MIXED STEADY CONVECTION AT A HORIZONTAL AXISYMMETRIC STAGNATION POINT

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Summary

We consider the mixed convective flow at an axisymmetric stagnation point on a heated horizontal boundary. The forced flow is moderated by a free-convective element whose origin is a pressure gradient induced by temperature variations along the boundary. The main aim of the investigation is to identify situations in which a steady flow is maintained.

1. Introduction

The concept of horizontal free convection was introduced by Stewartson (**1**) and Gill *et al.* (**2**) for a flow over a heated horizontal semi-infinite plate due to an induced pressure gradient. Amin and Riley (**3**) also demonstrated that on an infinite horizontal plane boundary, a varying wall temperature would induce a pressure gradient. This was illustrated by a quadratic wall temperature, symmetric about a stagnation line, which proved to be a line of attachment when the temperature increased away from it, and a line of separation when the temperature increased towards it. No steady flow is possible in the latter case. An extension of this work by the same authors (**4**) introduced the classical two-dimensional stagnation point flow of either attachment or separation. They were apparently able to find, in all cases, a steady flow for sufficiently large values of the temperature gradient at the boundary. In this paper, we extend the earlier work both to axisymmetric flow and to asymptotically large and small values of the Prandtl number.

Competitive free and forced convective flows have been studied by other authors. For the classical free-convective flow past a vertical semi-infinite flat plate, Merkin (**5**) introduces a uniform free stream. For the case in which buoyancy aids the motion, the velocity increases along the plate but when buoyancy opposes the motion the flow eventually separates. By contrast, Daniels (**6**) places a thermally insulated semi-infinite flat plate horizontally in a thermally stratified onset flow that results in a horizontal pressure gradient. In certain circumstances, this can lead to a singular breakdown of the solution of the boundary-layer equations at a finite distance from the leading edge of the plate.

The present paper is organized as follows. In the next section, we introduce the boundary-layer equations based upon scalings associated with a suitably defined large Grashof number. From these equations, we can determine the pressure gradient that demonstrates the interplay between the forced convective flow and the thermally induced pressure gradient. We restrict our attention to the case of a surface temperature that decreases radially from the stagnation point. In the following

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section, we consider solutions of these equations; in the absence of an outer flow, no steady solution is possible as the flow converges upon and erupts from the stagnation point. When classical forced convection is introduced, and its 'strength' exceeds a certain critical value, steady solutions are possible and we determine this critical value for a range of values of the Prandtl number. For a stagnation point flow of separation, as for example, the rear stagnation point of a sphere, Howarth (**7**) shows that there is no steady solution. In our situation, one might suppose that a sufficiently large positive temperature gradient would suppress the eruption of flow from the rear stagnation point as outlined by Howarth. But this proves not to be the case and is at variance with the two-dimensional analogue. We have supplemented the numerical solution of our equations with analyses for asymptotically small and large values of the Prandtl number. Principle amongst the results we have obtained is the role of the induced pressure gradient and its variation with the Prandtl number.

2. Problem formulation

We consider the steady flow at an axisymmetric stagnation point on a horizontal plane boundary which is maintained at a temperature T_w that is different from the ambient T_∞ with $T_\infty < T_w$. In the absence of any external flow, it has been shown (**4**), for two-dimensional flow, that variations in the surface temperature induce a horizontal pressure gradient that in turn results in a flow along the horizontal surface. The effect of an outer stagnation flow, which may counter or augment this pressure gradient, has been considered in (**4**). In this paper, we consider the equivalent scenario at an axisymmetric stagnation point. The flow is characterized by the Grashof number, Gr, which we assume to be large, where

$$\mathrm{Gr} = \beta g a^3 (T_0 - T_\infty)/\nu^2. \tag{2.1}$$

In this definition β is the coefficient of thermal expansion ($= T_\infty^{-1}$ for a perfect gas), g is the acceleration due to gravity, ν is the kinematic viscosity, $T_0 = T_w(0)$ is a reference temperature which we take as the temperature of the boundary at the stagnation point itself and a is a length associated with the length scale of variations in temperature at the boundary. For $\mathrm{Gr} \gg 1$, and introducing the Boussinesq approximation, the Navier–Stokes equations reduce to their boundary-layer form

$$\frac{1}{\tilde{r}}\frac{\partial(\tilde{r}\tilde{u})}{\partial \tilde{r}} + \frac{\partial \tilde{v}}{\partial \tilde{z}} = 0, \tag{2.2}$$

$$\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{r}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{z}} = -\frac{\partial \tilde{p}}{\partial \tilde{r}} + \frac{\partial^2 \tilde{u}}{\partial \tilde{z}^2}, \tag{2.3}$$

$$\frac{\partial \tilde{p}}{\partial \tilde{z}} = \tilde{\theta}, \tag{2.4}$$

$$\tilde{u}\frac{\partial \tilde{\theta}}{\partial \tilde{r}} + \tilde{v}\frac{\partial \tilde{\theta}}{\partial \tilde{z}} = \frac{1}{\sigma}\frac{\partial^2 \tilde{\theta}}{\partial \tilde{z}^2}. \tag{2.5}$$

In these equations $a\tilde{r}$ measures radial distances along the boundary and $a\mathrm{Gr}^{-\frac{1}{5}}\tilde{z}$ normal to it. The velocity components parallel and normal to the surface are $\nu a^{-1}\mathrm{Gr}^{\frac{2}{5}}\tilde{u}$ and $\nu a^{-1}\mathrm{Gr}^{\frac{1}{5}}\tilde{v}$. The pressure is $\rho\nu^2 a^{-2}\mathrm{Gr}^{\frac{4}{5}}\tilde{p}$, where ρ is the fluid density, the temperature is $T_\infty + (T_0 - T_\infty)\tilde{\theta}$ and $\sigma = \nu/\kappa$, with κ the thermal diffusivity, is the Prandtl number.

The boundary conditions for equations (2.2)–(2.5) are

$$\tilde{u} = \tilde{v} = 0, \quad \tilde{\theta} = \tilde{\theta}_w(\tilde{r}) \quad \text{at } \tilde{z} = 0, \tag{2.6}$$

and

$$\tilde{u} \to \tilde{u}_e(\tilde{r}), \quad \tilde{\theta} \to 0 \quad \text{as } \tilde{z} \to \infty, \tag{2.7}$$

where $\tilde{\theta}_w$ and $\tilde{u}_e$ are prescribed quantities with $\tilde{\theta}_w(0) = 1$. We note at this point that if $d\tilde{\theta}_w/d\tilde{r} < 0$, the induced pressure gradient is driving fluid towards the stagnation point, while for positive values fluid is driven away from it.

In the context of our stagnation-point flow with $\tilde{u}_e = \tilde{\lambda}\tilde{r}$, it is convenient to assume that

$$\tilde{\theta}_w = 1 - b\tilde{r}^2. \tag{2.8}$$

For reasons we shall outline below we are concerned for the most part with $b > 0$. The forms of $\tilde{u}_e$, $\tilde{\theta}_w$ lead us to seek a solution of the form

$$\tilde{u} = \tilde{r}\tilde{u}_0(\tilde{z}), \quad \tilde{v} = \tilde{v}_0(\tilde{z}), \quad \tilde{\theta} = \tilde{\theta}_0(\tilde{z}) + \tilde{r}^2\tilde{\theta}_1(\tilde{z}), \quad \tilde{p} = \tilde{p}_0(\tilde{z}) + \tilde{r}^2\tilde{p}_1(\tilde{z}). \tag{2.9}$$

The equation for $\tilde{\theta}_0$ has a solution only for values of $\lambda > 0$, which is a requirement anyway as we see later, and can be determined once $\tilde{v}_0$ has been found; the solution for $\tilde{p}_0$ follows. Neither of these quantities has any dynamical consequence. We see from (2.5) and (2.8) that the problem under consideration is characterized by the three parameters b, $\tilde{\lambda}$ and σ. The first of these can be eliminated if we write

$$\tilde{u}_0 = b^{\frac{2}{5}}u_0, \quad \tilde{v}_0 = b^{\frac{1}{5}}v_0, \quad \tilde{p}_0 = b^{-\frac{1}{5}}p_0, \quad \tilde{\theta}_1 = b\theta_1, \quad \tilde{p}_1 = b^{\frac{4}{5}}p_1, \quad \tilde{r} = r, \quad \tilde{z} = b^{-\frac{1}{5}}z. \tag{2.10}$$

The problem then becomes, for u_0, v_0, p_1 and θ_1,

$$2u_0 + \frac{dv_0}{dz} = 0, \tag{2.11}$$

$$u_0^2 + v_0\frac{du_0}{dz} = -2p_1 + \frac{d^2u_0}{dz^2}, \tag{2.12}$$

$$\frac{dp_1}{dz} = \theta_1, \tag{2.13}$$

$$2u_0\theta_1 + v_0\frac{d\theta_1}{dz} = \frac{1}{\sigma}\frac{d^2\theta_1}{dz^2}, \tag{2.14}$$

with

$$\left.\begin{aligned} &u_0 = v_0 = 0, \ \theta_1 = -1 \quad \text{at } z = 0, \\ &u_0 \to \lambda, \ \theta_1 \to 0 \quad \text{as } z \to \infty, \end{aligned}\right\} \tag{2.15}$$

where $\lambda = b^{-\frac{2}{5}}\tilde{\lambda}$.

From the above equations, we are able to infer that the scaled pressure gradient p_1 may be determined as follows:

$$p_1 = -\frac{1}{2}\lambda^2 - \int_z^\infty \theta_1\, dz. \tag{2.16}$$

With $\theta_1(0) = -1$ and the reasonable assumption that $\theta_1 < 0$ throughout, an assumption justified *a posteriori*, (2.16) illustrates clearly the competition between the induced and imposed pressure gradients.

3. Results

First let us consider the possible consequences of the pressure gradient in (2.16) above. With $\lambda = 0$, and $\theta_1 < 0$ for all z, the induced pressure gradient is positive. In that case, the fluid flow will converge upon the stagnation point $r = 0$, leading to an eruption of fluid from the stagnation point; as a consequence, as in the two-dimensional case (**4**), no steady solution is available. However, clearly, with λ sufficiently large, the pressure gradient will be sufficiently large, and negative, throughout the bulk of the boundary layer, to result in a net, steady, radial flow. There will be a critical value, λ_c say, below which the steady flow cannot be maintained. Consider next the case $\lambda < 0$ such that the outer flow is converging upon the stagnation point. In the absence of any induced pressure gradient, that is with $b \equiv 0$ in (2.8), we know (**7**) that no steady solution exists. If we take $b < 0$ so that there is an induced flow radially outwards we may reasonably suppose that, again, a steady flow may be attained, but this proves not to be the case. To integrate the system of (2.11)–(2.14) with the boundary conditions (2.15), we have both developed a second-order accurate fully implicit finite difference code and employed the Matlab routine *bvp4c* as a check. In both cases, the domain $[0, z_\infty]$ has to be prescribed, but the latter method has the advantage that the step length is automatically adjusted according to the local rate of change of the developing solution.

In applying the outer boundary condition for u_0 in (2.15), we may set the prescribed value of λ at $z = z_\infty$ or, alternatively, set $du_0/dz = 0$ there to achieve the same solution. As we have already remarked, for $b > 0$, solutions exist for all λ greater than some critical value λ_c. For $\lambda < \lambda_c$, our methods fail to yield a solution. In Fig. 1, we represent velocity and pressure profiles for different values of the external flow velocity λ. As λ_c is approached the pressure gradient becomes unfavourable at the wall leading to a small region of reversed flow near the boundary. In Fig. 2,

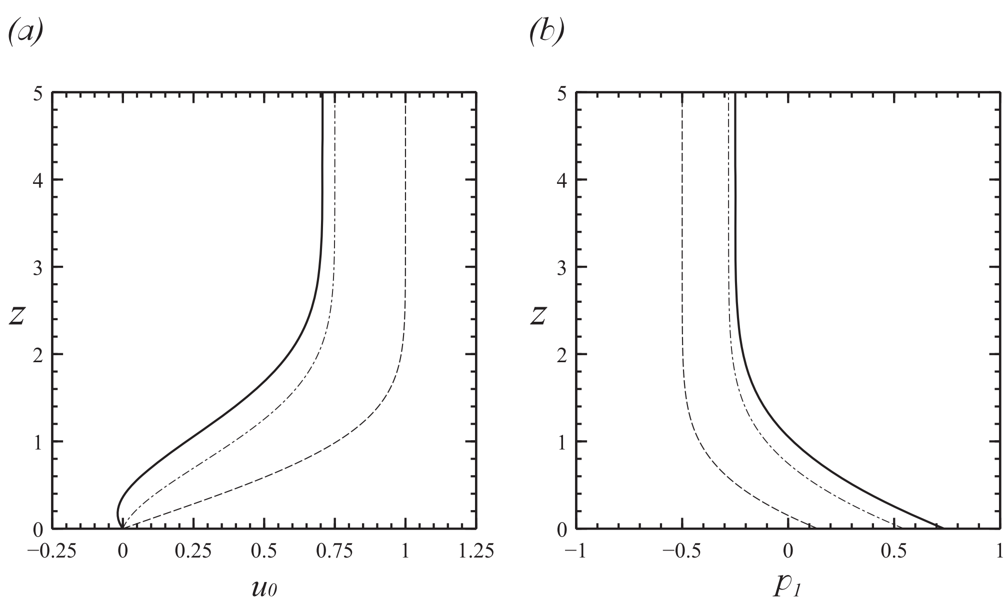

Fig. 1 (a) Profiles of the velocity component u_0 and (b) p_1 for values of $\sigma = 1$: (i) $\lambda = 1.0$ − − − −, (ii) $\lambda = 0.75$ · − · − ·−, (iii) $\lambda = \lambda_c$, ———

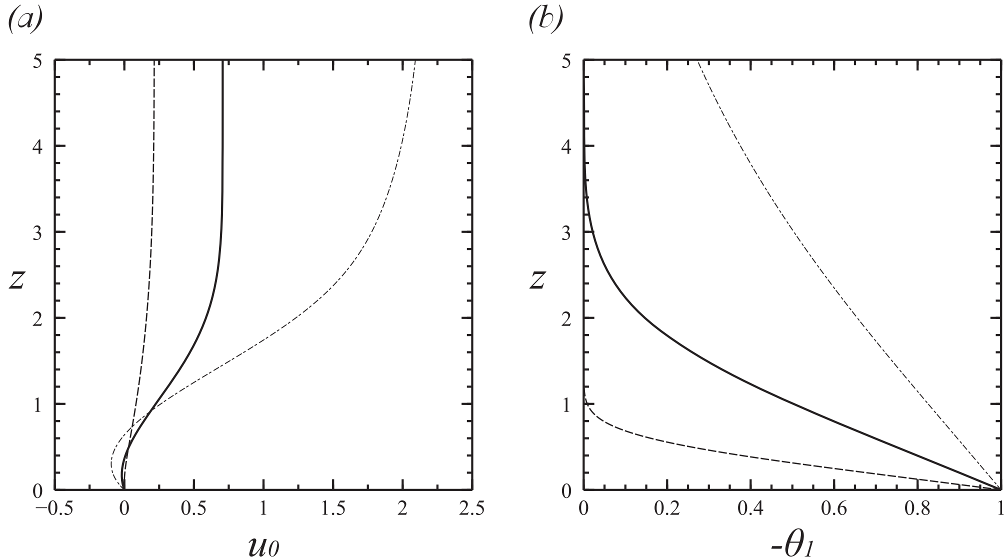

Fig. 2 (a) Profiles of the velocity component u_0 and (b) $-\theta_1$ for values of σ: (i) $\sigma = 0.01$ · — · — ·—, (ii) $\sigma = 1.0$ ———, $\sigma = 100.0$, — — —–. In each case, $\lambda = \lambda_c$

we show velocity and temperature profiles for representative values of the Prandtl number σ. We see, unsurprisingly, that as σ decreases, the thermal boundary layer thickness increases with the consequence, see (2.16), that the induced pressure gradient increases which leads in turn, as we see in Fig. 3(a), to an increase in λ_c. The induced pressure gradient at the boundary $p_{1i} = -\int_0^\infty \theta_1 \, dz$ is shown in Fig. 3(b). For the case when $b < 0$ in (2.8), following the same procedures leads to no steady solutions for $\lambda < 0$. With λ fixed at $z = z_\infty$, no converged solutions were obtained. Setting $du_0/dz = 0$ at z_∞ yielded a converged solution, but with a value $u_0(z_\infty) = \lambda' = -\lambda$. A further investigation of the unsteady analogue of our equations failed to approach a steady solution. This is at variance with the results of Amin and Riley (**4**) in the two-dimensional case, whose results we have been unable to reproduce. This anomaly may be resolved as follows. From (2.11), we see that as $z \to \infty$, $u_0 \to \lambda$ and $v_0 \sim -2\lambda z$. With these forms for u_0 and v_0 (2.14) show that

$$\theta_1 \sim C_1 z + C_2 z^{-2} \, e^{-\lambda z^2/2} \quad \text{as } z \to \infty, \tag{3.1}$$

and the difficulty associated with $\lambda < 0$ is apparent. A similar difficulty arises in (**4**).

We now supplement the numerical solutions of the governing equations by considering the asymptotic limits $\sigma \to 0$, $\sigma \to \infty$, respectively.

3.1 *The case $\sigma \ll 1$*

For the case of classical free convection from a semi-infinite flat plate, Kuiken (**8**) has shown that the flow regime divides into two parts. The same is true for the situation under consideration here. From (2.14) we have, as a leading order solution that satisfies the condition at $z = 0$,

$$\theta_1 = -1 + \alpha z, \tag{3.2}$$

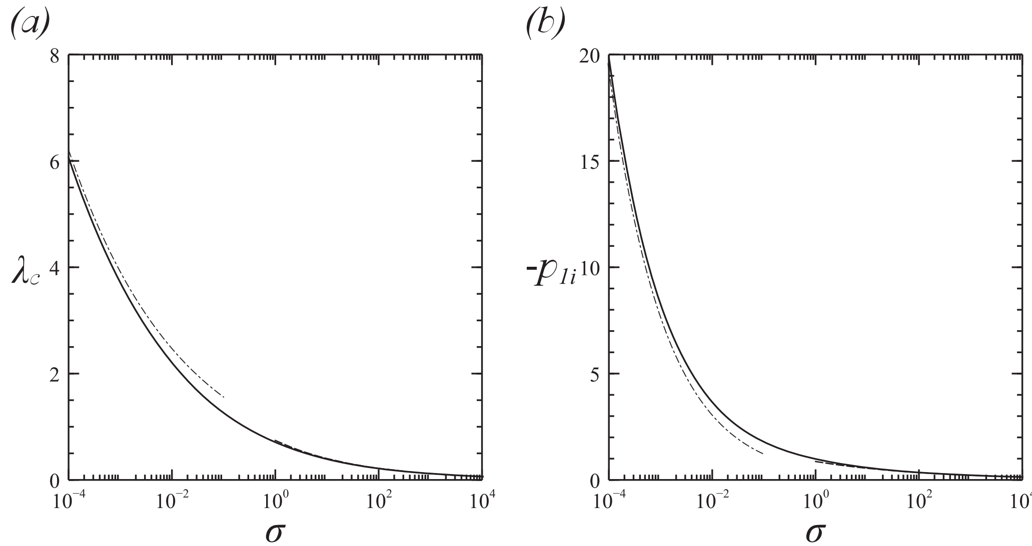

Fig. 3 (a) The critical value of $\lambda = \lambda_c$: (i) from the numerical solution of (2.11)–(2.15) ———, (ii) the asymptotic result $\lambda_c \sim 0.9833\sigma^{-1/5}$ as $\sigma \to 0$, $\cdot - \cdot - \cdot -$, (iii) the asymptotic results $\lambda_c \sim 0.7427\sigma^{-4/15}$ as $\sigma \to \infty$ $- - - -$ (b) The induced pressure gradient at the boundary $p_{1i} = -\int_0^\infty \theta_1\, dz$: (i) from the numerical solution of (2.11)–(2.15), ———, (ii) the asymptotic result $p_{1i} \sim 0.4827\sigma^{-2/5}$ as $\sigma \to 0 \cdot - \cdot - \cdot -$, (iii) the asymptotic result $p_{1i} \sim 0.866\sigma^{-1/5}$ as $\sigma \to \infty$, $- - - -$

where α may be expected to be negligibly small, consistent with the calculations carried out for small σ. This in turn leads to, from (2.13),

$$p_1 = -z - \beta, \tag{3.3}$$

where β is unknown. For the velocity components u_0, v_0 we then have

$$2u_0 + \frac{dv_0}{dz} = 0, \quad u_0^2 + v_0\frac{du_0}{dz} = 2z + 2\beta + \frac{d^2u_0}{dz^2}. \tag{3.4}$$

At the boundary, we require $u_0 = v_0 = 0$, but clearly there is no solution of (3.4) that satisfies the condition $u_0 \to \lambda$ as $z \to \infty$. Indeed, as $z \to \infty$ we find

$$u_0 \sim 6^{\frac{1}{2}} z^{\frac{1}{2}} + \left(\frac{3}{2}\right)^{\frac{1}{2}} \beta z^{-\frac{1}{2}} + \cdots, \quad v_0 \sim -4\left(\frac{2}{3}\right)^{\frac{1}{2}} z^{\frac{3}{2}} - 4\left(\frac{3}{2}\right)^{\frac{1}{2}} \beta z^{\frac{1}{2}} + \cdots. \tag{3.5}$$

This solution is therefore only valid in an inner region. In an outer region, heat diffusion and convection must be comparable, so that all terms in (2.14) are comparable in order of magnitude. This leads to the scaling

$$u_0 = \sigma^{-\frac{1}{5}} U_0, \quad v_0 = \sigma^{-\frac{3}{5}} V_0, \quad p_1 = \sigma^{-\frac{2}{5}} P_1, \quad z = \sigma^{-\frac{2}{5}} \zeta. \tag{3.6}$$

Of course, θ_1 remains $O(1)$, and our (2.11)–(2.14) become

$$2U_0 + \frac{dV_0}{d\zeta} = 0, \tag{3.7}$$

$$U_0^2 + V_0\frac{dU_0}{d\zeta} = -2P_1, \tag{3.8}$$

$$\frac{dP_1}{d\zeta} = \theta_1, \tag{3.9}$$

$$2U_0\theta_1 + V_0\frac{d\theta_1}{d\zeta} = \frac{d^2\theta_1}{d\zeta^2}. \tag{3.10}$$

From (3.5), the matching condition between the outer and inner solutions requires that, together with $\theta_1 = -1$,

$$U_0 \sim 6^{\frac{1}{2}}\zeta^{\frac{1}{2}}, \quad V_0 \sim -4\left(\frac{2}{3}\right)^{\frac{1}{2}}\zeta^{\frac{3}{2}} \quad \text{as } \zeta \to 0. \tag{3.11}$$

In addition, we require that

$$U_0 \to \Lambda, \quad \theta_1 \to 0 \quad \text{as } \zeta \to \infty, \tag{3.12}$$

where we have written $\lambda = \sigma^{-\frac{1}{5}}\Lambda$. Now, it is not possible to obtain a solution of the inviscid (3.7) and (3.8) consistent with the matching conditions and the condition at infinity for a prescribed value of Λ. However, treating Λ as a free parameter, it is possible to find a unique value of it such that, with $V_0(0) = 0$, $\zeta^{-\frac{1}{2}}U_0 \to 6^{\frac{1}{2}}$ as $\zeta \to 0$. That value, which corresponds to our critical value, is $\Lambda_c = 0.9833$ from which we deduce that $\lambda_c \sim 0.9833\sigma^{-\frac{1}{5}}$ as $\sigma \to 0$. This asymptotic expression is included in Fig. 3(a).

We may note that the constant β is not determined at leading order. The matching condition shows, from (3.5), that a perturbation of relative order $\sigma^{\frac{2}{5}}$ in the outer solution introduces the constant β and also indicates that the constant α in (3.2) is $O(\sigma^{\frac{2}{5}})$ which is consistent with our solutions of the full equations.

We have already noted from our calculations that as σ decreases, with the concomitant increase in the thermal layer thickness, the induced pressure gradient at the boundary increases. From our analysis for $\sigma \ll 1$, we see that this asymptotes as $p_{1i} \sim -0.4827\sigma^{-\frac{2}{5}}$, a result that is included in Fig. 3(b).

3.2 *The case $\sigma \gg 1$*

We turn now to the other limiting case of σ large compared to unity. Figures 2(a, b) show that as σ increases then, not unexpectedly, the thermal boundary layer diminishes in thickness, and within it velocities and the induced pressure gradient also diminish. The scale of this thin inner region is determined by ensuring that all the terms in the energy equation (2.14) are comparable in order of magnitude. This requires the following scaling:

$$u_0 = \sigma^{-\frac{3}{5}}U_0, \quad v_0 = \sigma^{-\frac{4}{5}}V_0, \quad p_1 = \sigma^{-\frac{1}{5}}P_1, \quad z = \sigma^{-\frac{1}{5}}\zeta, \tag{3.13}$$

so that we now have in this inner region,

$$2U_0 + \frac{dV_0}{d\zeta} = 0, \tag{3.14}$$

$$0 = -2P_1 + \frac{d^2U_0}{d\zeta^2}, \tag{3.15}$$

$$\frac{dP_1}{d\zeta} = \theta_1, \tag{3.16}$$

$$2U_0\theta_1 + V_0\frac{d\theta_1}{d\zeta} = \frac{d^2\theta_1}{d\zeta^2}. \tag{3.17}$$

Boundary conditions require

$$\left.\begin{aligned} &U_0(0) = V_0(0) = 0, \quad \theta_1(0) = -1, \\ &\theta_1 \to 0 \quad \text{as } \zeta \to \infty. \end{aligned}\right\} \tag{3.18}$$

From the velocity profiles in Fig. 2(a), we see that $du_0/dz < 0$ at $z = 0$ and decreasing in magnitude as σ increases. In the numerical solutions of the above equations, it proves convenient to specify $dU_0/d\zeta = \Lambda$, say, at large values of ζ. In this way, we find a critical value of $\Lambda = \Lambda_c = 0.8397$ below which there are no solutions with $dU_0/d\zeta = -0.25453$ at $\zeta = 0$.

Beyond this region of thickness $O\left(\sigma^{-\frac{1}{5}}\right)$, $\theta_1 = 0$ and there must be a balance of the convective and diffusive terms in the momentum equation. To achieve this, we write

$$u_0 = \sigma^{-2\gamma}U_1, \quad v_0 = \sigma^{-\gamma}V_1, \quad p_1 = -\frac{1}{2}\sigma^{-4\gamma}\tilde{\lambda}^2, \quad z = \sigma^{\gamma}\xi, \tag{3.19}$$

so that, from (2.11) and (2.12),

$$2U_1 + \frac{dV_1}{d\xi} = 0, \quad U_1^2 + V_1\frac{dU_1}{d\xi} = \tilde{\lambda}^2 + \frac{d^2U_1}{d\xi^2}. \tag{3.20}$$

To determine the exponent γ, it is necessary to match the solution in this region with that of the inner region. This is done most easily by matching the velocity gradients to give

$$\sigma^{-\frac{2}{5}}\Lambda = \sigma^{-3\gamma}\left.\frac{dU_1}{d\xi}\right|_{\xi=0}$$

or $\gamma = \frac{2}{15}$. In the numerical solution of (3.20), there will be a critical value of $\tilde{\lambda} = \tilde{\lambda}_c$ that corresponds to the gradient Λ_c. Thus, $\tilde{\lambda}_c = 0.7427$ which in turn corresponds to $\lambda_c = 0.7427\sigma^{-\frac{4}{15}}$. This result is included in Fig. 3(a) and shows very good agreement with the numerical solutions of the full equations. For $\lambda > \lambda_c$, we can obtain solutions. These correspond to values of $\Lambda > \Lambda_c$ which in turn leads to values of $dU_0/d\zeta > 0$.

From this asymptotic solution for large σ, we find the induced pressure gradient at the boundary, for the critical value λ_c, $p_{1i} \sim -0.866\sigma^{-\frac{1}{5}}$, which is included in Fig. 3(b). As we have noted earlier the induced pressure gradient is influenced by the thickness of the thermal boundary layer which in this large Prandtl number case is very small, $O\left(\sigma^{-\frac{1}{5}}\right)$.

4. Conclusions

In this paper, we have studied the steady flow at a planar, horizontal, axisymmetric stagnation point. The classical stagnation-point flow is influenced by an induced pressure gradient that arises due to variations in the temperature of the boundary. In particular, we have considered radially quadratic variations in the wall temperature. If the temperature gradient is positive, then the stagnation-point flow is enhanced. However, if the temperature gradient is negative, then the induced pressure gradient is negative, and this may overwhelm the classical stagnation-point pressure gradient to the extent that no steady-state solution is available. Our numerical solutions for various values of the Prandtl number σ have been augmented by asymptotic solutions for large and small values of σ. It has not been possible to maintain steady flow at a rear stagnation point by introducing a wall temperature that increases radially.

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