test offers in front of the ADF is that there is no need to specify the number of lags to test the regression and that it is robust to general forms of heteroskedasticity in the error term.

14.4 Stationary test: Kwiatkowski-Phillips- Schmidt-Shin test (KPSS)

The difference with this test is that the null hypothesis shows the time series is $I(0)$, a stationary process. Kwiatkowski, Phillips, Schmidt and Shin (1992) proposed an LM test for testing trend and level stationarity. The test is derived by starting with the following model del Barrio Castro, T.(n.d.):

$$y_t = \beta' D_t + \mu_t + u_t \qquad u_t \sim I(0)$$

$$\mu t = \mu t-1 + \varepsilon t, \qquad \varepsilon t \sim WN(0, \sigma 2\, \varepsilon)$$

> Where D_t is a deterministic component, either constant or deterministic trend, the error term might be heteroskedastic and the component μ_t is a pure random walk with variance σ_ε^2. In this test the null hypothesis is defined as follows:
>
> H0: $\sigma_\varepsilon^2 = 0$, implying that μ is a constant, then y_t is a stationary process, $I(0)$
> H1: $\sigma_\varepsilon^2 > 0$, implying there is a unit root and $y_t \sim I(1)$
>
> The KPSS test statistic is calculated as the Lagrange Multiplier (LM) for testing the null hypothesis, this statistic is given by:
>
> $$\mathrm{KPSS} = \left(T^{-2}\sum_{t=1}^{T}\hat{S}_t^2\right)/\hat{\lambda}^2$$
>
> Where $\hat{S}_t = \sum_{j=1}^{t} \hat{u}_j{}'\hat{u}_t$ is the residual of a regression of y_t on D_t and $\hat{\lambda}^2$ is an estimate of the long run variance of u_t using $\hat{u}_t$. (del Barrio Castro, T., n.d. Part 4. Stationary Tests)

del Barrio Castro, T.(n.d.) explains that:

> KPSS statistic converges to a function of standard Brownian motion that depends on the form of the deterministic terms D_t.
>
> So for example, if $D_t = 1$, then:
>
> $$\mathrm{KPSS} \rightarrow \int_0^1 V_1(r)\,dr$$
>
> Where $V_1(r) = W(r) - rW(1)$, called as a standard Brownian bridge for $r \in [0,1]$.